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A B S T R A C T

In this paper we shall consider the relations between the scattering lengths and the effective radii in the S,P states, following from the dispersion relations for NN scattering. The estimations of the contribution of $N\bar{N}$ interaction to the dispersion relations for NN scattering were obtained by means of experimental data for np and pp scattering at low energies. This contribution is not large. Its value depends on a sign of scattering lengths in the S - states.

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Introduction

The analysis of dispersion relations for NN scattering and their application for the analysis of experimental data is difficult because of the presence of the contribution of antinucleon-nucleon interaction. It concerns both the energy dependence of the total cross sections of $N\bar{N}$ interactions and especially, the unobservable region which remains even in the forward scattering.

One may hope that the influence of nucleon- antinucleon interaotion does not play a great part in the region of low energies. Under this assumption one may try to obtain[I,2] the approximate dispersion relations for NN scattering. They do not oontain an unobservable region.

To olear up the role of nucleon- antinucleon interaction is interesting both for obtaining the approximate relations and from the general point of view. In the present paper an attempt is made to estimate the contribution of $N\bar{N}$ interaction to the NN scattering at low energies.

The approach applied earlier[3] in the analysis of the dispersion relations for scattering and basing on "effective range theory"[4] is used for this purpose.

The relations between the scattering lengths and effective radii in different states were obtained for πN scattering in [3]. They made a direct check possible because the values determined directly from the experiment entered into the final relations. It was shown that the result of the analysis depends strongly on the small phase shifts.

Analogous relations for NN scattering include the unknown characteristies of the $N\bar{N}$ interaction. The use of the data on NN scattering at low energies (and the energy dependence of total cross sections) makes it possible to estimate the contribution of $N\bar{N}$ interaction to the dispersion relation for NN scattering.

We shall consider the dispersion relation for NN forward scattering. Having written the NN scattering amplitude in the form:

$$M=\frac{m}{\omega_6}\left\{\alpha+\beta(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})+i\gamma(\vec{\sigma}_1+\vec{\sigma}_2,\vec{n})+\delta(\vec{\sigma}_1\vec{m})(\vec{\sigma}_2\vec{m})+\varepsilon(\vec{\sigma}_1\vec{\ell})(\vec{\sigma}_2\vec{\ell})\right\}, \tag{I}$$

where $\vec{\ell}, \vec{m}, \vec{n}$ – unit vectors in the directions $\vec{k}_\ell \pm \vec{k}_\ell'$ and $\vec{k}_\ell \times \vec{k}_\ell'$, accordingly, and $\omega_\ell^2 = m^2 + k_\ell^2$ – one particle energy in the centre of mass system, we shall take only the dispersion relations for the quantity:

$$\alpha(\omega) = \frac{\omega_\ell}{m} \, \frac{1}{4} \, S_p M(0^\circ)$$

The dispersion relations for NN scattering were considered by different authors[5,6,7,8]. They were discussed in more detail by Goldberger, Nambu, and Oehme[8]. A number of interesting considerations was given by Ioffe[6].

=Neutron – Proton Scattering

The forward dispersion relation for the quantity $\alpha_{np}(\omega)$ may be written in the form:

$$\mathrm{Re}\left[\alpha_{np}(\omega) - \frac{1}{2}\left(1+\frac{\omega}{m}\right)\alpha_{np}(m) - \frac{1}{2}\left(1-\frac{\omega}{m}\right)\alpha_{n\tilde{p}}(m)\right] =$$

$$= (\omega^2 - m^2)\left\{\frac{f^2}{4\pi}\cdot\frac{1}{2}\,\frac{1}{(\omega_\mu+\omega)(\omega_\mu^2 - m^2)} + \Gamma_\alpha(0)\left(\frac{\omega_d + m}{2m}\right)\frac{1}{(\omega_d - \omega)(\omega_d^2 - m^2)} + \right. \tag{2}$$

$$\left. + \frac{P}{8\pi^2}\int_m^\infty \frac{d\omega'}{K'}\left[\frac{\sigma_{np}(\omega')}{\omega' - \omega} + \frac{\sigma_{n\tilde{p}}(\omega')}{\omega' + \omega}\right] + \frac{1}{\pi}\int_{\omega(2\mu)}^{m} d\omega' \, \frac{\mathrm{Im}\,\alpha_{np}(\omega')}{(\omega+\omega')(\omega'^2 - m^2)}\right\}$$

where w – total nucleon energy in the laboratory system.

