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Laboratory of Theoretical Physics

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ON A SUPERFLUID STATE OF THE ATOMIC NUCLEUS

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I. INTRODUCTION

It is known /I/ that weak interactions of electrons with the equal and opposite momenta near the Fermi surface lead to the metal superconductivity. The superfluid state is energetically lower than the state of a purely degenerate Fermi-gas which we shall call the normal one. Basing upon some similarity of the Fermi-systems of the nucleus and metal /2/ we shall consider the nucleon interactions in complex nuclei and investigate a possibility of appearing such state of the atomic nucleus which will be energetically lower than the normal state. We shall call this state the superfluid state of the atomic nucleus.

We intend to investigate the appearance of the superfluid state of the atomic nucleus using the Bogolubov variational principle /3/ which is the generalization of the well-known Fock method /4/ and by means of mathematical methods developed in the theory of superconductivity. /I/

Basing upon the nuclear shell model and having in view the medium and heavy nuclei we consider the weak interactions of protons /or neutrons/ situating on the same shell near the Fermi surface energy. We characterize the proton state by a set of the quantum numbers S determining the shell and by the quantum number m of the angular momentum projection along the symmetry axis of the nucleus. We assume the nuclear field to deviate somewhat from the central symmetrical one, therefore, there will be no energetic degeneration over m.

2. Ground Superfluid State.

Let us consider weak interactions of protons /or neutrons/ which are situated on the outer shell of the nucleus. The nucleons forming the closed internal shells create the central -symmetrical field which is somewhat distorted by the nucleons of the outer shell.

The Hamiltonian describing the interaction of protons situating on the same shell may be written as follows:

$$H = \sum_{s,m,\sigma} \{E(s,m) - E_F\} a^+_{m\sigma}(s)\, a_{m\sigma}(s) +$$

$$+\frac{1}{2N} \sum_{\substack{s,\sigma_1,\sigma_2,m_1,m_2,m_1',m_2' \\ m_1+m_2=m_1'+m_2';\ m_1\neq m_1'}} J(s|m_1,m_2;m_1',m_2')\, a^+_{m_1\sigma_1}(s)\, a^+_{m_2\sigma_2}(s)\, a_{m_2'\sigma_2}(s)\, a_{m_1'\sigma_1}(s), \qquad /I/$$

where N is the number of levels, $\sigma = \pm 1$ characterizes the spen direction, $a^+_{m\sigma}$, $a_{m\sigma}$ are the operators of proton production and absorption. $E(s,m) = E(s,-m)$ is the proton energy in the state s , m , E_F- is the parameter which plays the role of the chemical potential, in the normal state it equals the Fermi surface energy. The function $J(s|m_1,m_2;m_1',m_2')$ is the real one and possesses the following properties:

$$J(s|m_1,m_2;m_1',m_2') = J(s|m_1',m_2';m_1,m_2),$$

$$J(s|m_1,m_2;m_1',m_2') = J(s|-m_1,-m_2;-m_1',-m_2'). \qquad /2/$$

Let us perform the linear canonical transformation of the Fermi amplitudes

$$a_{\sigma m,\sigma}(s) = u_m(s)\, d_{m,-\sigma}(s) + \sigma v_m(s)\, d^+_{m\sigma}(s), \qquad /3/$$

in order that it would not violate their commutation properties it is necessary that the condition

$$\eta_m(s) = u_m(s)^2 + v_m(s)^2 - 1 = 0 \qquad /4/$$

would be fulfilled. Let us determine the vacuum state C_o

$$d_{m\sigma} C_o = 0 \qquad /5/$$

and find the mean value of H by the new vacuum state, and namely :

$$\overline{H} = \langle C_0^* H C_0 \rangle =$$

$$= 2 \sum_{s,m} \left\{ E(s,m) - E_F \right\} v_m(s)^2 +$$

$$+ \frac{1}{N} \sum_{\substack{s,m,m' \\ m \neq m'}} \left\{ J(s|m,-m;m',-m')\, u_m(s)\, v_m(s)\, u_{m'}(s)\, v_{m'}(s) - \right. \qquad /6/$$

$$\left. - J(s|m,m';m',m)\, v_m(s)^2\, v_{m'}(s)^2 \right\} \equiv \mathcal{E}(u,v)$$

