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Laboratory of Theoretical Physics

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IMPULSE APPROXIMATION AND DISPERSION RELATIONS FOR PION-DEUTERON SCATTERING *

Nucl. Phys., 1958, v 8, n 3, p 303-309.

Dubna, 1958.

* Submitted to Nuclear Physics.

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IMPULSE APPROXIMATION AND DISPERSION RELATIONS FOR PION-DEUTERON SCATTERING *

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Abstract: The recently derived dispersion relation for elastic forward scattering of pions by unpolarized deuterons is compared with the experimental data in the sense that the results of Chew's impulse approximation are adopted. It is shown that the dispersion relation is consistent with these data. But for a more detailed test of the theory it is necessary to consider other π-D processes for which the contribution from the non-observable part in the dispersion relations is no small quantity. It is indicated that for this the consideration of the elastic forward scattering amplitude for spin-flip is sufficient.

I. Introduction

In a preceding paper[I)] we have derived (under some definite assumptions) the following dispersion relation for the elastic forward scattering of pions by unpolarized deuterons (I(65))

$$\bar{D}(E) - \bar{D}(m) =$$

$$= \frac{1}{2\pi^2}(E^2-m^2)\int_m^\infty dE' \frac{E'\sigma_D(E')}{(E'^2-E^2)(E^2-m^2)} + \frac{f^2}{m^3}(E^2-m^2)\left[F(E)-\frac{4}{3}G(E)\right] \qquad (I)$$

$\bar{D}(E)$ is connected with the hermitic part $D(E)$ of the scattering matrix by

$$\bar{D} = \frac{1}{3}\,\mathrm{Sp}\,D \qquad (2)$$

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x) Submitted to Nuclear Physics

I) F. Kaschluhn, Zs.f.Naturforsch. (in print); hereafter referred to as I.

and the total cross section $\sigma_D(E)$ for pion scattering on unpolarized deuterons with the antihermetic part A(E) of the scattering matrix by the optical theorem

$$\sigma_D = \frac{1}{3} S_P A \tag{3}$$

E is the laboratory energy of the incoming pion with rest mass m and f=gm/2M. The dimensionless quantities F and G are given by the expressions I(60-63), but for our purposes it is sufficient to use the approximation I(64) accurate within 30% (decidedly for IO Mev and more laboratory kinetic energy; the connection between F,G and X,Y is F=2X and G=2Y, i.e. the connecting factor is 2 and not 0,45 as stated in I after (65))

$$F = -0{,}5\,\frac{m^2}{E^2} \qquad \frac{1}{3}G = 0 \tag{4}$$

The comparison of the relations (I) - (4) with the experimental scattering data is impaired by the following two difficulties: I) whereas already total cross section determinations have been performed at a variety of energies in π-D scattering (see ref.[3]), there are carried out till now only two measurements of the cross sections and angular distributions for the individual processes, i.e. the investigations of Nagle[2] at II9 Mev and especially those of Rogers and Lederman[3] at 85 MeV laboratory kinetic energy, which, however, are not sufficient for a test of the theory according to (I); 2) these measurements are all related to the scattering by unpolarized deuterons for which the theory of dispersion relations (without the aid of further relations) yields only an unequality as a test relation (see section 2).

But in the case of π-D scattering we may use sufficiently

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2) D.E. Nagle, Phys.Rev. 97,480 (I955)

3) K.C. Rogers, L.M. Lederman, Phys.Rev. I05, 247 (I957)

Chew's impulse approximation[4] which we adopt in the form of the pure-scattering model suggested by Fernbach, Green and Watson[5]. The accuracy of this approximation exceeds very likely the present experimental possibilities[6] and we can suppose that the errors connected with the neglect of binding effects, multiple scattering and absorption phenomena are not much larger than 20% [7]. Moreover in the special case of forward scattering we have the simple criterion for the availability of the impulse approximation (see section 3) that the elastic π-D scattering amplitude and the total cross section should be the direct sum of the scattering amplitudes respectively the total cross sections for the scattering by the free nucleons. This is fulfilled with the above assumed accuracy even in the resonance region[8].

In section 2 the relations will be derived necessary for the comparison with the experimental data and in section 3 it will be shown that the impulse approximation allows a real test of the dispersion relation (I).

2. Consequences from the Spin-Structure of the Scattering Matrix

Cheyshvili[9] has recently determined the general spin-structure of the scattering matrix including the corresponding

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4) G.F. Chew, Phys. Rev. 80, 196 (1950), G.F. Chew, G.C. Wick, Phys. Rev. 85, 636 (1952); G.F. Chew, L.M. Goldberger, Phys. Rev. 87, 778 (1952).