m, μ – nucleon and pion masses, $\omega_\mu = \dfrac{\mu^2}{2m} - m$

$B = \dfrac{\varkappa^2}{m}$ – deuteron binding energy, $\omega_d = m - 2B + B^2/2m$

$\Gamma_\alpha(0) = 3\dfrac{\varkappa}{m} \cdot 1/_{1-\varkappa r_{ot}}$

The effective radius r_{ot} was determined by the relation (4.40) from /8/, the notations used here are, in general, the same. The term proportional to the integral from $\omega(2\mu)$ to m (the unobservable region) and those including $\alpha_{np}(m)$ and $\sigma_{np}(\omega')$ will be considered as the contribution of nucleon- antinucleon interaction in (2).

Taking an interest in the values $\mathrm{Re}\,\alpha_{np}(\omega)$ at small $\eta^2 = (\frac{k}{m})^2$ we shall present the dependence of the phase shifts on the energy in form[4]

$$\eta_\ell^{2L+1}\,\mathrm{ctg}\,\delta(^{2S+1}L_J) = \frac{1}{a_L} + \frac{1}{2}r_o^{(L)}\eta_\ell^2 + Q_L\eta_\ell^4 \equiv A(^{2S+1}L_J), \tag{3}$$

where $a_L \equiv a(^{2S+1}L_J)$ scattering length (at $\eta \to 0$) in state with total momentum J, parity $(-1)^L$ and spin S ; $k_\ell = \eta_\ell \cdot m$ –nucleon momentum in the centre of mass system, and

$$\frac{k}{k_\ell} = \frac{\eta}{\eta_\ell} = \left(2 + 2\frac{\omega}{m}\right)^{1/2} \cong 2 \tag{4}$$

The scattering lengths in $^3S_1$ and $^1S_0$ states are sometimes denoted as $a_t \equiv a(^3S_1)$, $a_s \equiv a(^1S_0)$. In accordance with the determination (3) $a_t < 0$ and $a_s > 0$. The quantities in the centre of mass are marked by the sign ℓ . Using the expression for α_{np} in terms of phase shifts it is possible to obtain:

$$\alpha_{np}^{(\ell)}(\omega) = \frac{\omega_\ell}{m}\cdot\frac{1}{4k_\ell}\cdot\sum_{J=0}^{\infty}(2J+1)e^{i\delta_J}\sin\delta_J = \frac{\hbar}{mc}\cdot\frac{\omega_\ell}{m}\,\frac{1}{4\eta_\ell}\cdot\sum_{J,\ell,S}(2J+1)e^{i\delta_J}\sin\delta_J \tag{5}$$

where. δ_J - phase shifts in the states with the total momentum J . (Mixing coefficients fall out of[5])

As :

$$\sigma_{np} = \frac{4\pi}{K_\ell} \cdot \frac{m}{\omega_\ell} \cdot Jm\, \alpha'^{(\ell)}_{np}(\omega_\ell) = \frac{8\pi}{\sqrt{\omega^2 - m^2}}\, Jm\, \alpha_{np}(\omega) \tag{6}$$

in the laboratory system $\alpha_{np}(\omega)$ is expressed through $\alpha^{(\ell)}_{np}(\omega_\ell)$ by the relation : $(\lambda_c = \frac{\hbar}{mc} = 2{,}1 \cdot 10^{-14} cm)$

$$\alpha_{np}(\omega) = \frac{1}{2}\frac{K}{K_\ell}\cdot\frac{m}{\omega_\ell}\cdot\alpha^{(\ell)}_{np}(\omega_\ell) = \left(\frac{1}{2}+\frac{1}{2}\frac{\omega}{m}\right)^{1/2}\frac{\lambda_c}{4\eta_\ell}\sum_{J,\ell,S}(2J+1)e^{i\delta_J}\sin\delta_J \tag{7}$$