Let us determine the functions $u_m(s), v_m(s)$ from the condition of the minimum of the form $\mathcal{E}(u,v)$ if there is auxiliary condition /4/. The corresponding equation is

$$\delta \widetilde{\mathcal{E}}(u,v) = \delta \left\{ \mathcal{E}(u,v) + \sum_{s,m} \lambda_m(s)\, \eta_m(s) \right\} = 0 \,, \qquad /7/$$

where $\lambda_m(s)$ is the Eiler factor, the variations $\delta u_m(s)$ and $\delta v_m(s)$ are considered as independent ones. As a result we obtain

$$\Xi_m(s)\, u_m(s)\, v_m(s) + \qquad /8/$$

$$+ \frac{u_m(s)^2 - v_m(s)^2}{2N} \sum_{m'} J(s|m,-m;m',-m')\, u_{m'}(s)\, v_{m'}(s) = 0$$

where

$$\Xi_m(s) = E(s,m) - E_F - \frac{1}{N} \sum_{m'} J(s|m,m';m',m)\, v_m(s)^2 . \qquad /9/$$

$\mathcal{E}(u,v)$ gives the energy of the ground state for $u_m(s), v_m(s)$ satisfying /8/. Equation /8/ admits the solution

$$u_m(s) = 1 - \theta_F(s,m) \,, \qquad /10/$$

$$v_m(s) = \theta_F(s,m)$$

which corresponds to the normal state. The function $\theta_F(s,m)=1$ if $E(s,m)<E_F$ and $\theta_F(s,m)=0$ if $E(s,m)\geq E_F$.

To solve /8/ we introduce the new function

$$C_m(s)=\frac{1}{N}\sum_{m'}J(s|m,-m;m',-m')\,u_{m'}(s)\,v_{m'}(s),$$

connected with $u_m(s)$ and $v_m(s)$ in the following way:

$$u_m(s)^2=\frac{1}{2}\left[1+\frac{\Xi_m(s)}{\mathcal{E}_m(s)}\right],\quad v_m(s)^2=\frac{1}{2}\left[1-\frac{\Xi_m(s)}{\mathcal{E}_m(s)}\right] \qquad /11/$$

$$u_m(s)\,v_m(s)=\frac{1}{2}\sqrt{1-\frac{\Xi_m(s)^2}{\mathcal{E}_m(s)^2}},\quad u_m(s)\,v_m(s)=-\frac{1}{2}\,\frac{C_m(s)}{\mathcal{E}_m(s)}$$

$$\mathcal{E}_m(s)=\sqrt{C_m(s)^2+\Xi_m(s)^2}\,. \qquad /12/$$

For $C_m(s)$ one may obtain the following equation

$$C_m(s)=-\frac{1}{N}\sum_{m'}J(s|m,-m;m',-m')\frac{C_{m'}(s)}{\sqrt{C_{m'}(s)^2+\Xi_{m'}(s)^2}}. \qquad /13/$$

Let us consider the interaction of protons situating on the outer shell characterized by $s=s_0$, the proton energy which is in the neighbourhood of the Fermi surface, i.e.,

$$E_F-\Delta_1\leqslant E(s_0,m)\leqslant E_F+\Delta_2$$

$$\Delta_1\ll E_F,\quad \Delta_2\ll E_F.$$

Let us now pass in /13/ from the sum to the integral and obtain

$$C_m(s_0)=-\frac{1}{2}\int_{m_1}^{m_2}dm'\,\rho_0(m')\,J(s_0|m,-m;m',-m')\frac{C_{m'}(s_0)}{\sqrt{C_{m'}(s_0)^2+\Xi_{m'}(s_0)^2}}, \qquad /13'/$$

where $\rho_0(m') = \frac{dn}{dm'}$ is the level density $E(s_0, m_1) = E_F - \Delta_1$, $E(s_0, m_0) = E_F$, $E(s_0, m_2) = E_F + \Delta_2$. To obtain the asymptotic form of the solution of /I3'/ at small J we pass to the approximate equation

$$C_m(s_0) = C_{m_0}(s_0) \cdot \ln \frac{C_{m_0}(s_0)}{\mu} \cdot \frac{\rho_0(m_0)}{\left\{ \frac{d\Xi_{m'}(s_0)}{dm'} \right\}_{m'=m_0}} J(s_0 | m, -m; m_0, -m_0) +$$

$$+ \frac{1}{2} \int_{m_1}^{m_2} dm' \ln \frac{|E(s_0, m') - E_F|}{\mu} \cdot \frac{d}{dm'} \left[J(s_0 | m, -m; m', -m') \, C_{m'}(s_0) \frac{\rho_0(m')}{\frac{d\Xi_{m'}(s_0)}{dm'}} \right], \qquad /I4/$$

