P-20

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RECONSTRUCTION OF THE SCATTERING MATRIX OF A TWO-NUCLEON SYSTEM

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NUCLEON SYSTEM

A b s t r a c t: The problem of the number of experiments which are necessary for the determination of the elements of the elastic scattering matrix is discussed. It is shown that in virtue of the unitarity condition the required number of experiments equals the number of complex functions entering the scattering matrix. In the case of nucleon-nucleon scattering the elasting matrix can be determined on basis of 5 experiments: measurement of the cross section, polarization, normal component of the polarization correlation tensor and the normal components of the triple scattering tensor (for both particles). It is shown that experiments with rotation of polarization by the external magnetic field are not necessary for phase-shift analysis.

I. Introduction

At the present a large amount of experimental data on nucleon-nucleon scattering in a very broad energy region has been published*.

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* A good review of recent data has been written by Wolfenstein[1]. The authors take the opportunity to thank Prof. Wolfenstein for sending his manuscript prior to publication.

A number of papers have appeared which are devoted to phase analysis of the experimental results. However, it has invariably been found in these investigations that the phase-shift analysis is not unique and usually the authors give several sets of phase-shifts consistent with the experimental results. Phase-shift analysis in these works starts by restricting the number of states involved in scattering. The next step is to set up algebraic equations for a finite number of phases. However, for particles possessing a spin it remains unclear which experimental data should be used to write the equations. A good example is proton-proton scattering at not too high energies.

In this case two states $^1S_0$ and $^3P_0$ are predominant in the scattering. These states cannot be separated on basis of cross section measurements or polarization measurements. On the other hand, they can easily be differentiated by measuring the polarization correlation. The necessity of more intricate experiments is apparent in this case. In other cases, when scattering is not isotropic and it may formally be possible to determine a finite number of phases, the question still remains whether the same phases are sufficient to describe more complex scattering phenomena, such as plural scattering.

In this connection it seemed important to attempt an analysis of all possible types of experiments and to find out which of them are independent in the sense that complete reconstruction of the scattering matrix would be possible if the results of these experiments were known. In order to illustrate the line of reasoning applied here we shall start with the two simplest cases-scattering of zero spin particles in a central force field and scattering of spin 1/2 particles on zero spin nuclei. After this, scattering of nucleons by nucleons will be considered.

The case of arbitrary spin, which at present is only of theoretical interest, and photon scattering, will be considered in further communications. We shall moreover restrict our attention to a scattering matrix for a given energy. Further investigation is required to clarify the problem of the energy dependence of the matrix elements.

2. Scattering of Zero Spin Particles

Measurement of the differential cross section of a zero spin particle yields a function $\sigma(\vartheta)$, which is the square of the modulus of the scattering amplitude

$$\sigma(\vartheta) = |f(\vartheta)|^2 \tag{2.1}$$

Evidently, if this quantity is measured at only one value of energy and angle ϑ the phase of the complex function $f(\vartheta)$ will remain completely indeterminate.

If, however, scattering is measured at all angles and is known to be elastic, the phase of $f(\vartheta)$ can be determined also. Indeed, the unitarity relation for the amplitude $f(\vartheta)$ can be written as follows[2)]

$$4\pi \, \mathrm{Im} f(\vartheta) = k \int f^*(\vartheta'') f(\vartheta') \, d\omega \tag{2.2}$$

Here ϑ' is the angle between $\vec{k}$ (initial wave vector) and the variable vector $\vec{k}''$ over direction of which $(d\omega)$ the integration is carried out. ϑ'' is the angle between the final wave vector $\vec{K}'$ and vector $\vec{K}''$. Denoting

$$f(\vartheta)=\sqrt{\sigma(\vartheta)}\exp[i\alpha(\vartheta)] \qquad (2.3)$$

we obtain from (2.2), if $\sigma(\vartheta)$ is known, an integral equation for function $\alpha(\vartheta)$

$$4\pi \sin\alpha(\vartheta)=K_0 \int\left[\frac{\sigma(\vartheta')\sigma(\vartheta'')}{\sigma(\vartheta)}\right]^{1/2}\cos[\alpha(\vartheta')-\alpha(\vartheta'')]\,d\omega \qquad (2.4)$$

