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PROPERTIES OF CHARGE SYMMERTRY AND REPRESENTATIONS OF THE EXTENDED LORENTZ GROUP IN THE THEORY OF ELEMENTARY PARTICLES

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PROPERTIES OF CHARGE SYMMERTRY AND REPRESENTATIONS OF THE EXTENDED LORENTZ GROUP IN THE THEORY OF ELEMENTARY PARTICLES

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БИБЛИОТЕКА

A N N O T A T I O N

The extended Lorentz group, involving the full Lorentz group and the operation of charge conjugation is discussed. It is shown, that the use of irreducible projective representations of this extended group requires the existence of charge multiples. Charge symmetry and associated production of strange particles follow from the invariance under reflections, charge conjugation and the laws of conservation of electrical and baryon charges. For free nucleons there exists the Pauli-Gürsey transformation. The requirement of the invariance under this transformation in interaction, too, leads to the isobaric invariance for all the particles in strong interactions.

I. I N T R O D U C T I O N

It is known, that strongly interacting particles unite in charge multiplets ($p, n; \pi^+, \pi^0, \pi^-; K^+, K^0$). Particles, belonging to a given multiplet, have almost equal masses, the same spin, but they have different electrical charges. According to the experiment, the hypothesis of charge symmetry and the more strong hypothesis of charge independence are well established. In the usual theory it is expressed by means of the invariance under rotations in some formal isobaric space. Particles of the given multiplet are considered as the states with different projections of the isobaric spin of the same particle of the corresponding isobaric spin. A proton and a neutron, for example, form a nucleon.

For the description of a nucleon the reducible eight-component representation of the proper Lorentz group is used. An analogous situation (the reducibility of the representation of the proper Lorentz group) also takes place for the other strongly interacting particles.

The following question arises;

Is it possible to enlarge the Lorentz group in such a way and find such irreducible representations of this extended group, so that the existence of charge multiplets automatically follows out of them ?

The present paper is devoted to the solution of this question.

We enlarge the Lorentz group in the following way:

In the quantum field theory wave-functions are complex. The operation of the charge conjugation C, transforming a particle into an antiparticle, is always represented as the product of a linear operator (matrix)by a nonlinear operator of the complex conjugation :

$$C: \qquad \psi_c = C_0 \psi^*, \tag{I}$$

where C_0 is determined in such a way, that ψ_c is transformed aocording to the same irreducible representation of the proper Lorentz group, as ψ .

We include the operation of the chage conjugation C , besides the proper Lorentz group($\mathcal{L}$) space (I) - and time (T) - reflections, into the extended Lorentz group.

Then, we consider the usual irreducible representations of the extended group as well as its projective irreducible representations*).

Gelfand and Tsetlin(I) pointed out that it was important to use the projective representation of the Lorentz group in connection with the parity-doublet theory of Lee and Yang. The possibility of the use of projective representations is connected with the indeterminancy of the phase factor by the wave-function in the quantum field theory. After Gelfand and Tsetlin projective representations of the full Lorentz group were considered by Taylor, and Mc Lennan(2). In Taylor's(2) paper the connection between these representations and the isobaric invariance is outlined. But since only the full Lorentz group is considered, the spaoe parities of $\pi^{\pm}$ and π^0 mesons and protons and neutrons are different. At the 7th Roohester Conference Salam and Pais also discussed the idea of the necessity of new determinations for the operations of space- time reflections.

In the present paper we shall not study all the irreducible projective representations of the extended Lorentz group, but we shall restrict ourselves to those of them, which are necessary for the description of strongly interacting partioles. It will be shown that if nuoleons, Ξ -particles and K-mesons are described by unusual, projective representations of the extended Lorentz group, and the rest of the particles are desoribed in the usual way, the existence of multiplets, the property of oharge symmetry and associated production of strange partioles follow from the standart laws of conservation of the number of baryons, electrical charge and invariance under the full Lorentz group and charge conjugation.

In this theory under discussion there exists the Pauli-Gürsey transformation for free nucleons, which is naturally connected with isobaric invariance. The isobaric invariance in strong interactions for all the particles follows from the requirement of the invariance of interaction lagrangian under the above-mentioned transformation for nucleons.

In this theory it is not difficult to write the lagrangian of the interaction with electromagnetic fields with the help of a charge operator. It appears, that Schwinger time reversal is not conserved for electromagnetic interactions, and only Wigner time reversal is conserved.