which at small J coincides with /I3'/ asymptotically, here μ is the proton mass. The approximate solution of /I4/ at small $C_m(s_0)$ may be found in the following form

$$C_m(s_0) = \omega \frac{J(s_0 | m, -m; m_0', -m_0')}{J(s_0 | m_0, -m_0; m_0, -m_0)} e^{-\frac{1}{G}} \qquad /I5/$$

where

$$G = J(s_0 | m_0, -m_0; m_0, -m_0) \frac{\rho_0(m_0)}{\left\{ \frac{d\Xi_{m'}(s_0)}{dm'} \right\}_{m'=m_0}}, \qquad /I6/$$

$$\ln \frac{\omega}{\mu} = \int_{m_1}^{m_2} dm' \frac{\left\{ \frac{d\Xi_m(s_0)}{dm} \right\}_{m=m_0}}{\rho_0(m_0)} \cdot \ln \frac{|E(s_0, m') - E_F|}{\mu} \cdot \qquad /I7/$$

$$\cdot \frac{d}{dm'} \left[J(s_0 | m, -m; m', -m') \, C_{m'}(s_0) \frac{\rho_0(m')}{\frac{d\Xi_{m'}(s_0)}{dm'}} \right].$$

It can be seen from the above calculations that to obtain the asymptotics of the superfluid state of the atomic nucleus only a small part of all the interaction $J(s|m_1, m_2; m_1', m_2')$ of the protons situating on the same shell is essential, and namely $J(s_0|m,-m; m',-m')$, i.e. the interactions of protons with the equal and opposite projections of the angular momentum on the summetry axis of the nucleus are essential. The rest part of the proton interactions should be taken into account according to the perturbation theory. Therefore, further investigations will be performed with the interaction $J(s|m,-m; m',-m')$ which is denoted as follows $J(s|m,-m; m',-m') \equiv J(s|m,m')$.

3. Ground and Excited States.

Let us calculate the energy of the excited state by taking into account the weak interactions of protons/or neutrons/ on the same shell with the equal and opposite values of the angular momentum projection on the symmetry axis of the nucleus.

In the considered approximation $\xi_m(s)$ assumes the value $E(s,m) - E_F$, whereas $\tilde{\mathcal{E}}_m(s)$ passes into $\varepsilon_m(s) = \sqrt{\{E(s,m)-E_F\}^2 + C_m(s)^2}$ The expression $\rho_0(m') \Big/ \frac{d\xi_{m'}(s)}{dm'}$ becomes equal to $\rho(E') = \frac{dn}{dE'}$, which at $s = s_0$, $m = m_0$ is the level density near the Fermi surface. Further let us write the expression for $C_m(s_0)$ in the following form :

$$C_m(s_0) = \omega \frac{J(s_0|m, m_0)}{J(s_0|m_0, m_0)} \cdot e^{\frac{1}{G}} , \qquad /15'/$$

where

$$G = J(s_0|m_0, m_0)\, \rho(E) \qquad /16'/$$

$$\ln\frac{\omega}{\mu} = \int_{m_1}^{m_2} \frac{dm'}{\rho(E_F)} \cdot \ln\frac{|E(s_0,m') - E_F|}{\mu} \cdot \frac{d}{dm'}\left[J(s_0|m,m')\, C_{m'}(s_0)\, \rho(E')\right] . \qquad /17/$$

Calculating the mean energy in the excited state $C_1 = \alpha^{+}_{m_0} C_s = \alpha^{+}_{m_1} C_s$ /further $\alpha_{m_0} \equiv \alpha_{m-}$, $\alpha_{m_1} \equiv \alpha_{m+}$/ we obtain

$$\langle C_s^{*} \alpha_{m_1}(s) H \alpha^{+}_{m_1}(s) C_s \rangle =$$

$$= 2 \sum_{m'} \{ E(s,m') - E_F \} v_{m'}(s)^2 + \{ E(s,m) - E_F \} \{ u_m(s)^2 - v_m(s)^2 \} +$$

$$+ \frac{1}{N} \sum_{m'm''} J(s|m',m'') \, u_{m'}(s) v_{m'}(s) u_{m''}(s) v_{m''}(s) -$$

$$- \frac{2}{N} \sum_{m'} J(s|m,m') \, u_{m'}(s) v_{m'}(s) \cdot u_m(s) v_m(s) . \qquad /18/$$