The solution of the set of equations (2.1) and (2.2) or equation (2.4) is an operation which is equivalent to phase-shift analysis. It is evident that representation of $f(\vartheta)$ in the form of a finite sum of Legandre polynomials is only one possible method of solving the problem which is useful when it be known that only a few phase-shifts are involved in the solution. In the general case straight-forward solution of (2.4) may turn out to be a more convenient procedure.

It can be seen from (2.4) that the equation is invariant with respect to the substitution

$$f(\vartheta)\to -f^*(\vartheta)$$

or, what is the same,

$$\alpha(\vartheta)\to\pi-\alpha(\vartheta). \qquad (2.5)$$

This transformation corresponds to change of the signs of all scattering phase shifts. Its existence signifies that there are two ways of reconstructing the scattering amplitude on basis of its modulus. As is well known, this ambiguity can be removed by considering interference with Coulomb scattering(for charged particles) or by studying the energy dependence of scattering at low energies. Thus, measurement of the scattering cross section at a given energy is complete in the sense that it permits one to reconstruct completely the scattering amplitude (perhaps with the aforementioned two possibilities remaining).

3. Scattering of Particles of Spin $\frac{1}{2}$

The scattering amplitude of a particle of spin $\frac{1}{2}$ can be written, as usual, in the form

$$\mathcal{M} = a(\vartheta) + b(\vartheta)\vec{\sigma}\cdot\vec{n} \qquad (3.1)$$

wher $\vec{\sigma}$ is the Pauli matrix and $\vec{n} = \frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}'|}$ is the normal to the scattering plane.

Experimentally, the cross section and polarization (double scattering) are measured, thus yielding two functions

$$\sigma(\vartheta) = |a|^2 + |b|^2 = \frac{1}{2}(|f_+|^2 + |f_-|^2),$$
$$\mathcal{P}(\vartheta)\sigma(\vartheta) = 2\,\mathrm{Re}\, ab^* = \frac{1}{2}(|f_+|^2 - |f_-|^2). \qquad (3.2)$$

Evidently, (3.2) defines two moduli*

* It should be noted that (3.3) are the scattering cross sections for polarized particles with spins directed "up" or "down" (with respect to $\vec{n}$).

$$|f_+|^2 = |a+b|^2,$$
$$|f_-|^2 = |a-b|^2. \qquad (3.3)$$

If the measurements are carried out at all possible angles and scattering is elastic the phase shifts of complex functions $f_+(\vartheta)$ and $f_-(\vartheta)$ will again be defined by the unitarity conditions

$$4\pi\, \mathrm{Im}\, a(\vartheta) = \kappa \int d\omega \left[a^*(\vartheta'') a(\vartheta') + b^*(\vartheta'') b(\vartheta') \frac{\cos\vartheta - \cos\vartheta' \cos\vartheta''}{\sin\vartheta' \sin\vartheta''} \right],$$

$$4\pi\, \mathrm{Re}\, b(\vartheta) = \kappa \int d\omega \left[2\, \mathrm{Im}\, a^*(\vartheta'') b(\vartheta') \frac{\cos\vartheta \cos\vartheta' - \cos\vartheta''}{\sin\vartheta \sin\vartheta'} + b^*(\vartheta'') b(\vartheta') \frac{1 + 2\cos\vartheta \cos\vartheta' \cos\vartheta'' - \cos^2\vartheta - \cos^2\vartheta' - \cos^2\vartheta''}{\sin\vartheta \sin\vartheta' \sin\vartheta''} \right]. \quad (3.4)$$

Equations (3.2) and (3.4) are also amenable to an invariant transformation.

In order to find the latter, we note that the transformation (inversion of sign of the scattering phase shifts)

$$a(\vartheta) \to -a^*(\vartheta), \quad b(\vartheta) \to b^*(\vartheta) \quad (3.5)$$

does not after the cross sections or unitarity conditions, but reverses the sign of the polarization.