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*) If the operator $R(g)$ corresponds to an element g of the group $\mathcal{G}$ and the operator $R(g_1 g_2)=\alpha(g_1,g_2)R(g_1)R(g_2)$ corresponds to the product of elements of the group $g_I\ g_2$, then, they say, that the projective representation of the group is defined. If $\alpha(g_1,g_2)=1$ then the projective representation will be usual. In the general case $\alpha(g_1,g_2)$ may be equal to $+1$ and - I. Thus, in the projective representation anticommuting operators of the representation may correspond to the commuting elements of the group. In particular, the usual spinor representation is projective, (the operations γ_4 and $\gamma_4\gamma_5$ anticommute, while spaoe - and time - reflections commute.

Weak interactions, where even the space parity is broken, are more complex, and they will not be discussed in this paper.

For concreteness, we shall suppose, that the relative space parity of all the baryons is the same and the reflection of the usual spinors is described with the help of the operator γ_4. All the bosons are considered pseudo-scalar.

Let us begin with nucleons.

2. Free Nucleon Field

For 4 - component spinors, if we require, that

$$I^2 = T^2 = C^2 = 1, \tag{2}$$

then, as it is easily to show, operators I, T and C are expressed as follows:

$$I: \quad \psi' = \gamma_4 \psi, \tag{3a}$$

$$T: \quad \psi' = i\gamma_4\gamma_5\psi, \tag{3b}$$

$$C: \quad \psi_c = i\gamma_2 \psi^*, \tag{3c}$$

where T is Schwinger spinor time reversal (3), and the matrixes γ_i are expressed in the Pauli representation. The following commutation relations hold for operators I, T and C

$$IT = -TI, \tag{4a}$$

$$IC = -CI, \tag{4b}$$

$$TC = -CT. \tag{4c}$$

In contradistinction to the usual theory, preserving relations (2), we change the sign of the relation (4a) for nucleons, so that:

$$IT=TI \quad (a), \qquad IC=-CI \quad (b),$$ (4)
$$TC=-CT \quad (c)$$

Only 8 x 8 matrixes satisfy the rules of commutation (2) and (4) :

$$I: \qquad \psi' = \tau_3 \times \gamma_4 \psi,$$

$$T: \qquad \psi' = 1 \times \gamma_4 \psi,$$ (5)

$$C \qquad \psi_c = i \tau_3 \times \gamma_2 \psi^*,$$

where τ — is Pauli matrixes.

These operators, together with the operators of the proper Lorentz group, in which everywhere it is necessary to replace γ_μ by $1 \times \gamma_\mu$, form the projective irreducible representation of the extended Lorentz group, and the spinors ψ have eight components. $\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$ The lagrangian for the free field ψ is uniquely determined*)

$$L = \bar{\psi}(1 \times \gamma_\mu \partial_\mu + i \tau_2 \times \gamma_5 m)\psi,$$ (6)

where $\bar{\psi} = \psi^{*T} 1 \times \gamma_4$, and the equation of the field has the following form:

$$1 \times \gamma_\mu \partial_\mu \psi = -i \tau_2 \times \gamma_5 m \psi$$ (7)

The lagrangian (6), as well as equation (7) are invariant, under two parameter groups of transformations.

$$\psi' = \exp(i\lambda)\psi$$ (8)

$$\psi' = \exp(i \tau_1 \times \gamma_5 \lambda)\psi$$ (9)

and the three-parameter group

$$\psi' = a\psi + b \tau_3 \times \gamma_5 \psi_c,$$ (10)

where $|a|^2 + |b|^2 = 1$

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*) Under the Schwinger inversion of time, the sign of transposing belongs to the operators of the Hilbert space 3.

The transformations (9) and (I0) are analogous to the Pauli transformations (4) for the neutrino and differ from them in the fact that γ_5 is replaced by $\tau_1 \times \gamma_5$ and $\tau_3 \times \gamma_5$, accordingly.

If we introduce the new four-component spinors,

$$\psi_p = \frac{1}{\sqrt{2}}(\psi_1 + \gamma_5 \psi_2), \qquad \psi_{pc} = \frac{1}{\sqrt{2}}(\psi_{c1} + \gamma_5 \psi_{c2}),$$
$$\psi_{nc} = \frac{1}{\sqrt{2}}(-\gamma_5 \psi_1 + \psi_2), \qquad \psi_n = \frac{1}{\sqrt{2}}(\gamma_5 \psi_{c1} - \psi_{c2}), \tag{II}$$

then these spinors satisfy the usual Dirac equation

$$\gamma_\mu \partial_\mu \psi = -m\psi, \tag{I2}$$

and under the transformation (9)

$$\psi_p' = \exp(i\lambda)\psi_p, \qquad \psi_n' = \exp(i\lambda)\psi_n$$
$$\psi_{pc}' = \exp(-i\lambda)\psi_{pc}, \qquad \psi_{nc}' = \exp(-i\lambda)\psi_{nc} \tag{I3}$$

Thus, we may consider the (9) as gauge transformation connected with the conservation of the number of baryons, and ψ_p, ψ_n, ψ_{pc}, ψ_{nc} connect with the proton, neutron, antiproton and antineutron fields, accordingly.

