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Richard Weiner

CHARGE DISTRIBUTION OF EXCITED ISOMERIS NUCLEI AND ATOMIC SPECTRA

(The Nuclear Isomeric Shift)

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(The Nuclear Isomeric Shift)

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**) A brief account along these lines is appearing in the J.Exptl. Theoretical Phys. (U.S.S.R.).

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CHARGE DISTRIBUTION OF EXCITED ISOMERIC NUCLEI AND ATOMIC SPECTRA

/The Nuclear Isomeric Shift/

by

Richard Weiner

A b s t r a c t

The attention is called on an electric characteristic of excited nuclei almost unstudied until present, neither theoretically nor experimentally - namely the charge distributions of two isomeric nuclei is automatically realized in atomic spectra giving rise to the nuclear isomeric shift on spectral lines.

Some general theoretical aspects of this effect are here discussed. Only odd nuclei are studied. It is assumed: (A) the transitions are single particle ones; (B) validity of the Ro-

senthal-Briet perturbation theory; (C9 at even-odd nuclei the effect is due to electron-neutron interaction.

Under these assumptions it is shown that except the sign, the effect is a pure single particle effect, given by the optical nucleons. The sign of the shift is generally intimately related to the whole nuclear configuration. In the case of two characteristic transitions, it is shown that the order of magnitude of the effect does not depend on the shape of nuclear potential. The general formulae of the shift for odd-even and even-odd nuclei are given. By specialising for two particular forms of nuclear potential (harmonic oscillator and infinite square well), it is shown that there exists a very simple relation between the shift and the characteristics of the two nuclear states involved. Numerical applications are given for $In^{115}\ \mathrm{III}$, $Au^{197}\ \mathrm{I}$, $Hg^{197}\ \mathrm{II}$ and $Pb^{207}\ \mathrm{IV}$ At the odd-even nuclei the theoretical value of the effect is surely within the reach of atomic spectroscopy, $(\geq 10^{-2}\ cm^{-1}$ at even-odd nuclei the shift predicted is at the limit of spectroscopical measurability $(\sim 10^{-4}\ cm^{-1})$ but the specific interest of the phenomenon lies here in the possible study of a pure bound neutron-electron interaction.

The importance of an experimental proof of the effect which become a new tool in the research of nuclear structure and a strong test of the validity of the hypotheses (A) and (C) is emphasized.

Introduction

Excited nuclei are usually studied by their radioactive products. But radioactivity is not the only characteristic of an excited nucleus. It is well known that electromagnetic properties like nuclear moments e.g., characterize a nuclear state, too. We should like to call the attention on another electric feature of excited isomeric nuclei, not studied until now in this connection, i.e. the charge distribution, respectively the electromagnetic radius of excited nuclei.

If the present experiments on a given nuclear state, cannot provide beside the nuclear radius more than a single parameter of the charge distribution, the comparison of distributions of two different states, may supply new information on this nuclear property in particular, and on the excitation process in general. This comparison is automatically realised in nature by a sui generis substraction process in the nuclear isomeric shift on spectral lines. In two previous notes*/ this shift was calculated for $In^{\overline{115}}_{\underline{111}}$ and it was shown that the theoretical magnitude of the shift is within the reach of spectroscopic measurability; for $Hg^{197} - Hg^{197}_{m}$ a shift of $\sim 10^{-4} cm^{-1}$ due to electron neutron interaction was also predicted.

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*/ R. Weiner; Nuovo Cimento, Serie X, Vol. 4, 1587 (1956); Studii şi Cercetări de Fizică, 7, 567 (1956).

In Section 1 of this paper the general formulae of the shift for odd-even nuclei are given and the dependence of the shift on the nuclear potential and configuration is studied, In Section 11 the similar effect at even-odd nuclei is discussed.

Only odd nuclei will be studied. This is the most important class from the experimental point of view because among them we find the absolute majority of isomeric nuclei which posess a sufficiently big lifetime for the experiences here proposed.