The same statement also applies to the Minami transformations[3,4)]

$$a(\vartheta) \to a(\vartheta)\cos\vartheta + i\, b(\vartheta)\sin\vartheta$$
$$b(\vartheta) \to -b(\vartheta)\cos\vartheta - i\, a(\vartheta)\sin\vartheta \quad (3.6)$$

Thus, the product of the two transformation

$$a(\vartheta) = -a^*(\vartheta)\cos\vartheta + i\, b^*(\vartheta)\sin\vartheta,$$
$$b(\vartheta) \to -b^*(\vartheta)\cos\vartheta + i\, a^*(\vartheta)\sin\vartheta \quad (3.7)$$

leaves all the quantities invariant.*

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* In terms of the phase shifts $\delta(j, \ell)$ this transformation is corresponded to by

$$\delta(j, j - 1/2) \rightleftarrows -\delta(j, j + 1/2)$$

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Therefore, in the case of particles of spin 1/2 a complete experiment consists in measurement of the cross section and polarization. Ambiguity in phase shift determinations due to the presence of transformation (3.7) can be removed by studying the energy dependence of the cross section at low energies. It should be mentioned that (as previously noted by Wolfenstein[5]) the ambiguity can also be removed by studying triple (in which the scattering planes are mutually perpendicular).

4. Scattering of Nucleons by Nucleons

Nucleon-nucleon scattering is described by the matrix

$$\mathcal{M} = \alpha(\vartheta) + \beta(\vartheta)\vec{\sigma}_1\cdot\vec{n}\,\vec{\sigma}_2\cdot\vec{n} + \gamma(\vartheta)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{n} + \delta(\vartheta)\vec{\sigma}_1\cdot\vec{m}\,\vec{\sigma}_2\cdot\vec{m} + \varepsilon(\vartheta)\vec{\sigma}_1\cdot\vec{\ell}\,\vec{\sigma}_2\cdot\vec{\ell}. \qquad (4.1)$$

Here $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli matrices of the two particles and $\vec{m}$, $\vec{\ell}$, $\vec{n}$ are Cartesian unit vectors which are correspondingly parallel to $\vec{K}-\vec{K}'$, $\vec{K}+\vec{K}'$, $\vec{K}\times\vec{K}'$ (in the c.m.s.)[6]. That this system is a convenient one can be seen from the fact that its unit vectors will be parallel to the laboratory system wave vectors of the two particles after scattering and to the normal to the scattering plane.

Expression (4.1) differs from the general scattering matrix for two spin 1/2 particles by the absence of a term which is proportional to $(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{h}$. This follows from identity of the particles of the p-p system and from charge invariance of the n-p system.

For identical particles the coefficients of matrix (4.1) possess the property of symmetry with respect to the substitution $\vartheta \to \pi - \vartheta$. They can also be conventiently formulated by introducing new functions which will also be useful in the future[7])

$$a=\alpha+\beta,\ b=\alpha-\beta,\ c=\delta+\varepsilon,$$
$$d=\delta-\varepsilon,\quad e=2\gamma \tag{4.2}$$

It can then be shown that for identical particles

$$a(\pi-\vartheta)=-a(\vartheta),\ b(\pi-\vartheta)=-c(\vartheta),$$
$$c(\pi-\vartheta)=-b(\vartheta),\ d(\pi-\vartheta)=d(\vartheta),$$
$$e(\pi-\vartheta)=e(\vartheta). \tag{4.3}$$

Thus for identical particles the measurements may be carried out only for angles $0 \leftrightarrow \frac{\pi}{2}$. In the case of scattering of neutrons by protons the measurement range should be doubled to $0 \leftrightarrow \pi$, which exactly corresponds to doubling of the number of states in this system.

Before using the unitarity condition we shall briefly describe the experiments required for a two-nucleon system.