The transformation

$$E: \qquad \psi' = \exp\left[\frac{i}{2}(1 \times 1 + \tau_1 \times \gamma_5)\lambda\right]\psi \tag{I4}$$

must be connected with the law of conservation of the electric charge.

Actually, under the transformation E

$$E: \quad \psi_p' = \exp(i\lambda)\psi_p, \qquad \psi_{pc}' = \exp(-i\lambda)\psi_{pc}$$
$$\psi_n' = \psi_n, \qquad \psi_{pc}' = \psi_{nc} \tag{I5}$$

The three-parameter transformation (I0) is isomorphous to the rotation in the isobaric space. In reality, if we form eight-component nucleon field $\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$ in the usual way, then under the transformation (I0)

$$\psi_N' = \exp[i\vec{\tau}\cdot\vec{\lambda}]\psi_N, \tag{I6}$$

where $\vec{\lambda}$ -is a vector with real components $\lambda_1, \lambda_2, \lambda_3$, τ-are the usual two-dimensional Pauli matrixes, and

$$a = \cos|\lambda| + i\frac{\lambda_3}{|\lambda|}\sin|\lambda|, \qquad b = \frac{\sin|\lambda|}{|\lambda|}(\lambda_2 - i\lambda_1) \tag{I7}$$

Gürsey (5) pointed out an analogous isomorfism by purely formal doubling of the number of components.

3. Interaction between Nucleons and Usual Bosons

First of all, let us consider interaction between nucleons and the neutral pseudoscalar field φ_0 with the positive time parity

$$\begin{aligned} I: &\quad \varphi_0' = -\varphi_0 , \\ T: &\quad \varphi_0' = \varphi_0 , \\ C: &\quad \varphi_{0c} = \varphi_0 \end{aligned} \tag{I8}$$

Requirements of the invariance under I, T and C and the transformations (9) and (E) simply lead to the lagrangian of interaction (only lagrangians of interaction without derivatives are further oonsidered).

$$\begin{aligned} L &= i g_0 \bar{\Psi} \tau_3 \times \gamma_5 \Psi \varphi_0 = i g_0 (\bar{p} \gamma_5 p - \bar{n} \gamma_5 n) \varphi_0 \\ &= i g_0 \bar{\psi}_N \gamma_5 \tau_3 \psi_N \varphi_0 ; \end{aligned} \tag{I9}$$

i.e. the sign of the constant of the interaction between mesons φ_0 and protons and neutrons has the same sign and φ_0 may be identified with the neutral π_0 -meson.

If the neutral meson φ_0' were space pseudoscalar, but had negative time parity, then the lagrangian of interaction follow.out in a unique way :

$$\begin{aligned} L &= g_0' \bar{\Psi} \tau_2 \times 1 \Psi \varphi_0' \\ &= i g_0' (\bar{p} \gamma_5 p + \bar{n} \gamma_5 n) \varphi_0' \end{aligned} \tag{20}$$

φ_0' may be oonnected with a problematic ρ_0 -meson.

For the lagrangian of interaction between nucleons and the charged pseudoscalar boson field φ

$$I: \varphi' = -\varphi ; \qquad T: \varphi' = \varphi ;$$
$$C: \qquad \varphi_c = \varphi^*$$

again it follows uniquelly that :

$$\begin{aligned} L &= i g (\bar{\Psi} \Psi_c \varphi^* - \bar{\Psi}_c \Psi \varphi) \\ &= 2 i g (\bar{p} \gamma_5 n \varphi^* + \bar{n} \gamma_5 p \varphi) \end{aligned} \tag{2I}$$

So, we may suppose, that φ (φ^*) describes π^- (π^+) mesons.