From (3) and (7)

$$Re\,\alpha_{np}(m) \equiv D_{np}(m) = \frac{\lambda_c}{4}(3a_t + a_s), \tag{8}$$

and the expression for $D_{np}(m)$

$$D_{n\bar{p}}(m) = \frac{\lambda_c}{4}\left\{3a_{n\bar{p}}(^3S_1)\exp[-2\beta_{n\bar{p}}(^3S_1)] + a_{n\bar{p}}(^1S_0)\exp[-2\beta_{n\bar{p}}(^1S_0)]\right\} \tag{9}$$

takes into account an inelastic process-annihilation by means of $\beta_{np}(^3S_1)$ and $\beta_{np}(^1S_0)$ which are imaginary parts of the phase shifts in the corresponding states of $n\bar{p}$ system.

2. As in paper/3/ we shall consider the relation, following from (2) if it is differentiated over η^2 and putting that $\eta^2 = 0$. Taking into consideration the available experimental data we shall restrict ourselves by one differentiation. Denoting a derivative over η^2 of $D_{np}(\omega)$ through $D'_{np}(\omega)$, and its value at w = m through $D'_{np}(m)$ and taking into account that all lengths are expressed through λ_c we shall obtain:

$$D'_{np}(m) + \frac{1}{4}\left[D_{n\bar{p}}(m) - D_{np}(m)\right] = \lambda_c\left\{-\frac{f^2}{4\pi}\left(\frac{m}{\mu}\right)^4 + \frac{3}{8}\cdot\left(\frac{m}{B}\right)^{3/2}\frac{1}{1-\varkappa r_{0t}} + \right. \tag{10}$$

$$\left. + \frac{1}{8\pi^2}P\int_1^\infty\frac{d\omega'}{\eta'}\cdot\left[\frac{\sigma_{np}(\omega')}{\omega'-1} + \frac{\sigma_{n\bar{p}}(\omega')}{\omega'+1}\right] + \frac{1}{\pi}\int_{\omega(2\mu)}^1 d\omega'\frac{Jm\,\alpha_{np}(\omega')}{(\omega'+1)(\omega'^2-1)}\right\}$$

The contribution of the deuteron state in (2) is calculated within the accuracy of the terms of order $\frac{B}{m}$, therefore, in (10) the terms of order $\frac{B}{m}$ if compared with those written out are omitted in the second term on the right.

The terms of order $\left(\frac{\mu}{2m}\right)^2$ are omitted in the term proportional to f^2.

For $D'_{np}(m)$ from (3) and (7) we have:

$$D'_{np}(m) = \frac{1}{8}D_{np}(m) - \frac{1}{16}\left\{a^2(^1S_0)\left[a(^1S_0) + \frac{1}{2}r(^1S_0)\right] + 3a^2(^3S_1)\left[a(^3S_1) + \right.\right. \tag{11}$$

$$\left.\left. + \frac{1}{2}r(^3S_1)\right] - a(^3P_0) - 3\left[a(^1P_1) + a(^3P_1)\right] - 5a(^3P_2)\right\}$$

According to the experimental data/9,10/

$$a(^3S_1) = -(0{,}537 \pm 0{,}004)10^{-12}cm = -(25{,}6 \pm 0{,}19)\lambda_c$$
$$a(^1S_0) = (2{,}373 \pm 0{,}007)\cdot 10^{-12}cm = (113 \pm 0{,}33)\lambda_c \qquad (I2)$$
$$r(^1S_0) \equiv r_{0s} = 2{,}7\cdot 10^{-13}cm = 12{,}84\,\lambda_c$$
$$r(^3S_1) \equiv r_{0t} = 1{,}7\cdot 10^{-13}cm = 8{,}1\,\lambda_c$$

from where

$$D_{np}(m) = (0{,}190 \pm 0{,}0025)\,10^{-12}cm = 9{,}05\,\lambda_c \qquad (I2')$$
$$D'_{np}(m) = -9{,}20\cdot 10^{9}\lambda_c$$