Suppose we are dealing with an unpolarized nucleon beam; the first scattering on hydrogen will define a differential cross section which is connected with the scattering matrix elements by the formula*

* The formulae of this section are familiar from the literature.

$$\sigma(\vartheta)=\frac{1}{2}\left(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2\right). \qquad (4.4)$$

The first scattering produces a beam polarized along $\vec{n}$. The magnitude of this polarization is measured on the second scattering

$$\sigma(\vartheta)\mathcal{P}(\vartheta)=\mathrm{Re}\, a e^*. \qquad (4.5)$$

Evidently any target with known properties (polarizer) can be used for measuring or for obtaining of the polarization. (In all experiments only one target might be a hydrogen one).

The polarization of recoil particles is also equal to $\mathcal{P}(\vartheta)$ (absence of $(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{n}$ in $\mathcal{M}$) and hence should not be measured.

The third target may be added in two ways.

Measurement of the component $\mathcal{P}_{nn}$ of the correlation tensor yields

$$\sigma(\vartheta)\mathcal{P}_{nn}(\vartheta)=\frac{1}{2}\left(|a|^2-|b|^2-|c|^2+|d|^2+|e|^2\right). \qquad (4.6)$$

In these measurements all three scattering events occur in a single plane. Moreover, the polarization after the second scattering on the hydrogen target can be measured. By using a third target to measure it (any analyser can be used for this) we obtain two quantities corresponding to the two particles participating in the second scattering*.

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* Since the inoident particle in this scattering was polarized, the polarization of the two particles after scattering will now be different.

If all three arrangements of scattering events occur in a single planoe, the measured quantities will be

$$\sigma(\vartheta)\mathcal{D}_{nn}(\vartheta)=\frac{1}{2}\left(|a|^2+|b|^2-|c|^2-|d|^2+|e|^2\right) \qquad (4.7)$$

(for the scattered particle)

$$\sigma(\vartheta)K_{nn}(\vartheta)=\frac{1}{2}\left(|a|^2-|b|^2+|c|^2-|d|^2+|e|^2\right) \qquad (4.8)$$

(for the recoil particle)

The difference between these two quantities is obvious in the case of neutron scattering; for identical particles the first quantity corresponds to measurement in the angular interval $0 \longleftrightarrow \frac{\pi}{2}$ and the second in the interval $\frac{\pi}{2} \longleftrightarrow \pi$.

The aforementioned experiments yield five equations from which the moduli of the five functions $a+e,\ a-e,\ b,\ c,\ d$ can easily be determined and we have to apply the considerations similar to the case of spin one half particles. In order to determine the phases of these complex functions measured at the various angles one appeals to the unitarity relation. It can be shown (see ref.[8]) that exactly five such relations exist. (These relations [see appendix 2] are invariant with respect to the substitution $\alpha(\vartheta)\to-\alpha^*(\vartheta)$, $\beta(\vartheta)\to-\beta^*(\vartheta)$, $\gamma(\vartheta)\to-\gamma^*(\vartheta)$, $\delta(\vartheta)\to-\delta^*(\vartheta)$, $\varepsilon(\vartheta)\to-\varepsilon^*(\vartheta)$ which is equivalent to simultaneous reversal of the signs of all phase shifts (as in the preceding case-this substitution changes the polarization sign); in the case of nucleon-nucleon scattering no substitution corresponding to the Minami transfromation exists[9]. This signifies that the set of experiments indicated above is a complete set.

An important feature of the preceding considerations is the conclusion that in order to determine the scattering matrix measurement of fourfold scattering of application of a magnetic field are not necessary and it should suffice to perform experiments with three targets and with parallel scattering planes.

Up to the present no such complete set of experiments has been performed for any single energy value and this is the reason of the ambiguity encountered in analyzing these experiments.