5) Fernbach, Green, Watson, Phys. Rev. 84, 1084 (1951).

6) The measurements of Rogers and Lederman for the elastic scattering are only accurate to 40-50% in the region of smaller angles so that the extrapolation to forward scattering is subjected to the corresponding uncertainties.

7) This is especially in accordance with the results from the work of Rockmore, Phys. Rev. 105, 256 (1957), that gives an impulse approximation analysis for the data of Rogers and Lederman.

8) H.A. Bethe, F. deHoffmann, Mesons and Fields (Row, Peterson and Company, Evanston, 1955), Vol. II, p.405.

9) O.D. Cheyshvili, JETF, 30, 1147 (1956).

phase shift relations for the elastic scattering of deuterons by nuclei with zero spin. These formula may be taken over directly if we replace the initial and final momentum of the deuteron appearing in[9] by the corresponding momenta of the pion. Then the scattering matrix has the following structure with respect to the deuteron spin $\vec{S}$

$$T = a + b\,\vec{S}\vec{n} + c\,(\vec{S}\vec{n})^2 + d\,\tfrac{1}{2}\{(\vec{S}\vec{q}_0)(\vec{S}\vec{q}) + (\vec{S}\vec{q})(\vec{S}\vec{q}_0)\} \tag{5}$$

$a, \dots, d$ are scalar amplitudes depending on the scattering angle and the energy of the incoming pion, the unit vectors $\vec{q}_0$ and $\vec{q}$ specify the directions of its initial and final momentum and $\vec{n}$ is the unit vector $\vec{q}_0 \times \vec{q} \,/\, |\vec{q}_0 \times \vec{q}|$ perpendicular to the scattering plane. In the case of forward scattering we get from the relations in[9]

$$b = c = 0 \tag{6}$$

and with $\vec{q}_0 = \vec{q}$ (5) becomes simply

$$T = a + d\,(\vec{S}\vec{q})^2 \tag{7}$$

(7) means that contrary to pion-nucleon scattering the forward scattering amplitude for $\pi - D$ scattering depends on the target spin $\vec{S}$. From this some complications follow for the experimental test of the dispersion relation (I) if one uses only experimental data for pion scattering by unpolarized deuterons.

The differential cross section for elastic scattering by unpolarized deuterons is given by

$$d\sigma/d\Omega = \frac{1}{3}\,\mathrm{Sp}\,T^{*}T \tag{8}$$

whence we get for forward scattering using (7)

$$d\sigma/d\Omega_0 = a^*a + \tfrac{2}{3}(a^*d + ad^*) + \tfrac{2}{3}d^*d \qquad (9)$$

On the other hand the optical theorem (3) appears in the form

$$\sigma_D = \tfrac{4}{3}\,\mathrm{Sp}A = \mathrm{Im}\,a + \tfrac{2}{3}\,\mathrm{Im}\,d \qquad (10)$$

We suppose the experimental quantities $d\sigma/d\Omega_0$ and σ_D to be known and may calculate a further quantity from the dispersion relation (I)

$$\bar{D} = \tfrac{1}{3}\,\mathrm{Sp}\,D = \mathrm{Re}\,a + \tfrac{2}{3}\,\mathrm{Re}\,d \qquad (11)$$

From (9), (10) and (11) it follows now

$$d\sigma/d\Omega_0 = |a + \tfrac{2}{3}d|^2 + \tfrac{2}{9}|d|^2 \geq |a + \tfrac{2}{3}d|^2 \qquad (12)$$

at which according to (10) and (11) we can only determine the last expression in (12). So we come to the result that a real test of the dispersion relation (without the aid of further relations) is not possible in general since the theory yields only an unequality as a necessary condition. Secure answers can only be given if the quantity $(2/9)|d|^2$ may be neglected in (12) (which according to section 3 is true for the results of the impulse approximation) or if the unequality (12) may be proved to be not fulfilled (in the last case the statement is clearly negative).