We shall restrict our considerations to low single-particle excitations for which we shall assume valid the shell model[2/](A).

The Breit-Rosental perturbation theory will be used, with electronic wave functions corrected as regards the finite extension of the nucleus (B). Only the most penetrating electrons will be considered.

Finally the electron-neutron interaction potential is taken the form

$$V_{e-n} = b\,\delta\left(\underline{r}_n - \underline{r}_e\right) \tag{1}$$

2/ In this paper we shall understand by the shell model, a single particle for deformed nuclei as well as for spherical ones. The parameters of the potential depend only on the nucleus considered and not on the nuclear states. (Velocity dependent potentials are not studied).-

where r_n and r_e are the position vectors of the nautron respectively the electron and b is a constant given by the scattering experiments (C).

For the cases above mentioned and with the assumptions /A,B,C/ we shall show that besides the sign, the isomeric shift is a pure single-particle effect, given only by the optical (external) nucleons . It is verified in two characteristio cases that the order of magnitude of the shift does not depend on the nuclear potential assumed in the shell model.

I. ODD-EVEN NUCLEI (OPTICAL NEUTRONS)

1. Perturbation Theory

The influence of the finite extension of the nucleus on the atomic spectra is reflected in various phenomena as isotopic shift, hyperfine-structure (h.f.s.) etc., and special methods were elaborated to interprete these effects theoretically. The above mentioned methods are based on the perturbation theory, either under the usual form developed by Rosenthal and Breit[3/], or under the form of boundary conditions perturbation.[4/5/]

3/ J. Rosental, G. Breit: Phys. Rev. 41, 459 (I932).
4/ J. Smorodinsky: J. Phys. (USSR) 10, 419 (I946).
5/ E. Broch: Arch. Mat. Naturvidensk 48, 25 (I946).

We shall apply the method given in [3] to calculate the electronic wave functions φ ; the charge distribution will be assumed uniform through the volume of the nuclaus. In this way we take in account grosso modo the influence of the finite nuclear extension on the electronic functions. But these functions must be considered unperturbed functions since they do not reflect the nonuniform character of the charge distribution and ellthe less the variation of the distribution by the excitation of the nucleus. The determination of the perturbed functions is equivalent with the rigorous solving of the problem of the electron motion in the field of the nonuniform distribution. This can be made only by numerical integration and is not of great interest, because of the approximative form of the nuclear charge distribution given by the shell model.

It is worthwhile to mention that the φ functions calculated starting from e uniform charge distribution give relatively small corrections (10-20%) in the isomeric shift at odd-even nuclei, in comparison with the Racah functions; this situation is changed at even-odd nuclei (See section II of this paper).

3/ J. Rosenthal, G. Breit: Phys. Rev. <u>41</u>, 459 (1932).

It is clear that the only part of φ interesting in our problem is the part corresponding to the interior of the nucleus. For Δ electrons, the uniform charge distribution leads to the Dirac wave function:

$$|\varphi|^2 = \varphi^2 = C\left[1 - (\gamma^2/2)x^2 + (\gamma^2/10 + 0{,}112\gamma^4)x^4\right] \qquad (2)$$

where $\gamma = Z\alpha$, $\alpha = e^2/\hbar c$, $x = r/R$, R being the nuclear radius. The constant C is determined by matching (2) to the exterior functions and by the normalization condition.