In connection with the problem of determining the scattering matrix, the question arises as to its connection with the potential. A salient feature is that the number of independent functions in the scattering amplitude is equal to the number of scalar functions in the interaction hamiltonian. Indeed, it is not difficult to show that the general form of interaction of (say) two protons is

$$V=V_1(\tau)+V_2(\tau)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_3(\tau)\vec{\sigma}_1\cdot\vec{\tau}\,\vec{\sigma}_2\cdot\vec{\tau}+$$
$$+V_4(\tau)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{L}+V_5(\tau)\vec{\sigma}_1\cdot\vec{L}\,\vec{\sigma}_2\cdot\vec{L}$$

where $\vec{L}$ is the orbital angular momentum. It would be very interesting to ascertain what restrictions are imposed on the scattering matrix if one or several $V_i(r)$ are zero. In other words, one is interested to ascertain whether any qualitative conclusions regarding the form of $\overset{\circ}{V}$ can be made on basis of the scattering matrix properties.

In conclusion the authors wish to thank Professor E. Segré and Professor O. Chamberlain for numerous interesting and enlightening discussions of nucleon scattering experiments, carried out in the summer of 1956.

A P P E N D I X I

List of Formulae for Possible Nucleon-Nucleon Scattering E x p e r i m e n t s

As the set of experiments (five in all) mentioned above for determination of the nucleon-nucleon scattering matrix is not the only possible one we shall present here a short survey of all conceivable experiments. The latter may differ from each other first of all in respect to the state of polarization of the primary beam and target and, secondly, in the nature of the measured quantities (cross section, polarization of scattered particle, polarization of recoil particle, correlation of polarization). Schematically, they may be represented by the following table:

Initial spin state / Result of experiment	A. Unpolarized beam-unpolarized target	B. Polarized beam-unpolarized target	C. Unpolarized beam-polarized target	D. Polarized polarized
1. Cross section	σ *	$\sigma_i^{(1)}$	$\sigma_k^{(2)}$	σ_{ik}
2. Polarization of scattered particle	$\mathcal{P}_p^{(1)}$ *	$\mathcal{D}_{ip}^{(1)}$ *	K'_{kp}	$T_{ikp}^{(1)}$
3. Polarization of recoil particle	$\mathcal{P}_q^{(2)}$	K_{iq} *	$\mathcal{D}_{kq}^{(2)}$	$T_{ikq}^{(2)}$
4. Correlation of polarizations	$\mathcal{P}_{pq}$ *	$\mathcal{P}_{ipq}^{(1)}$ *	$\mathcal{P}_{kpq}^{(2)}$	T_{ikpq}

Here each column corresponds to a certain initial spin state of the two-nucleon system and the scattering process characteristics to be measured are given in the horizontal rows. Subscript i indicates the direction of initial polarization of the projective, K refers to the initial target polarization, P refers to the measured component of the scattered particle polarization and q to the measured component of the recoil particle polarization. In the future we shall denote each experiment by a letter (column) and number (row) which will indicate the initial spin state of the system and the measured characteristic of the scattering process. Experiment B2, for example, consists in the measurement of a set of quantities $\mathcal{D}^{(1)}_{ip}$ which determine the influence of the i- polarization component of the incident particles on the p - polarization component of the scattered particles, etc.

Not all experiments in the table are different. Due to the symmetry properties some of them are actually identical. Thus, in the absence of singlet-triplet transitions (absence of term $(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{n}$) experiments A2 and A3, B1 andC1, B4 and C4, D2 and D3, B2 and C3 are essentially the same. In the case of identical particles (see symmetry properties (4.3)), experiments B2 and B3 yield the same quantities (B2) but for complementary angles (ϑ and $\pi-\vartheta$). A similar relation exists between experiments C3 and C2.

In virtue of invariance of matrix $\mathcal{M}$ with respect to reversal of time, the sets of experiments in those table cells which are symmetrical with respect to the main diagonal are equivalent (equivalency of experiments A2 and B1, A4 and D1, etc.). Thus the only experiments which are different are those denoted by the asterisk in the table. It can immediately be seen that a polarized target is required only in the most complicated experiment. However, for actual determination of the scattering matrix these experiments are also superfluous.

Consider now the different characteristics determined in the various experiments.