The difference $\Delta E'$ between the energies in the states C_1 and C_s may be found in the form

$$\Delta E'_m(s) = \langle C_s^{*} \alpha_{m_1}(s) H \alpha^{+}_{m_1}(s) C_s \rangle - \langle C_s^{*} H C_s \rangle = \mathcal{E}_m(s) . \qquad /19/$$

It is worth noting that $C_1 = \alpha^{+}_{m_0} C_0$ is a superposition of the states with the odd number of particles. Therefore, to avoid the transition from the even-even nucleus to the odd one-one should make the calculations with the excited state $C_2 = \alpha^{+}_{m_1} \alpha^{+}_{m_0} C_s$.

Calculating the difference ΔE^{I} between the energy of the first excited and ground superfluid state of the even-even nucleus we find /7/ that

$$\Delta E^{I}_m(s) = \langle C_s^{*} \alpha_{m_0}(s) \alpha_{m_1}(s) H \alpha^{+}_{m_1}(s) \alpha^{+}_{m_0}(s) C_s \rangle -$$

$$- \langle C_s^{*} H C_s \rangle = 2 \, \mathcal{E}_m(s) . \qquad /20/$$

and at $s = s_0$, $m = m_0$

$$\Delta E^{I}_{m_0}(s_0) \approx 2 \omega \, e^{\frac{1}{G}} . \qquad /20'/$$

It can be seen from here that the first excited state is separated from the ground state by the gap /20'/. It should be noted that in the odd nuclei the energetic gap must not appear since there will be observed the transitions between the states inside the outer shell, there is also no energetic gap in the excitation of the normal state both in the even-even and in the odd nuclei.

The difference ΔE between the superfluid and the normal states may be obtained as follows /7/:

$$\Delta E_m(s) = \langle C_s^* H C_s \rangle - \langle C_n^* H C_n \rangle =$$

$$= -\frac{1}{2}\,\mathcal{E}_m(s) - \frac{1}{2}\,\frac{\{E(s,m)-E_F\}^2}{\mathcal{E}_m(s)} + \{E(s,m)-E_F\}\cdot\{1-2\,\theta_F(s,m)\} \qquad /21/$$

at $s = s_0,\ m = m_0$

$$\Delta E_{m_0}(s_0) = -\frac{1}{2}\,C_{m_0}(s_0)\ . \qquad /21'/$$

It can be seen from here that the superfluid state is energetically lower and separated from the normal state by the gap.

Thus, the interactions of the protons situating on the same shell with equal and opposite angular momentum projections lead to the appearance of the superfluid state of the atomic nucleus. The presence of the energetic gap between the first excited state and the ground superfluid one is confirmed by the considerations suggested by A. Bohr, Mottelson and Pines /2/ about the possibility of explaining the energetic gap in the heavy even-even nuclei in such a way.

4. Conditions for Appearing the Superfluid State of the Atomic Nucleus.

Using the variational principle suggested by Bogolubov one may obtain the conditions for appearing the superfluid state of the system.

In /8/ the method for investigating the condition of the superfluidity of the nuclear matter is proposed, whereas in /5/ it is shown that the nuclear matter posseses the superfluidity property if the attraction forces at the Fermi surface energy predominate. Let us investigate the conditions for appearing the superfluid state in the assumed model of describing the nuclear properties.

In order to determine when the energy of the normal state C_n is not be minimum, and, therefore, the superfluid state C_s will appear, let us find the second variation of $\mathcal{E}(u,v)$ if the condition /4/ is fulfilled, i.e.,

$$\delta^2 \tilde{\mathcal{E}}(u,v) = \delta^2 \left\{ \mathcal{E}(u,v) + \sum_{s,m} \lambda_m(s)\, \eta_m(s) \right\}. \qquad /22/$$

If expression /22/ is less than zero for the solution corresponding to the normal state C_n it means that the superfluid state C_s will appear. As a result of the calculations we obtain that the superfluid state of the atomic nucleus exists in the case when there are solutions of the equation

$$2\left|E(s_0,m)-E_F\right| \Psi_m(s_0) + \frac{1}{N}\sum_{m'}{}' J(s_0|m,m')\,\Psi_{m'}(s_0) = E\,\Psi_m(s_0) \qquad /23/$$

with the negative eigenvalues $E=-2\delta,\ \delta>0$.