One obtains[6/]:

$$C = \alpha^2 a_H^2 \varphi_{sch}^2(0) L^{-2}/4R^2 \qquad (3)$$

with

$$L = x_1(1)\,\Gamma(2\sigma)(\xi + \sigma - 1)\,y_0^{-\sigma} \qquad (4)$$

a_H is the radius of the first Bohr orbit, $\varphi_{sch}(0)$ - the value of the Schrödinger wave function in the origin. The further notations are: $\sigma = (1 - \alpha^2 Z^2)^{1/2}$, $y_0 = 2ZR/a_H$

$\xi = -\alpha Z x_2(1)/x_1(1)$ where x_1 and x_2 are the dimensionless Dirac wave functions $\varphi^2 = C\left[x_1^2 + x_2^2\right]$

Let us put $\Delta\bar{\rho}$ for the charge densities difference of the two nuclear states considered. The corresponding difference of the electronstatic potential will be

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6/ See e.g. W. Humbach: Zs. f. Phys. 133, 589 (1952).

$$\Delta\varphi = \int \frac{\Delta\bar{\rho}(\underline{r}')d\underline{r}'}{|\underline{r}'-\underline{r}|} \tag{5}$$

and the respective shift of the atomic (electronic) level:

$$\Delta E = -e\int \varphi^2 \Delta\varphi \, d\underline{r} \tag{6}$$

In (6) the integration over the angles concerns only $\Delta\varphi$. This enables us to average apriori $\Delta\bar{\rho}$ on these variables and (5) becomes:

$$\Delta\varphi = 4\pi\left[\int_r^\infty \Delta\bar{\rho}(r')r'dr' - 1/r\int_r^\infty \Delta\bar{\rho}\, r'^2 dr'\right] \tag{5'}$$

Introducing (5') and (2) in (6)one obtains after partial integration:

$$\Delta E = 16\pi^2 e^2 \sum_i \frac{a_i}{(i+2)(i+3)} \int_0^\infty \Delta\rho \, r^{i+4} dr \tag{7}$$

with the condition 7/

$$\lim_{u\to\infty} u^k \int_u^\infty \Delta\rho \, v^m dv = \lim_{r\to\infty} r^{k+m+1} \Delta\rho(r) = 0 \tag{8}$$

(2) has been written under the form

$$\varphi^2 = \sum_i a_i r^i \tag{2'}$$

7/ This condition is satisfied by all charge distributions with physical meaning, for arbitrary k and m.

ρ is the charge distribution related to the charge density $\bar{\rho}$ by the equation

On account of the numerical value of the coefficients $a_i/[(i+2)(i+3)]$, only the first term in (8) is important (the next terms give corrections less than 5%). In this approximation (8) becomes

$$\Delta E = \frac{2}{3}\pi e^2 a_0 \,\Delta < R^2 > = \frac{2}{3}\pi e^2 C \,\Delta < R^2 > \tag{9}$$

where $< R^2 >$ denotes the nuclear electromagnetic average radius in a given state. This result does not depend on a special nuclear model. Such a model must be applied to calculate $\Delta < R^2 >$,

2. THE SHELL MODEL

For single-particle levels of odd nuclei the excitation of the nucleus means by definition the transition of the optical nucleon from the ground level to the corresponding excited level.

Neglecting interparticle interactions the protonic charge distribution is given by

$$\rho^p = \sum_{i=1}^{Z} \varphi_i^{p*} \varphi_i^p \tag{10}$$

where φ_i^p are the proton wave functions given by the model and the summation in (10) takes account on the exclusion principle.

The last occupied level defines the nuclear state. By excitation, in an odd-even nucleus only the optical proton changes its state. This means:

$$\Delta \rho^{p} = \rho^{p}_{exc} - \rho^{p}_{gr} = \pm \left(\varphi^{p*}_{exc} \varphi^{p}_{exc} - \varphi^{p*}_{gr} \varphi^{p}_{gr} \right) \qquad (11)$$

where the indexes exc and p respectively gr and p denote the excited respectively ground level of the proton. From (11) and (7) follows the fundamental result that the absolute magnitude of the shift $|\Delta E|$ depends only on the charge distributions difference of the optical protons. In other words the shift reslizes a "filtration" of the last two single-particle states of the nucleus. This direct consequence of assumptions (A) and (B) distinguishes the isomeric shift from all the other phenomena which depend on the nuclear charge distribution and may become of importance in the study of nuclear structure and especially in the research of the nuclear surface layer.