A.1. Measurement of cross section

$$\sigma(\vartheta)=\frac{1}{4}\,Sp\,\mathcal{M}\mathcal{M}^+=\frac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2)$$

A.2. Measurement of polarization

$$\sigma(\vartheta)\mathcal{P}(\vartheta)=\frac{1}{4}\,Sp\,\vec{\sigma}_1\cdot\vec{n}\,\mathcal{M}\mathcal{M}^+=Re\,a\,e^*.$$

B.2. Determination of the tensor

$$\sigma(\vartheta)\,\mathcal{D}_{ip}(\vartheta)=\frac{1}{4}\,Sp\,\mathcal{M}\,\sigma_{1i}\,\mathcal{M}^+\sigma_{1p}=$$
$$=\frac{1}{2}(|a|^2+|b|^2-|c|^2-|d|^2+|e|^2)n_i n_p+Re(a^*b+c^*d)m_i m_p+$$
$$+Re(a^*b-c^*d)\ell_i\ell_p-Im\,b^*e(m_i\ell_p-\ell_i m_p).$$

From experiments on triple scattering with parallel or perpendicular planes the following components are determined:

$$\sigma(\vartheta)\mathcal{D}_{nn}(\vartheta)=\frac{1}{2}(|a|^2+|b|^2-|c|^2-|d|^2+|e|^2),$$
$$\sigma(\vartheta)\mathcal{D}_{xm}(\vartheta)=-\cos\frac{\vartheta}{2}\,Re(a^*b+c^*d)+Im\,b^*e\,\sin\frac{\vartheta}{2}.$$

For determination of the components

$$\sigma(\vartheta)\mathcal{D}_{zm}(\vartheta)=\sin\frac{\vartheta}{2}\,\mathrm{Re}\,(a^*b+c^*d)+\cos\frac{\vartheta}{2}\,\mathrm{Im}\,b^*e,$$

$$\sigma(\vartheta)\mathcal{D}_{x\ell}(\vartheta)=\sin\frac{\vartheta}{2}\,\mathrm{Re}\,(a^*b-c^*d)+\cos\frac{\vartheta}{2}\,\mathrm{Im}\,b^*e,$$

either fourfold scattering must be measured or else triple scattering with a magnetic field between the targets. Unit vectors $\vec{z}$ and $\vec{x}$ are directed along $\vec{K}$ and $\vec{n}\times\vec{K}$ respectively.

B.3. Measurement of tensor

$$\sigma(\vartheta)K_{iq}(\vartheta)=\frac{1}{4}\,Sp\,\mathcal{M}\,\sigma_{1i}\,\mathcal{M}^+\,\sigma_{2q}=$$

$$=\frac{1}{2}(|a|^2-|b|^2+|c|^2-|d|^2+|e|^2)n_i n_q+$$

$$+\mathrm{Re}\,(a^*c+b^*d)m_i m_q+\mathrm{Re}\,(a^*c-b^*d)\ell_i\ell_q-\mathrm{Im}\,c^*e(m_i\ell_q-\ell_i m_q).$$

Components K_{nn} and $K_{x-\ell}$ can be determined from triple scattering and K_{xm} and $K_{z-\ell}$ from fourfold scattering.

In the case of identical particles this is equivalent to measurement of the corresponding $\mathcal{D}_{ip}$ components at complementary angles ($\vartheta\to\pi-\vartheta$), where

$$\mathcal{D}_{nn}(\pi-\vartheta)=K_{nn}(\vartheta),\ \mathcal{D}_{xm}(\pi-\vartheta)=K_{x-\ell}(\vartheta)=-K_{x\ell}(\vartheta),$$

$$\mathcal{D}_{zm}(\pi-\vartheta)=-K_{z-\ell}(\vartheta),\ \mathcal{D}_{x\ell}(\pi-\vartheta)=-K_{zm}(\vartheta).$$