Let us investigate the asymptotic form of solutions of /23/ if is tending to zero when J is also tending to zero being negative.

We assume

$$\Psi_m(s) = \frac{\theta_m(s)}{\left|E(s,m)-E_F\right| + \delta}$$

and pass from the sum to the integral and obtain

$$2\,\theta_m(s_0) + \int_{m_1}^{m_2} dm'\, \rho_0(m')\, J(s_0|m,m')\, \frac{\theta_{m'}(s_0)}{|E(s_0,m') - E_F| + \delta} = 0 . \qquad /24/$$

Taking into account that at $\delta \to 0$ the integral in /24/ becomes logarithmically divergent near the Fermi surface we pass to the consideration of the approximate equation

$$\theta_m(s_0) + \ln\frac{\Delta}{\delta} \cdot \rho(E_F)\, J(s_0|m,m_0)\, \theta_{m_0}(s_0) -$$

$$-\frac{1}{2}\int_{m_1}^{m_2} dm' \ln\frac{|E(s,m') - E_F|}{\mu} \cdot \frac{d}{dm'}\left[J(s_0|m,m')\,\rho(E')\,\theta_{m'}(s_0)\right] = 0 \quad , \qquad /25/$$

which at small J coincides asymptotically with /24/, in /25/ $\Delta = \Delta_1 = \Delta_2 \gg \delta$. Let us introduce a new function

$$f_m(s_0) = \frac{\theta_m(s_0)}{\theta_{m_0}(s_0)}\, \frac{1}{\ln\frac{\Delta}{\delta}}$$

$$f_{m_0}(s_0) = \frac{1}{\ln\frac{\Delta}{\delta}} > 0 .$$

The equation for $f_m(s_0)$ at $m=m_0$ may be obtained as follows :

$$f_{m_0}(s_0) + \rho(E_F)\, J(s_0|m_0,m_0) -$$

$$-\frac{1}{2}\int_{m_1}^{m_2} dm' \ln\frac{|E(s,m') - E_F|}{\mu} \cdot \frac{d}{dm'}\left[J(s_0|m_0,m')\,\rho(E')\,f_{m'}(s_0)\right] = 0 . \qquad /26/$$

If J tends to zero equation /26/ has the solution in the case if

$$\rho(E_F)\, J(s_0|m_0,m_0) < 0 . \qquad /27/$$

Indeed, $f_{m_0}(s_0) > 0$, whereas the last term in /26/ is the value of higher order of smallness, if $J(s_0|m_0,m')$ does not change rapidly in the interval m' from m_1 up to m_2. Since the level density $\rho(E_F) > 0$ then the superfluidity condition of the atomic nucleus may be obtained

$$J(s_0|m_0,m_0) < 0 , \qquad /27'/$$

i.e. the attraction forces must predominate between the protons situating on the same shell.

5. Temperature of Phase Transition.

To determine the critical temperature of the phase transition of the nucleus from the superfluid to the normal state we make use of the statistical variational principle. /9/ The statistical variational principle suitable for the determination of the thermodynamic quantities both at zero temperature and at that different from zero is the generalization of the variational principle suggested by Bogolubov /3/.

As a result of the calculations we obtain that the superfluid state of the atomic nucleus at the temperature θ different from zero exists in the case when there are solutions of the equation

$$2\left|E(s_0,m)-E_F\right| \, \mathrm{cth}\, \frac{\left|E(s_0,m)-E_F\right|}{2\theta}\, \varphi_m(s_0) + \qquad /28/$$

$$+ \frac{1}{N} \sum_{m'}{}' J(s_0|m,m')\, \varphi_{m'}(s_0) = E\, \varphi_m(s_0)$$

with the negative eigenvalues $E < 0$. It is easily seen that at $\theta = 0$ equation /28/ passes into /23/. If the eigenvalue $E > 0$ then the normal state is a ground one. If at $\theta = 0$ there exists a superfluid phase then with the temperature θ increase E will increase also. In the transition from the superfluid phase to the normal one at the critical temperature θ_0 the eigenvalue E must vanish. One may obtain from here the equation for the determination of the critical temperature, and namely :

$$2\left|E(s_0,m)-E_F\right| \, cth \frac{\left|E(s_0,m)-E_F\right|}{2\theta_0} \, \varphi_m(s_0) + \\ + \frac{1}{N'} \sum_{m'}{}' \, J(s_0|m,m') \, \varphi_{m'}(s_0) = 0 \, . \qquad /29/$$