From (11) follows immediately that $\Delta\langle R^2\rangle$ reduces to $\Delta\langle r^2\rangle p$ where $\langle r^2\rangle p$ is the quadratic mean radius of the optical proton, and formula (9) becomes:

$$\Delta E = 2/3\, \pi e^2 C \Delta \langle r^2 \rangle p \qquad (12)$$

The sign in (11) depends on the concrete nuclear configuration and cannot be predicted apriori except in two cases:

1. if there exists a single particle out of closed shells (or subshells) the sign is + ; 2. if there exists a single hole the sign is −.[8/,9/].

THE NUCLEAR POTENTIAL

To obtain from (12) some more concrete informations about the shift, we must calculate $\Delta\langle r^2\rangle_p$. This calculation makes necessary the knowledge of the nuclear single-particle functions φ^p which depend on the nuclear potential. In this connection one must distinguish spherical potantials and nonspherical ones. For deformed nuclei we shall consider only the Nilsson nonspherical potential; this is the most complete nonspherical potential; for which concrete single-particle wave functions were available. We shall see that in our particular problem the deformation corrections given by this potential are negligible. As concerns the spherical potentials, we shall deal with the harmonic oscillator (osc), the rectangular infinite well (i.w.) and the diffuse well (d.w.) in the form studied by Ross, Mark and Lawson.

8/ This question will be discussed by the author at greater length in another paper.

9/ In|1| the influence of the configuration on the sign of Δ_p respectively ΔE was not taken in account. The correct signs are those given here.

Two main conclusions can be drawn from the study of these potantials:

1. The sign of the shift does not depend on the concrete form of the nuclear potential.

2. There exists a very simple relation between the magnitude and the sign of the shift on one hand, and the quantum numbers of the nuclear states, on the other; this relation enables the formulation of a sign rule and may perhaps be used to determine from future experimental data, unknown nuclear states.

a. The Nilsson potential. The Nilsson functions[10/] $|N\Omega\alpha>$ may be expressed by the harmonical oscillator functions $|N\ell,\Omega\pm 1/2>$ as follows:

$$|N\Omega\alpha> = \sum_{\ell} a_{\ell,\Omega\pm 1/2} \, |N\ell,(\Omega\pm 1/2)\mp> \tag{13}$$

where N is the total quantum number of the oscillator, Ω the component of the total angular momentum along the nuclear axis and α designs the proper values. The coefficients $a_{\ell\Omega}$ are normalized to 1:

$$\sum a^2_{\ell,\Omega\pm 1/2} \tag{14}$$

From (13) and (14) we get:

$$<r^2>_{Nilsson} = \sum_{\ell} a^2_{\ell\Omega\pm 1/2} <r^2> \tag{15}$$

10/ S. Nilsson: Kongl. Danske Vidensk. Selsk. Mat. Fys. Medd. 29, Nr. 16, 1955.

on account of the independence of $\langle r^2 \rangle_{osc}$ on ℓ (see point b of this paragraph). We see thus that in the approximation (12) the Nilsson potential gives the same ΔE as the osc. The deformation corrections appear only in the superior powers of r, and are negligible.[8]

b. The harmonic oscillator potential. In this case $\Delta \langle r^2 \rangle_p$ can be calculated immediately. We have:

$$\langle U \rangle = \tfrac{1}{2} \mathcal{E} \qquad (16)$$

where U is the potential energy[11] and $\mathcal{E} = \hbar\omega(N+3/2)$ the osc. quantum energy. From the particular form of U, follows:

$$\Delta \langle r^2 \rangle_p = r_0^2 \Delta N = \frac{\Delta \mathcal{E}}{m\omega^2} \qquad (17)$$

where $r_0 = (\hbar/m\omega)^{1/2}$ is the characteristic length of the osc., ω the associated frequence, m the nuclear mass and $\Delta \mathcal{E}$ the excitation energy.