A.1. The tensor of polarization correlation is measured:

$$\sigma(\vartheta)\mathcal{P}_{pq}(\vartheta)=\frac{1}{4}\,Sp\,\sigma_{1p}\,\sigma_{2q}\,\mathcal{M}\mathcal{M}^+=$$

$$=\frac{1}{2}(|a|^2-|b|^2-|c|^2+|d|^2+|e|^2)n_p n_q+\mathrm{Re}\,(a^*d+b^*c)m_p m_q+$$

$$+\mathrm{Re}\,(b^*c-a^*d)\ell_p\ell_q+\mathrm{Im}\,d\,e^*(\ell_p m_q+m_p\ell_q).$$

The component

$$\sigma(\vartheta)\mathcal{P}_{nn}(\vartheta)=\frac{1}{2}(|a|^2-|b|^2-|c|^2+|d|^2+|e|^2)$$

can be determined in an experiment in which the planes of the analyzing scatterings coincide with the plane of predominant scattering.

In order to determine the components

$$\sigma(\vartheta)\mathcal{P}_{em}(\vartheta)=\mathrm{Im}\, d e^*$$

an experiment is required in which the planes of the analyzing scatterings are perpendicular to those of predominant scattering.

Experiments in which the planes of the analyzing scatterings are perpendicular to the plane of predominant scattering and in which a magnetic field perpendicular to the first scattering plane is applied in front of one of the analyzers, yield the components

$$\sigma(\vartheta)\mathcal{P}_{mm}(\vartheta)=\mathrm{Re}(a^*d+b^*c),$$

$$\sigma(\vartheta)\mathcal{P}_{ee}(\vartheta)=\mathrm{Re}(b^*c-a^*d).$$

B.4. Measurement of tensor

$$\begin{aligned}\sigma(\vartheta)\mathcal{P}_{ipq}(\vartheta)&=\frac{1}{4}\mathrm{Sp}\,\mathcal{M}\sigma_{1i}\mathcal{M}^+\sigma_{1p}\sigma_{2p}=\\ &=n_i n_p n_q \mathrm{Re}\, a e^*+[m_i m_p n_q+\ell_i \ell_p n_q]\mathrm{Re}\, b e^*+\\ &+m_i \ell_p n_q \mathrm{Im}(a^*b-c^*d)-\ell_i m_p n_q \mathrm{Im}(a^*b+c^*d)+\\ &+m_i n_p \ell_q \mathrm{Im}(a^*c-b^*d)-\ell_i n_p m_q \mathrm{Im}(a^*c+b^*d)+\\ &+n_i m_p \ell_q \mathrm{Im}(a^*d-b^*c)+n_i \ell_p m_q \mathrm{Im}(a^*c-b^*d)+\\ &+[m_i n_p m_q+\ell_i n_p \ell_q]\mathrm{Re}\, c e^*\\ &+[n_i m_p m_q-n_i \ell_p \ell_q]\mathrm{Re}\, d e^*.\end{aligned}$$

D.4. In order to perform this experiment a polarized target is required. The components of the following tensor are measured