The $^1S_0$ state gives the main contribution when calculating $D'_{np}(m)$. Triplet S-scattering gives a contribution not exceeding 5% of that of a singlet one while an inaccuracy in the value a_s results in an inaccuracy in the determination of $D'_{np}(m)$ about I%. The contribution of scattering in p-states turns out to be smaller. If we make use of the predictions of Gammel, Christian, and Thaler's potential/II/, for a rough orientation, the p-state contribution does not exceed a value determined by an inaccuracy in the calculation of the deuteron term and will be comparable with $D_{np}(m)$. The estimation based on the Signell and Marshak's potential/I2/ gives :

$$a(^1P_1) = -146{,}4\,\lambda_c\,,\ a(^3P_0) = 57{,}2\,\lambda_c\,,\ a(^3P_1) = -46{,}5\,\lambda_c\,,\ a(^3P_2) = 27{,}8\,\lambda_c$$

The estimation of different terms in (IO) shows that the numerical contribution of deuteron state turns out to be more essential than the onemeson term which played an important part for πN scattering. In practice the presence of $D_{np}(m)$ (in IO) in the considered low energy region does not influence the result as one can see from (II) and (I2). The quantity $D_{np}(m)$ does not exceed errors when calculating the deuteron state contribution. The smallness of this term confirms the supposition about a small role of "subtraction" in the nonrelativistic region. Besides, this fact makes it possible to suppose that the contribution $D_{np}(m)$ is not large. For the time being it is not possible to estimate it directly.

The contribution of the deuteron (in the righthand side at B =2,2 MeV, $1-\varkappa r_{0t}=0{,}608$) is +(5450 ± I3). The contribution of the one meson state - I62 at $f^2/4\pi = 0{,}08$ and - I84 at $f^2/4\pi = 0{,}09$.

The main difference between (II), (I2) and analogous formulas in paper/3/ consists in quite a different role of the S and P-states in np and πN scattering. The presence of "resonance" πN interaction in the P-state leads to the following: $D_{\pi N}(\omega)$ is an increasing function at low energies, while the "resonance" nucleon interaction in S state makes $D_{np}(\omega)$ decreasing at low energies. A sign of $a(^1S_0)$ is essential here.

3. The calculation of a dispersion integral

$$J_{np}(m) = \frac{1}{8\pi^2}\,P\int_1^\infty \frac{d\omega'\cdot\sigma_{np}(\omega')}{\sqrt{\omega'^2-1}\,(\omega'-1)} = \frac{F_{np}(m)}{8\pi^2} \qquad (I3)$$

leads to the value :

$$J_{np}(m) = -\frac{7{,}70\cdot 10^6}{8\pi^2} = -9{,}65\cdot 10^4 \qquad (14)$$

The integral $F_{np}(m)$ is considered as a limit :

$$F_{np}(m) = \lim_{\omega_0\to 1} P\int_1^\infty \frac{\sigma_{np}(\omega')d\omega'}{(\omega'-\omega_0)(\omega'^2-1)^{1/2}} = \lim_{\omega_0\to 1}\left[F_s(\omega_0)+3F_t(\omega_0)\right] \qquad (15)$$

All integration interval is divided into sections. On each section the function $\sigma_{np}(\omega')$ is approximated by a simple expression. The Smorodinsky's formula (13) is used in the region of kinetic energies up to 20 MeV.

$$\sigma_{np}(E_0) = 1{,}3\cdot 10^{-24}\left\{\frac{3}{(1{,}22-0{,}06E_0)^2+\frac{E_0}{2}} + \frac{1}{(0{,}27+0{,}06E_0)^2+\frac{E_0}{2}}\right\} cm^2 \qquad (16)$$