Charge symmetry i.e., the possibility of simultaneous substitution ($p \rightleftarrows n$, $\pi^+ \rightleftarrows \pi^-$, $\pi^0 \rightleftarrows -\pi^0$) of the interactions (I9) and (2I) is obvious. The general lagrangian of interaction with π -mesons has the following form :

$$\mathcal{L}_\pi = i g_0 (\bar{p}\gamma_5 p - \bar{n}\gamma_5 n)\pi^0 + 2ig(\bar{p}\gamma_5 n\,\pi^+ + \bar{n}\gamma_5 p\,\pi^-). \tag{22}$$

If we now require, that not only the free lagrangian of nucleons (22), but also the lagrangian of interaction should be invariant under transformations of three-parameter group, then

$$g = g_0/\sqrt{2} = g_\pi/\sqrt{2},$$
$$\mathcal{L}_\pi = i g_\pi \bar{\Psi}_N \gamma_5 (\vec{\tau}\cdot\vec{\pi})\Psi_N \tag{23}$$

Under the transformation (IO) meson fields are transformed in the following way :

$$(\vec{\tau}\cdot\vec{\pi})' = \exp[i\,\vec{\tau}\cdot\vec{\lambda}](\vec{\tau}\cdot\vec{\pi})\exp[-i\,\vec{\tau}\cdot\vec{\lambda}], \tag{24}$$

and the masses of mesons π^+, π^- and π^0 must be equal.

So, we have come to the usual symmetric theory of interaction between π -mesons and nucleons.

4. Free K- Mesons

A usual representation of the extended Lorentz group for bosons is exhausted by π-mesons. We shall describe K-mesons with a projective representation where

$$I^2 = 1;\quad C^2 = 1,\quad T^2 = -1.$$
$$IT = TI,\quad IC = CI,\quad TC = CT. \tag{25}$$

The simpliest irreducible representation, where these rules of commutation are accomplished, is two-dimensional and operators I, T and C have the following form :

$$\begin{aligned} I:&\quad \varphi' = -\varphi,\\ T:&\quad \varphi' = i\tau_2\varphi,\\ C:&\quad \varphi_c = \varphi^* \end{aligned} \tag{26}$$

Let us identify K^+ -meson with φ_1, $\bar{K^o}$-meson with φ_2, K^--meson with φ_1^*, K^o- meson with φ_2^*

Then, the transformation

$$E: \qquad \varphi' = exp\left[\frac{i}{2}(1+\tau_3)\lambda\right]\varphi \tag{27}$$

may be connected with the law of conservation of electrical charge.

In fact, under this transformation

$$\begin{aligned} K^{+\prime} &= exp(i\lambda)K^+ , & K^{-\prime} &= exp(-i\lambda)K^-, \\ K^{o\prime} &= K^o , & \bar{K}^{o\prime} &= \bar{K}^o \end{aligned} \tag{28}$$

The transformation $\varphi' = exp\,(i\tau_3\lambda)\varphi$, under which

$$\begin{aligned} K^{+\prime} &= exp\,(i\lambda)K^+ , & K^{o\prime} &= exp\,(i\lambda)K^o \\ K^{-\prime} &= exp\,(-i\lambda)K^- , & \bar{K}^{o\prime} &= exp(-i\lambda)\bar{K}^o. \end{aligned} \tag{29}$$

corresponds to the law of conservation of hypercharge.

5. Interaction between K-Mesons and Nucleons. Λ and Σ -Particles

Let us proceed to the study of interaction between K-mesons and nucleons. Both K-mesons and nucleons are transformed according to the projective representation of the extended Lorentz group and the law of conservation of the baryon charge is held. Therefore, one more baryon must participate in the interaction and this baryon must be described with a usual representation. This requirement with necessity leads to assosiated production of strange particles.

Let us first discuss the case of the neutral baryon. We have already said before, that for the sake of definition the relative space parity of all the baryons is supposed to be the same

$$Y_o' = \gamma_4 Y_o \tag{30}$$

There are two possibilities T

$$Y_o' = -\gamma_4\gamma_5 Y_{oc} , \tag{31}$$

$$T: \qquad Y_o' = \gamma_4\gamma_5 Y_{oc} . \tag{32}$$

for the baryon under discussion

It is necessary to introduce the antibaryon Y_{0c}, in equations (3I) and (32), because the transformation T for nucleons anticommutes with the transformation of conservation of the baryon charge (9).