$$
\begin{aligned}
&\sigma(\vartheta)\, T_{ikpq}(\vartheta)=\frac{1}{4}\, Sp\, \mathcal{M}\, \sigma_{1i}\, \sigma_{2k}\, \mathcal{M}^{+}\, \sigma_{1p}\, \sigma_{2p} = \\
&= n_i n_k n_p n_q \frac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+ \\
&+(m_i m_k m_p m_q + \ell_i \ell_k \ell_p \ell_q)\frac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2-|e|^2)+ \\
&+(n_i \ell_k n_p \ell_q + \ell_i n_k \ell_p n_q)\, Re\,(a^* b + c^* d)+ \\
&+(n_i m_k n_p m_q + m_i n_k m_p n_q)\, Re\,(a^* b - c^* d)+ \\
&+(m_i \ell_k m_p \ell_q + \ell_i m_k \ell_p m_q)\frac{1}{2}(|a|^2+|b|^2-|c|^2-|d|^2-|e|^2)+ \\
&+(\ell_i m_k m_p \ell_q + m_i \ell_k \ell_p m_q)\frac{1}{2}(|a|^2-|b|^2+|c|^2-|d|^2-|e|^2)+ \\
&+(m_i m_k \ell_p \ell_q + \ell_i \ell_k m_p m_q)\frac{1}{2}(-|a|^2+|b|^2+|c|^2-|d|^2+|e|^2)+ \\
&+(\ell_i n_k n_p \ell_q + n_i \ell_k \ell_p n_q)\, Re\,(a^* c + b^* d)+ \\
&+(n_i m_k m_p n_q + m_i n_k n_p m_q)\, Re\,(a^* c - b^* d)- \\
&-(\ell_i \ell_k n_p n_q + n_i n_k \ell_p \ell_q)\, Re\,(a^* d + b^* c)+ \\
&+(n_i n_k m_p m_q + m_i m_k n_p n_q)\, Re\,(a^* d - b^* c)+ \\
&+(m_i n_k n_p \ell_q + n_i m_k \ell_p n_q - n_i \ell_k m_p n_q - \ell_i n_k n_p m_q)\, Im\, e^* c+ \\
&+ n_i n_k m_p \ell_q + n_i n_k \ell_p m_q - m_i \ell_k n_p n_q - \ell_i m_k n_p n_q)\, Im\, e^* d+ \\
&+(\ell_i m_k m_p m_q + m_i \ell_k m_p m_q + \ell_i \ell_k m_p \ell_q + \ell_i \ell_k \ell_p m_q - \\
&- m_i \ell_k \ell_p \ell_q - \ell_i m_k \ell_p \ell_q - m_i m_k m_p \ell_q - m_i m_k \ell_p m_q)\, Im\, a^* e+ \\
&+(\ell_i n_k m_p n_q + n_i \ell_k n_p m_q - n_i m_k n_p \ell_q - m_i n_k \ell_p n_q)\, Im\, b^* e.
\end{aligned}
$$

A P P E N D I X II

Unitarity Relation

We present below the unitarity relation for nucleon-nucleon scattering

$$4\pi \operatorname{Im} \alpha(\vartheta) = K\int \frac{1}{4} Sp[M^{+}(\vec{K}', \vec{K}'') M(\vec{K}, \vec{K}'')] d\omega_{\vec{K}''},$$

$$4\pi \operatorname{Im} \beta(\vartheta) = K\int \frac{1}{4} Sp[\vec{\sigma}_1 \cdot \vec{n}\, \vec{\sigma}_2 \cdot \vec{n}\, M^{+}(\vec{K}', \vec{K}'') M(\vec{K}, \vec{K}'')] d\omega_{\vec{K}''},$$

$$4\pi \operatorname{Re} \gamma(\vartheta) = iK\int \frac{1}{8} Sp[(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{n}\, M^{+}(\vec{K}', \vec{K}'') M(\vec{K}, \vec{K}'')] d\omega_{\vec{K}''},$$

$$4\pi \operatorname{Im} \delta(\vartheta) = K\int \frac{1}{4} Sp[\vec{\sigma}_1 \cdot \vec{m}\, \vec{\sigma}_2 \cdot \vec{m}\, M^{+}(\vec{K}', \vec{K}'') M(\vec{K}, \vec{K}'')] d\omega_{\vec{K}''},$$

$$4\pi \operatorname{Im} \varepsilon(\vartheta) = K\int \frac{1}{4} Sp[\vec{\sigma}_1 \cdot \vec{\ell}\, \vec{\sigma}_2 \cdot \vec{\ell}\, M^{+}(\vec{K}', \vec{K}'') M(\vec{K}, \vec{K}'')] d\omega_{\vec{K}''}.$$

Evaluation of the spurs involved in the unitarity relation is elementary but yields cumbersome expressions which will not be given here. It should be mentioned that if one uses the expression for M given in ref.[5] the unitarity relation for coefficient $\overset{\circ}{B}(\vartheta)$ (singlet scattering) takes the form of the unitarity relation for scattering of spinless particles.

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