From (17) and (12) follows directly that the measurement of ΔE is equivalent with the measurement of $\Delta \mathcal{E}$. It is difficult however to accord great importance to this result since we know very well how poor are the nuclear energy-predictions of the osc. From (16) we get also

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11/ We neglect the Coulomb interaction in the nuclear potential U_{osc}.

the following sign rule: at a given nuclear configuration the sign of Δ E depends only on the relation N $\gtrless$ N'.

In isomeric single-particle transitions Δ N is usually $\pm$ 1.[12] and thus $\Delta \langle r^2 \rangle = \pm r_0^2$.

c. <u>The rectungular infinite well.</u> Using the Bessel eigenfunctions of this potential and applying the Schaffheitlin integration formulae we get:

$$\Delta \langle r^2 \rangle_{iw} = \tfrac{2}{3} R^2 \Delta \frac{\ell^2 + \ell - 3/4}{\omega^2} \tag{18}$$

where $\omega_{n\ell}$ is related to the i.w. energy $\varepsilon_{n\ell}$ by the equality:

$$\omega_{n\ell} = \left(2 m \varepsilon_{n\ell} R^2 / \hbar^2 \right)^{1/2} \tag{19}$$

In a first approximation we may put $\omega_{n\ell} \simeq \omega^r_{n'\ell'}$; then (18) becomes

$$\langle r^2 \rangle_{iw} = \frac{2}{3} \frac{R^2}{\omega^2} \Delta \left[\ell (\ell + 1) \right] = \frac{2}{3} \frac{R^2}{\hbar^2 \omega^2} \Delta (M^2); \; \Delta E_{iw} \sim \Delta (M^2) \tag{20}$$

where M^2 is the orbital angular momentum of the optical proton. Formula (19) is analogous to (17); a measurement of Δ E is on principle a measurement of M. Formula (20) represents also a new aspect of the sign rule ($\ell \gtrless \ell'$).

The rectangular finite well given a similar result but

12/ M. Korsunsky: The isomerism of atomic nuclei (in russian) G.I.T.T.L. Moscow 1954.

the corresponding formulae become far more complicated. The quantitative results do not differ by more than little percent from those given by (20), showing that the contribution of the region corresponding to $r > R$ is effectively neglegible.

d. <u>Diffuse (realistic) potential. Numerical applications</u>[13/]

The harmonic oscillator and the square well are the two extremities of the nuclear potential. It has been pointed out[14/] that the "real" potential should have a diffuse character.

Ross, Bark and Lawson have solved at the UCRL differential analyzer the Schrödinger equation for the potential

$$V = -V_0 / [1 + \exp \alpha (r - R)] \qquad (21)$$

taking in account the spin orbit interaction and the Coulombian repulsion for protons[15/]. This is the most complete d.w. potential for which eigen-functions are available and therefore we have compared the numerical values of Δ E given by these eigen-functions with those corresponding to osc. and i.w.[8/] Two characteristic transitions have been considered here. The first

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13/ The numerical results given here have only an illustrative purpose; as long as no experimental data exist, it seems a little exaggerated to enter in more quantitative details.

14/ See for example: W. Heisenberg "Theorie des Atomkerns" Göttingen 1951.

15/ A. Ross, H. Mark, R. Lawson: Phys. Rev. 102, 1613 (1956) and private communication.

$(2p_{1/2} - 1g_{9/2})$ is characteristic for medium isomers with single-particle levels; the second one $(1h_{11/2} - 2d_{3/2})$ for heavy nuclei. The parameter r_0 of osc. has been calculated for In from the total nuclear distribution as in[1]. For the other elements r_0 has been taken[16] $\sim R^{1/2}$ starting from the value of $r_0 = 2{,}5 \cdot 10^{-13}$ cm In for In.