3. Comparison with the Impulse Approximation

The impulse approximation[4] assumes that the amplitude T for scattering of particles by a bound system can be written as the sum of the amplitudes t_j for scattering by the corresponding free target particles, i.e. for the case of scattering by deuterons

$$T = t_P + t_N \tag{I3}$$

If one considers the matrix elements of T between the initial state ϕ_a and final state ϕ_b of the whole system

$$\langle \phi_b | T | \phi_a \rangle = \langle \phi_b | t_P | \phi_a \rangle + \langle \phi_b | t_N | \phi_a \rangle \tag{I4}$$

it may be seen [4] that this approximation represents the scattering of the incoming particles by a wave packet of free target particles which has the same impulse distribution as the real bound state. In the pure-scattering model of Fernbach, Green and Watson[5] for $\pi - D$ scattering the expression (I4) may be further appriximated by

$$\langle \phi_b | T | \phi_a \rangle = t^P_{ab} J^P_{ab} + t^N_{ab} J^N_{ab} \tag{I5}$$

where t^P_{ab} and t^N_{ab} are the matrix elements for pion scattering by the free proton and neutron but their spin states are related to the whole P-N system and J^P_{ab} and J^N_{ab} are some integrals depending on the special transition. The matrices t^P_{ab} and t^N_{ab} have the well-known spin-structure

$$t^i = u^i + i v^i \vec{\sigma}^i (\vec{q}\,' \times \vec{q}) \tag{I6}$$

where u^i and v^i are scalar amplitudes, $\vec{\sigma}^i$ the spin vectors of proton and neutron and $\vec{q}$ and $\vec{q}\,'$ initial and final momentum of the pion.

In the case of elastic forward scattering the integrals J^P and J^N turn to the normalisation integrals, i.e. simply to one, whereas the spin dependence of the matrices t^P and t^N disappear due to (I6). So we get

$$\langle \phi_b | T | \phi_a \rangle = u^P_{ab} + u^N_{ab} \tag{I7}$$

and especially it follows from this for the total cross section according to the optical theorem (3).

$$\sigma_D = \sigma_P + \sigma_N \tag{I8}$$

the approximative features of which had been discussed in the introduction. We note that the spin dependence of the scattering amplitude disappeared and consequently the difficulties discussed in section 2.

The comparison of the dispersion relation with the results of the impulse approximation is now nearly trivial since according to (I7) containing only spin independent quantities it follows from (2)

$$\overline{D} = D_P + D_N \tag{I9}$$

Taking into account the charge independence of the theory we may write (I8) and (I9) in the form

$$\overline{D} = D_+ + D_- \qquad \sigma_D = \sigma_+ + \sigma_- \tag{20}$$

where D_+, D_- and σ_+, σ_- are the hermitic parts of the scattering matrix and the total cross section for scattering of π^+- or π^--mesons by protons.

Now the amplitudes D_+ and D_- satisfy the Goldberger relations[I0)]

$$D_+(E) + D_-(E) - [D_+(m) + D_-(m)] =$$
$$= \frac{1}{2\pi^2}(E^2 - m^2)\int_m^\infty dE' \frac{E'[\sigma_+(E') + \sigma_-(E')]}{(E'^2 - E^2)(E'^2 - m^2)} + 4f^2 \frac{E^2 - m^2}{E^2 - \left(\frac{m^2}{2M}\right)^2} \frac{1}{2M} \tag{2I}$$

I0) Goldberger, Miyazawa, Oehme, Phys. Rev. 99, 986 (I955).

$$D_+(E) - D_-(E) - \frac{E}{m}\left[D_+(m) - D_-(m)\right] = \tag{22}$$
$$= \frac{1}{2\pi^2}(E^2-m^2)E\int_m^\infty dE' \frac{\sigma_+(E') - \sigma_-(E')}{(E'^2-E^2)(E'^2-m^2)} + 4f^2 \frac{E^2-m^2}{E^2-\left(\frac{m^2}{2M}\right)^2} \frac{E}{m^2}$$

According to (20), i.e. within the limits of the impulse approximation, the relation (2I) represents also the dispersion relation for π^+-D scattering and the comparison with the dispersion relation (I) requires that the contribution from the non-observable part must be the same in both equations. We show that this is indeed the case within the limits of our approximation.

The contribution of the non-observable part in the dispersion relation (I) becomes using (4)

$$\frac{f^2}{m^3}(E^2-m^2)\left[F(E) - \frac{1}{3}G(E)\right] = -f^2 \frac{E^2-m^2}{E^2} \frac{3,4}{M} \tag{23}$$

whereas the corresponding contribution in (2I) may be written in the form

$$4f^2 \frac{E^2-m^2}{E^2-\left(\frac{m^2}{2M}\right)^2} \frac{1}{2M} = f^2 \frac{E^2-m^2}{E^2} \frac{2}{M} \tag{24}$$

where we have neglected the small quantity $(m^2/2M)^2$ against E^2 ($E^2 \geq m^2 \gg (m^2/2M)^2 = I/I3,5^2$). Furthermore we take into account the fact that (24) and consequqntly (23) are only small quantities their values are still within the limits of the present experimental works or of the impulse approximation respectively. We notice that in the case of π-P scattering an experimental determination of the coupling constant f is only possible due to the relative large contribution of the non-observable part in (22): the ratio of the corresponding contributions in (22) and (2I) is $E2M/m^2 \geq I3,5$ and the sum $D_+ + D_-$ and the difference $D_+ - D_-$ in (2I) and (22) are in general of the same order of magnitude [II].