(E_0 - kinetic energy of neutron in the laboratory system in MeV). In other region more rough approximation is used. The roughness of the approximation $\sigma_{np}(\omega)$ at high energies does not introduce any appreciable error, because the region $\omega'\sim\omega_0$ play the principal part when calculating (15). When the dependence of σ_{np} on energy is given by (16) the contribution of this region is :

$$F_1(m) = \lim_{\omega_0\to 1} F_1(\omega_0) = \lim_{\omega_0\to 1}\left\{3F_{1t}(\omega_0)+F_{1s}(\omega_0)\right\},$$

where, for example,

$$F_{1s}(\omega_0) = P\int_1^{1+\frac{\varkappa^2}{2}} \frac{d\omega'}{\eta'}\cdot\frac{\sigma_{np}^{(s)}(\omega')}{\omega'-\omega} = \pi a^2({}^1S_0)\left\{-\frac{1}{K_0}\ln\left|\frac{K_0+\varkappa}{K_0-\varkappa}\right| + \right. \qquad (17)$$

$$\left. + \left(\frac{K_0^2+K_{2s}^2}{K_{1s}^2-K_{2s}^2}\right)\frac{2}{K_{1s}}\,\mathrm{arctg}\,\frac{\varkappa}{K_{1s}} - \left(\frac{K_0^2+K_{1s}^2}{K_{1s}^2-K_{2s}^2}\right)\frac{2}{K_{2s}}\,\mathrm{arctg}\,\frac{\varkappa}{K_{2s}}\right\}$$

and analogous expression for triplet state contribution. In (17) K_{1s}^2 and K_{2s}^2 are roots of the equation :

$$\left(\frac{0{,}27}{\sqrt{m}} + 0{,}06\sqrt{m}\,\frac{K_s^2}{2}\right)^2 + \frac{K_s^2}{4} = 0 \qquad (K_1^2 > K_2^2)$$

By means of (17) and (15) we obtain directly :

$$F_{np}^{(1)}(m) = -\frac{2}{\varkappa}\sigma_{np}(m) - 6\pi a_t^2\left\{\frac{K_{1t}^2\,\mathrm{arctg}\frac{\varkappa}{K_{2t}}}{K_{2t}(K_{1t}^2-K_{2t}^2)} - \frac{K_{2t}^2\,\mathrm{arctg}\frac{\varkappa}{K_{1t}}}{K_{1t}(K_{1t}^2-K_{2t}^2)}\right\} - \qquad (18)$$

$$-2\pi a_s^2\left\{\frac{K_{1s}^2\,\mathrm{arctg}\frac{\varkappa}{K_{2s}}}{K_{2s}(K_{1s}^2-K_{2s}^2)} - \frac{K_{2s}^2\,\mathrm{arctg}\frac{\varkappa}{K_{1s}}}{K_{1s}(K_{1s}^2-K_{2s}^2)}\right\} = -7{,}70\cdot 10^6$$

In (18) K_{1t}^2 and K_{2t}^2 are roots of the equation :

$$\left(\frac{1{,}22}{\sqrt{m}} - 0{,}06\sqrt{m}\,\frac{K_t^2}{2}\right)^2 + \frac{K_t^2}{4} = 0$$

The contribution of the whole region of energies exceeding 20 MeV is no more than 0,3 % of the value in (18). In an auxiliary region an interval of the energies > 100 MeV gives about 12% of the contribution of this region.