For i.w. the eigen-values ω_{nl} given by Feenberg[17] have been used. The nuclear radius has been taken as in[1]: R = 1,2 x $10^{-13} \cdot A^{1/3}$ cm For In^{115} (first kind of transition)[18] we get:

$$\Delta E_{d.w.}/\Delta E_{osc} \simeq 1.1 \quad ; \quad \Delta E_{i.w}/\Delta E_{osc} \simeq 1.0 \tag{22}$$

For Au^{197} (second kind of transition) we get:

$$\Delta E_{d.w}/\Delta E_{osc} \simeq 1.3 \quad ; \quad \Delta E_{i.w.}/\Delta E_{osc} \simeq 1.2 \tag{23}$$

Taking in account the errors in the choice of the parameters involved in these potentials, the results (22) and (23) must be considered quite satisfactory.

To obtain absolute values for ΔE , $\psi^2_{Sch}(0)$ must be de-

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16/ M. Mayer, J. Jensen: Elementary theory of nuclear shell structure - N. York-London 1955, page 236.

17/ E. Feenberg: Shell theory of the nucleus. Princeton 1955 page 15.

18/ For this element the corresponding Cd^{116}_{50} d.w. functions have been used.

termined from experimental spectroscopical data. We have used the h.f.s. data of the ground state given in [19] for In^{115}_{III} and in[20] for Au^{197}_{I} applying the corresponding Goudsmit-Fermi-Segre formulae. For In^{115}_{III} the shift of the $5s\ ^2S_{1/2}$ electronic ground state term has been calculated; for Au^{197}_{I} the shift of the $6s\ ^2S_{1/2}$ ground state term.

The configuration for In^{115} is known in both nuclear states. The incomplete shell has the following occupation numbers :

$$(1f\,7/2)^8,\ (1f\,5/2)^6,\ (2p\,3/2)^4,\ (2p\,1/2)^2,\ (1g\,9/2)^2\ (g.r\ state),$$

$$\text{"}\qquad\text{"}\qquad\text{"}\qquad (2p\,1/2)^1\ (1g\,9/2)^{10};\ (en\ state) \qquad (24)$$

This given[9] with the osc. potential

$$\Delta E\ In^{115} = E_{exc} - E_{gr} \simeq +4 \times 10^{-2}\ cm^{-1} \qquad (25)$$

For Au^{197} only the ground state configuration is known[21]:

$$(1g\,7/2)^8,\ (2d\,5/2)^6,\ (3s\,1/2)^2,\ (1h\,11/2)^{10};\ (2d\,3/2) \qquad (26)$$

The excited state may be either;

19/ J. Campbell, J. Davis: Phys. Rev. 55, 1125 (1939).
20/ G. Welsel , H. Zew: Phys. Rev. 92, 641 (1953).
21/ H. Zeldes: Nucl. Phys. 2,1, (1956)/57).

$$(1g_{7/2})^8, (2d_{5/2})^6, (3s_{1/2})^2, (1h_{11/2})^{11}, (2d_{3/2})^2 \tag{27}$$

or:

$$(1g_{7/2})^8, (2d_{5/2})^6, (3s_{1/2})^2, (1h_{11/2})^9, (2d_{3/2}) \tag{28}$$

and the sign of ΔE cannot be given apriori. The numerical result is, using the osc. potential,

$$\Delta E \simeq \pm 10^{-1} cm^{-1} \tag{29}$$

II. EVEN - ODD NUCLEI

(ELECTRON -NEUTRON INTERACTION)

According to assumption (A) the excitation of these nuclei is realised by the transition of the optical neutron from the ground state to the excited state. This gives rise to a variation of the neutron distribution perfectly analogous to (11):

$$\Delta \rho^n = \pm \left(\varphi_{exc}^{n\,*} \varphi_{exc}^{n} - \varphi_{gr}^{n\,*} \varphi_{gr}^{n} \right) \tag{30}$$