4. Conclusion

In the last section it was shown that the dispersion relation (I) for the elastic forward scattering of pions by unpolarized deuterons is consistent with the results of the impulse approximation adopted by us as the experimental scattering data. But it turned out in detail that the contribution in the dispersion relation resulting from the integration over the non-observable part is only a small quantity in the whole range of energy (this is by no means trivial for it is quite impossible to make a simple estimation of its order of magnitude because of the appearing divergences and the necessity to carry out explicitly the analytic continuation [I2]). So it is desireable to investigate other $\pi - D$ processes for which a more detailed test of the theory is possible, i.e. a quantitative comparison of the contribution from the non-observable part. The inelastic and charge exchange $\pi - D$ scattering, which also can be handled in the scheme of the impulse approximation, are not very suitable since their forward scattering amplitudes vanish (so it would be necessary to consider non-forward scattering or derivatives of the scattering matrix in forward direction). But it can be shown that the more simple consideration of the elastic forward scattering amplitude for spin-flip is sufficient. For this we remember first

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II) Compare for the details Figs.3,4 and 5 in the work of G.Puppi, A. Stanghellini, Nuovo Cim. 5, I305 (1957).

I2) R. Oehme, Phys. Rev.I00, I503 (I955); see also W.C.Davidon, M.L.Goldberger, Phys. Rev. I04, III9 (I956) and N.N.Bogoliubov, B.V. Medvedev, M.K. Polivanov, Problems of the Theory of Dispersion Relations, Gostekhidat Moscow (in print); shortened translation into German in Fortschr. d. Phys., Akademie-Verlag Berlin (in print).

that contrary to the case of no-spin-flip scattering the contribution from the non-observable part is large in the dispersion relation for the sum $D_+^s + D_-^s$ and small in that for the difference $D_+^s - D_-^s$, where D_+^s and D_-^s are the hermitic parts of the spin-flip amplitudes for $\pi^+ - P$ and $\pi^- - P$ scattering[I2]. Then we consider the general case of non-forward scattering and suppose the deuteron to be polarized perpendicular to the scattering plane. From (I6) and (I5) it follows for the spin-flip amplitudes using charge independence of the theory if finally we restrict us to forward scattering again

$$v_+ + v_- = \frac{1}{2i\,|\vec{q}\,'\times\vec{q}\,|}\left\{\langle\chi_{1,1}|T|\chi_{1,1}\rangle - \langle\chi_{1,-1}|T|\chi_{1,-1}\rangle\right\}\Big|_{\vec{q}\,'=\vec{q}} \tag{25}$$

where $\chi_{1,1}$ and $\chi_{1,-1}$ are the usual spin functions for the triplet-states. The corresponding dispersion relation for $D_+^s + D_-^s$ are given in [I2] (in the last cited work even for non-forward scattering, but we can restrict us for practical purposes to forward scattering). On the other hand we can derive for this case dispersion relations along the lines of[I], and it can be shown that now the resulting contribution from the non-observable part has the same sign as the corresponding contribution in the dispersion relation for $D_+^s + D_-^s$ (that must be so if the theory is consistent since now the contribution from the non-observable part is testable). But the explicit calculation of the appearing integrals must be carried out once more much which shall be done elsewhere[I3].

I3) So we may exspect that it is possible to determine the pion-nucleon coupling constant from the dispersion relation for the elastic spin-flip amplitude of pion-deuteron scattering (whereas we can conlude from the dispersion relation for the corresponding no-spin-flip amplitude only an upper limit for the order of magnitude of the coupling constant corresponding to $f^2 = 0,I$).

The author wishes to express his gratitude to Prof.Bogoliubov for his interest in this work and to the members of the Institute for Theoretical Physics for valuable discussions, especially to Dr.Lapidus. He would also like to thank Prof.Pomeranchuk and Dr.Ioffe for interesting talks.

Note: In the meantime the author had worked out the suggested case of elastic spin-flip forward scattering of pions by deuterons along the lines of I. The coupling constant was determined as $g^2/\hbar c = 18$ using Chew's impulse approximation with an error which is, of course, a little larger than that following from determination by means of pion-nucleon dispersion relations. The details shall be published in near future in Nuclear Physics.