4. Collecting results, carrying known quantities on one side (in 10), we obtain

for a contribution of the nucleon-antinucleon interaction :

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\frac{\sigma_{n\bar{p}}(\omega')}{\omega'+1} + \frac{1}{\pi}\int_{\omega(2\mu)}^1 d\omega' \frac{Jm\,\alpha_{n\bar{p}}(\omega')}{(\omega'+1)(\omega'^2-1)} - \frac{1}{4}D_{n\bar{p}}(m) = -1000 \tag{I9}$$

Thus, a direct comparison of dispersion relations with experimental data on np scattering at low energies shows, that the contribution of nucleon-antinucleon interaction to the nucleon scattering in this region is small. Hence it appears, in particular, that for this region of energies it is possible without a large error to replace the exact dispersion relation (2) by an approximate relation

$$D_{np}(\omega) - D_{np}(m) = (\omega^2 - m^2)\left\{\left(\frac{\omega d + m}{2m}\right)^{1/2}\frac{\Gamma_\alpha(0)}{(\omega d-\omega)(\omega^2 d - m^2)} + \frac{1}{8\pi^2}P\int_1^\infty \frac{d\omega'}{\eta'}\cdot\frac{\sigma_{np}(\omega')}{\omega'-\omega}\right\} \tag{20}$$

A relative role of the unobservable region decreases with the increase of energy. To obtain data on its contribution at different energies one can employ the relation (76) of paper /8/.

Proton - Proton Scattering

The amplitude α_{np} can be expressed in terms of amplitudes of NN scattering in states with certain values of isotopic spin α_0 and α_1 by the relation :

$$2\alpha_{np} = \alpha_0 + \alpha_1 = \alpha_0 + \alpha_{pp} \tag{2I}$$

As a result we shall obtain a relation for pp scattering instead of (I0). It does not include a deuteron state contribution and the numerical coefficient before f^2 is changed.

An unobservable region includes also states of several mesons, with T = I.

Instead of (8) and (II) we have

$$D_{pp}(m) = \frac{1}{2}a(^1S_0) = 56{,}5\,\lambda_c \tag{22}$$

$$D'_{pp}(m) = \frac{1}{8}\left\{a_s - a_s^2(a_s + \frac{1}{2}r_{0s}) + a(^3P_0) + 3a(^3P_1) + 5a(^3P_2)\right\} \cong -19{,}00\cdot 10^4\lambda_c$$

Employing an isotopic invariance and calculating only the contribution of the region up to 20 MeV for a corresponding integral by means of Smorodinsky's formula, we have :

$$J_{pp}(m) \cong -\frac{a_s^2}{2\pi}\left\{\frac{1}{\varkappa} + \frac{\operatorname{arctg}\frac{\varkappa}{K_{2s}}}{K_{2s}}\right\} = -18{,}75\cdot 10^4 \tag{23}$$

The relation (I9) is replaced by :

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\cdot\frac{\sigma_{p\bar{p}}(\omega')}{\omega'+1} + \frac{1}{\pi}\int_{\omega(2\mu)}^1 d\omega' \frac{Jm\,\alpha_{p\bar{p}}(\omega')}{(\omega'+1)(\omega'^2-1)} - \frac{1}{4}D_{p\bar{p}}(m) = -2500, \tag{24}$$

what points out one scale of the corresponding quantities in pp and np scattering. In the analysis made this was the consequence of a fundamental role of the singlet scattering in (I9) and (23). Thus, the approximate dispersion relation of type (20) in which $\Gamma_\alpha(0) = 0$ takes place also for pp scattering.

Nucleon Scattering in the T= 0 State

Using (2I) it is possible to obtain a dispersion relation for the scattering in the states with T = 0. It will have the form of (2) in the right-hand side of which one meson state contribution is absent and a deuteron contribution is doubled. The relation (IO) is transformed accordingly.