If we choose the law (3I) for T, then the only form for the lagrangian, invariant under transformations of the extended Lorentz group and the transformations (9) and (E) is :

$$\mathcal{L} = ig[\bar{\Psi}(1\times\gamma_5-\tau_1\times 1)(1\times 1+\tau_3\times 1)\varphi\times Y_0$$
$$-\bar{\Psi}_c(1\times\gamma_5+\tau_1\times 1)(1\times 1-\tau_3\times 1)\varphi^*\times Y_0] + \text{эрм. сопр.} \qquad (33)$$

If we proceed from Ψ and φ to operators of the nucleon field and the K-meson field, then we shall obtain the usual form of the lagrangian of the interaction between nucleons and Λ_0 particles:

$$\mathcal{L} = ig_\Lambda(\bar{p}\gamma_5\Lambda_0 K^+ + \bar{n}\gamma_5\Lambda_0 K^0) + \text{эрм. сопр.} \qquad (34)$$

The law (32) for T leads to the lagrangian which differs from expression (33) only in the sign + between the two members in expression (33) and corresponds to Σ_0-particle:

$$\mathcal{L} = ig_{\Sigma_0}(\bar{p}\gamma_5\Sigma_0 K^+ - \bar{n}\gamma_5\Sigma_0 K^0) + \text{эрм. сопр.} \qquad (35)$$

So, we come to conclusion, that by the transformation T for nucleons the laws of transformations for Λ_0 and Σ_0 differ in a sign.

Now, if we consider the interaction between nucleons and charged baryons, it is possible to find the lagrangian invariant with respect to I, T, C and the laws of conservation of electrical and baryon charges only under the condition:

$$T: \begin{array}{l} \Sigma^+ \to -\gamma_4\gamma_5\Sigma_c^-, \\ \Sigma^- \to -\gamma_4\gamma_5\Sigma_c^+, \end{array} \qquad (36)$$

that provides the equality of masses of Σ^+ and Σ^--particles. This lagrangian has the following form:

$$\mathcal{L} = -ig[\bar{\Psi}(1\times 1-\tau_1\times\gamma_5)(1\times 1-\tau_3\times 1)\varphi^*\times\Sigma^+ +$$
$$+\bar{\Psi}_c(1\times 1+\tau_1\times\gamma_5)(1\times 1+\tau_3\times 1)\varphi\times\Sigma^-] + \qquad (37)$$

It may be expressed in the usual notations by means of the expression :

$$\mathcal{L} = i g_{\Sigma\pm} (\bar{p}\gamma_5 \Sigma^+ K^0 + \bar{n}\gamma_5 \Sigma^- K^+) + \text{эрм. сопр.} \tag{38}$$

Charge symmetry is obvious.

Now if we require, that transformation of the Pauli-Gürsey type (I0) leaves the lagrangian of interaction also invariant

then

$$g_{\Sigma\pm} = \sqrt{2}\, g_{\Sigma_0} = \sqrt{2}\, g_{\Sigma}\,, \tag{39}$$

masses Σ^+, Σ^- and Σ_0 must be equal, and we have the usual interaction lagrangian :

$$\mathcal{L} = i g_{\Sigma} (\bar{p}\gamma_5 \Sigma_0 K^+ - \bar{n}\gamma_5 \Sigma_0 K^0 + \sqrt{2}\,\bar{p}\gamma_5 \Sigma^+ K^0 + \tag{40}$$

$$+ \sqrt{2}\,\bar{n}\gamma_5 \Sigma^- K^+) + \text{эрм. сопр.}$$

7. Ξ Particles

In the framework of the representations under discussion there exists a place for one and the only one additional particle. Namely, in projective representation (5) having left the same transformation (9) for the law of conservation of baryons, it is possible to replace the sign between I x I and $\tau_1 \times \gamma_5$ in transformation (I4) which is connected with the conservation of electrical charge :

$$E: \quad \psi' = \exp\left[-\frac{i}{2}(1\times 1 - \tau_1 \times \gamma_5)\lambda\right]\psi \tag{4I}$$

Then it is necessary simply to substitute Ξ^0 for p, Ξ^- for n, $\bar{K}^0$ for K^+, K^- for K^0 in all the formulae .

We again obtain properties of charge symmetry and if there is the invariance under the Pauli transformation in interaction too, usual interaction invariant lagrangians.

CONCLUSION

We have accomplished the programm, planned at the beginning of the paper. Introducing a new irreducible projective representation of the extended Lorentz group for nucleons, and, proceeding from it, it is really shown, that properties of charge symmetries, assosiated production of strange particles and the existence of charge multiplets follow out of the standard laws of conservation and the isobaric invariance - of the Pauli-Gürsey transformation for free nucleons.

The description of weak interactions, from this point of view, was not discussed here. This task is much more difficult because of the violation of the conservation of the space parity.

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