Taking into account that $\left|E(s_0,m)-E_F\right|$ is rather small, and passing from the sum to the integral we obtain the approximate equation

$$\varphi_m(s_0) + \frac{1}{4\theta_0} \int_{m_1}^{m_2} dm' \, \rho_0(m') \, J(s_0|m,m') \, \varphi_{m'}(s_0) = 0 \, . \qquad /30/$$

In order that /30/ would have the solution different from zero it is necessary that the determinant $\mathcal{D}$ would be equal to zero. Taking into account the weakness of the interaction we may obtain the expression for the temperature of the phase transition

$$\theta_0 = -\frac{1}{4} \int_{m_1}^{m_2} dm' \, \rho_0(m') \, J(s_0|m',m') \qquad /31/$$

or

$$\theta_0 \approx -\frac{1}{4} \, \bar{\rho}_0 \, \bar{J} \, . \qquad /31'/$$

from the equation $\mathcal{D} = 0$

6. Rough Approximation.

In Section 2 we have obtained the asymptotic form of the solution of the ground equation /13/. Let us consider two more rough approximate solutions of this equation.

Let us somewhat simplify equation /13'/ and write it as follows :

$$C_m(s_0) = -\frac{1}{2}\int\limits_{E_F-\Delta}^{E_F+\Delta} dE'\,\rho(E')\,J(s_0|m,m')\cdot\frac{C_{m'}(s_0)}{\sqrt{C_{m'}(s_0)^2+(E'-E_F)^2}} \qquad /32/$$

We assume that $\rho(E')$ and $J(s_0|m,m')$ are weakly dependent upon (E'), in this case $C_{m'}(s)$ will depend only upon S . As a result we obtain

$$1 = -\frac{1}{2}\,\bar{\rho}\,\bar{J}\int\limits_{E_F-\Delta}^{E_F+\Delta} dE'\,\frac{C(s_0)}{\sqrt{C(s_0)^2+(E'-E_F)^2}} \qquad /33/$$

Taking into account that $C(s_0) \ll \Delta$ we find that

$$C(s_0) = 2\,\Delta\,e^{\frac{1}{\bar{\rho}\,\bar{J}}} \qquad /34/$$

In this case

$$\mathcal{E}_m(s_0) = \sqrt{\{E(s_0,m)-E_F\}^2 + 4\,\Delta^2\,e^{\frac{2}{\bar{\rho}\,\bar{J}}}}\;,$$

at $\bar{J} \to 0$

$$\mathcal{E}_m(s_0) \approx \Delta\, e^{\frac{1}{\bar{\rho}\,\bar{J}}}$$

Thus, in this rough approximation there is also a gap between the excited and the ground superfluid states, as well as between the superfluid and normal states. The superfluidity condition in this case is reduced to the requirement $\overline{J} < 0$, whereas the temperature of the phase transition is determined by the formula /31'/.

As another rough approximation we can consider the case, when $E(s,m)$ is independent of m, i.e., there a degeneration over the quantum number m. Then for the shell $s=s_o$ we have $E(s_o)=E_F$ and from the equation /13/ we obtain that

$$C_m(s_o) = -\frac{1}{N} \sum_{m'}{}' J(s_o|m,m') \qquad /35/$$

further

$$U_m(s_o)^2 = V_m(s_o)^2 = \frac{1}{2}, \quad \mathcal{E}_m(s_o) = C_m(s_o).$$

In this case the difference between energies in the excited and ground superfluid states will be proportional to J.

7. Conclusion.

We have considered a rather idealized model of the atomic nucleus. Analogous calculations may be made for other nuclear models in which the interactions of all the nucleons are taken into account. The account of n-p interactions together with p-p and n-n interactions as is shown in [5] for case of nuclear matter leads to great complications and requires the modification of the mathematical methods.

The obtained results confirm the considerations set forth in [8,2] about the physical analogy between the Fermi-systems of metal and nucleus and illustrate the fruitfulness of application of the mathematical methods developed in the theory of superconductivity [1] for studying the properties of the atomic nucleus[x/].

In conclusion the author expresses the gratitude to N.N. Bogolubov for his constant interest to the present work and valuable remarks.

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x/ After this paper had been completely prepared for publication the author got the preprint by Belyaev /10/ who obtained some similar results using the method /1/ but in somewhat different manner.

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