Instead of (8) we have :

$$D^{(0)}_{NN}(m) = \frac{3}{2}a(^3S_1) = -38{,}4\,\lambda_c, \tag{25}$$

instead of (II) :

$$D^{(0)'}_{NN}(m) = \frac{1}{4}D^{(0)}_{NN}(m) - \frac{3}{8}\left\{\frac{a_t}{2} + a_t^2\left(a_t + \frac{1}{2}r_{0t}\right) - a(^1P_1)\right\} \cong 5160\lambda_c \tag{26}$$

Here the relative role of the p-states is higher, certainly, than in the cases considered earlier. However, a role of $D(m)$ is not significant as before. If calculating a dispersion integral

$$J^{(0)}_{NN}(m) = \frac{1}{8\pi^2}P\int_1^\infty \frac{d\omega'}{\eta'}\cdot\frac{\sigma^{(0)}_{NN}(\omega')}{\omega'-1}$$

we shall limit ourselves to a contribution of region of kinetic energies, below 20 MeV, then

$$J^{(0)}_{NN}(m) = -\frac{3}{2}\cdot\frac{a_t^2}{\pi}\left\{\frac{1}{\varkappa} + \frac{k_{1t}^2\,\mathrm{arctg}\,\frac{\varkappa}{k_{2t}}}{k_{2t}(k_{1t}^2-k_{2t}^2)} - \frac{k_{2t}^2\,\mathrm{arctg}\,\frac{\varkappa}{k_{1t}}}{k_{1t}(k_{1t}^2-k_{2t}^2)}\right\} = -8{,}35a_t^2 = -5450$$

The deuteron state contribution is now IO,900 and by analogy with (I9) and (24) :

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\cdot\frac{\sigma^{(0)}_{N\bar N}(\omega')}{\omega'+1} + \frac{1}{\pi}\int_{\omega(2\mu)}^{1} d\omega'\,\frac{\mathrm{Im}\,\alpha^{(0)}_{N\bar N}(\omega')}{(\omega'+1)(\omega'^2-1)} - \frac{1}{4}D^{(0)}_{N\bar N}(m) = +300 \tag{27}$$

The approximate dispersion relation differs from (20) by its numerical factor before $\Gamma_\alpha(0)$.

D I S C U S S I O N

As a result of this paper we shall consider the estimations of type (I9), (24), (27) and the foundation of the approximate relations of type (20). The estimation (27) is less reliable, moreover all three relations show a magnitude of contribution of nucleon-antinucleon interaction to dispersion relation for NN scattering.

The existence of approximate relations (20) can be useful when analysing the experimental data on NN scattering in region of low energies, especially, when more detailed data on NN scattering in p-states will be obtained.

For the first time, the approximate dispersion relation for NN scattering was postulated in Blank and Isaev's paper /I/.*) In this paper it is founded by a direct comparison with experimental data. The main result of this paper - relations, type (IO) - can be employed to obtain data on contribution of unobservable region in the future, when data on antinucleon -nucleon interaction in a wide region of energies will be available.

It would be interesting to note that the conclusion about small contribution of $N\tilde{N}$ interaction is connected with a sign of a_s and a_t.

The dispersion relations for pion-nucleon scattering were used at one time by different authors to determine a positive value of $D_{\pi+p}$ and α_{33} below a resonance. In this paper when calculating the signs of NN scattering lengths a_s and a_t were taken from data on scattering of neutrons in parahydrogen and orthohydrogen (see, for ex. /IO/).

As $D'_{pp}(m)$ is determined completely and $D'_{np}(m)$ is determined by a singlet S -scattering, a change of sign of $a(^1S_0)$ would result in an important role of nucleon-antinucleon interaction both at low energies. It can be interesting to pay an attention to the fact that a positive value of $a(^3S_1)$ is determined, from point of view of dispersion relations, by the presence of real deuteron state, as this follows from "theory of effective range" (see, for ex./I4/).

This analysis may be interesting for a discussion of data on nucleon-antinucleon interaction at low energies. The dispersion relation for $N\tilde{N}$ scattering is obtained from (2), if we shall replace formally $\omega \longrightarrow -\omega$ and α_{np} by α_{np} in certain places. The role of one meson and deuteron states is decreased sharply but that of unobservable region is increased noticeably. In presence of necessary data the relation of type (IO) can be used to obtain an estimation of contribution of an unobservable region in this case.

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*) Paper /I/ includes an unnecessary section on the symmetrization of the scattering amplitude.

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