Although the excitation does not modify the protonic charge distribution, the electron-nucleus interaction energy however is changed, due to the electron-neutron (e-n) interaction. If we assume for this interaction the potential (1), the variation of the electron-nucleus energy by excitation, i.e. the isomeric shift, will be

$$\Delta E = b \int \Delta \rho_n \varphi^2 d\underline{r} \tag{31}$$

Using for φ^2 the expression (2), we find that the first term of (2) vanishes in (31) and disregarding the third term in comparison with the second for the same reasons as in the corresponding formula (12) of Section I we get

$$\Delta E = -\frac{1}{2} b C \gamma^2 \Delta < r^2 >_n \quad . \tag{32}$$

This expression is very similar to the above quoted one (12), and the same considerations may be made here on the general trends of the shift. In eddition it must be emphasized here that the isomeric shift at even-odd nuclei, is by the same substraction process (30) a pure $e-n$ effect, and thus distinct from the other phenomena studied until present where this weak interaction is masked by other, much stronger effects. (In the isotopic shift by the ceulombian electron-nucleus interaction, in the scattering phenomena by the nuclear interaction).

To obtain an order of magnitude for ΔE, we assume that b has the value given by neutron-electron scattering. An average value of 4000 eV for the well depth of the interaction potential with the radius equal to the classical electron radius[22/], gives $b \simeq -6 \times 10^{-46} erg \cdot cm^3$ erg. cm^3. Using the osc. potential and the electron data from[23/] for Hg II (for the isotope I99) and from[24/]) for $Pb^{210}\ \underline{IV}$ we obtain for the electronic ground state $6s\ S_{1/2}$ the following shifts:

$$\Delta E_{Hg} \simeq 10^{-4} cm^{-1}; \quad \Delta E_{Pb} \simeq 2 \cdot 10^{-4} cm^{-1} \tag{33}$$

In connection with the above values the remark[I3] is in so far more important here as the electronic data used in this section are probably affected by greater errors; the same concerns the constant the constant. In addition to this quantitative aspect of the problem, it must be emphasized that the above qualitative considerations have a rather speculative character as compared with those of the first Section.

This is clear from the the following arguments:

a) The e-n interaction potential is a very poor description of the real phenomena with which we are here faced. Besides that, the strictly local character of the e-n potential makes the value of E very sensitive to the form of the electronic functions. It is of interest to mensition for example that the Racah functions give a Δ E increased by a factor 3-5 in comparison with (32). This fact may be partly explained by the weak divergence of the Racah functions in

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22/ B.Feld in "Experimental Nuclear Physics" Vol. II(Editor E. Segre) N. York-London (1953).

23/ H. Kopfermann- Kernmomente - Second edition. Frankfurt a. M. (I956) page 123.

24/ H. Kopfermann - Kernmomente - First edition. Leipzig (I940) page 77.

the origin, but it is also not impossible that our perturbation mechanism is not applicable in this particular case, although the interaction is weak.

b) The constant b can calculated at present only from scattering experiments where the discussed interaction is a free neutron-electron interaction. In the isomeric shift (and in the isotopic shift) we have a bound neutron-electon interaction and it is probable that the corresponding interaction constant shell have a different value in our case. This results from the existence of an effective nucleon mass in the nucleus, different from the free nucleon mass and from the fact that the main part of the e-n interaction is due to the magnetic moment of the neutron[25/]. The many-body interaction in the nucleus gives rise to a modified magnetic moment of the nucleon[26/] and thus the e-n interaction in the nucleus will also be modified. A satisfactory quantitative approach to this question does not yet exist.

c) It is possible that the exchange effects should not be rigorously compensated in (30) respectively (11). At even-odd nuclei this may be of greater importance, as the transition

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25/ L. Foldy: Phys. Rev. 87, 693 (1952).

26/ J. Bell, R. Eden, T. Skyrme: Nucl. Phys. 2, 586 (1956/57), J. Bell: Nucl. Phys. 4, 295, (1957).

probabilities suggest.

Although these three arguments do not exhaust the criticism of our approach, we shall confine ourselves to these ones because of the same reason as that quoted in[13/]. In conclusion it can be said that the arguments a) b) and c) and the fact that we might here be faced with a pure e-n effect, show tha great interest of an experimental test of the isomeric shift at even-odd nuclei.

III. ON THE EXPERIMENTAL PROOF OF THE EFFECT. FINAL DISCUSSION

The experimental study of the short and medium-life nuclear states is a rather complicated problem both for nuclear and atomic spectroscopy. As concerning the latter two specific difficulties (these are not the single ones!) may be quoted: 1. the relatively big amount of substance necessary in these experiments; 2. the overlapping of the h.f.s. spectra corresponding to the excited and ground nuclear states. In the special case of the isomeric shift at even-odd nuclei. a third difficulty must be added: the probable smallness of the effect.

. As the author knows the first and until now the only atomic spectrum of an <u>excited isomer</u> has been obtained experimentally by Bitter and collaborators (Hg^{197m}, halflife ~ 23 h) by a very ingenious optical and magnetic scanning method[27/]. But on account of the overlapping of the Hg^{197m} and Hg^{197} h.f.s. spectra on one hand, and the spectra of the other Hg isotopes on the other, the h.f.s. of Hg^{197m} could not be resolved satisfactorily and no conclusion on the excited state, except the probable 13/2 value of the spin, could be drawn. It is clear heerfrom that no question of isomeric shift could be raised, because the h.f.s. is a necessary condition for the measurment of the isomeric shift-which concerns the gravity centers of the h.f.s. corresponding to the nuclear states involved.

Further work along these lines has been done by the Berkeley group using a magnetic resonance method with atomicbeams, but again no attempt to detect an isomeric shift was or could be made in these experiments[28/].

27/ F. Bitter, H. Plotkin, B. Richter, A. Teviotdsle,J. Young: Phys. Rev. <u>91</u>, 421, (1953); F. Bitter, S. Davis, B. Richter, J.Young: Phys.Rev. <u>96</u>, 1531 (1954).

28/ J. Hobson, H. Hubles, W. Nierenberg, H. Silsbee, R.Sunderland: Phys.Rev. <u>104</u>, 101, (1956); G. Brink, J. Hubbs,W.Nierenberg, J. Worcester: Phys. Rev. <u>107</u>, 1891, (1957).

29). J. Brossel, F. Bitter: Phys. Rev. <u>86</u>, 308 (1952).

30). F. Bitter - private communications.

It is possible that some progress in this direction could be obtained by a chemical sppparation of <u>isomers.</u>

At present F. Bitter and collaborators study the h.f.s. of Hg^{197m} and Hg^{197} by a "double resonance" method similar to that described in [29]. They intend also to get some information on this occasion on the isomeric shift.[30]

The experimental research of the iomeric shift may bring new data on the excitation mechanism and on the nuclear configuration and may be a strong test of the validity of assumptions A and C. By this method one could obtain also for the first time some information on the nuclear readii of expited states. But the experimental study of the effect, presents, also per se, great interest since this effect, togehter with the n.f.s. effect could lay the basis for a new method in the investigating of nuclear excited states characteristics. In spite of the technical difficulties, some of which were quoted above, the methods of atomic spectroscopy are simpler than the corresponding nuclear ones.

We did not intend to discuss all the various problems which

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29) J. Brossel, F. Bitter: Phys. Rev. <u>86</u>, 308 (1952).

30) F. Bitter - private communications.

may be put in connection with the isomeric shift, but confined ourselves only to those which seemed the most striking in the present stage when no experimental data are yet available. We hope that the present study will incite such experiments.

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