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PHENOMENOLOGICAL ANALYSIS OF REACTIONS OF THE
$a + a' \rightarrow b + b'$ TYPE

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<u>Abstract</u>: Conditions for reconstruction from experimental data of the matrix for reactions of the $a + a' \rightarrow b + b'$ type are considered on basis of general quantum mechanical principles. The reaction matrix M is expanded in a complete set of irreducible tensor operators $T^{JM}(j_b, j_a)$ and the number of complex scalar functions which determine it is computed for the case when invariance under space rotations and reflections in taken into account. Time reversal invariance of the interaction leads to relations between polarization effects in the direct and inverse reactions. The number of experiments required for complete reconstruction of the reaction matrix in the presence of several channels can be determined on basis of unitarity of the S matrix.

The general form of the azimuthal dependence of the angular distribution of the reaction products (for arbitrary spins) is derived in the appendix.

I. Introduction

It has been shown in ref.[I,2] that in a phenomenological analysis of the experimental data on elastic scattering the conditions of space rotation and reflection invariance, symmetry under time reversal and also the unitarity conditions permit one to determine the number of experiments required for complete reconstruction of

the scattering matrix. A similar analysis is carried out below for
an arbitrary reaction of the type

$$a + a' \rightarrow b + b' \tag{I}$$

The spins of the particles are denoted here as i_1 and i_2 for
a and a' and I_1 and I_2 for b and b'. The analysis is
carried out by application of the method of construction of the
reaction amplitudes as functions of the initial and final wave
vectors and spin operators; this method is a direct extension of
the procedure proposed by Wolfenstein and Ashkin[3], Dalitz[4] and
Lakin[5].

2. Construction of Reaction Matrix

By definition, the reaction matrix after acting on the initial
spin function χ_a yields the amplitude of the outgoing wave in
the final state

$$f_b = \sqrt{\frac{v_a}{v_b}} \, M_{ba}(\vec{n}_b, \vec{n}_a) \, \chi_a \tag{2}$$

Here $\vec{n}_a$ and $\vec{n}_b$ are unit vectors directed along the relative
momenta of the initial and final states and v_a and v_b are the
corresponding velocities. M is a scalar for reaction in which the
relative intrinsic parities of the particles do not change and is
a pseudoscalar if the intrinsic parity changes. For convenience of
investigation of the **symmetry** properties of matrix M we make use
of the complete set of operators $T^{JM}(j_b, j_a)$ which transform
according to the irreducible representation of weight ($2J+1$)
of the three dimensional rotation group

$$T^{JM}(j_b, j_a) = A(j_b, j_a, J) \sum_{m_a m_b} (j_a J m_a M | j_a J j_b m_b) \, \chi_{j_b m_b} \, \chi^{+}_{j_a m_a} \tag{3}$$

where j_a and j_b are the total spins of the system in the
initial and final states. $\chi_{j_a m_a}$ and $\chi_{j_b m_b}$ denote the

respective spin functions. Coefficients $A(j_\beta, j_a, j)$
satisfy the condition

$$A^*(j_\beta j_a j)\sqrt{2j_\beta+1} = (-1)^{j_a-j_\beta}\sqrt{2j_a+1}\, A(j_a j_\beta j) \tag{4}$$

which for $j_a = j_\beta$ reduces to the requirement that $A(j_a j_a j)$
be real. Otherwise the value of $A(j_\beta j_a j)$ depends on how the
tensors $T^{JM}(j_\beta j_a)$ are normalized.

Condition (4) follows from the requirement

$$T^{+\,JM}(j_\beta j_a) = (-1)^M\, T^{J-M}(j_a j_\beta) \tag{5}$$

Under space reflections the $T^{JM}(j_\beta j_a)$ transforms as J rank
tensors for even J and as J rank pseudotensors for odd J

Expanding M in terms of the operators in (3) we get

$$M_{\beta a}(\vec{n}_\beta, \vec{n}_a) = \sum_{JM\, j_a j_\beta} \Phi^{JM}_{j_\beta j_a}(\vec{n}_\beta, \vec{n}_a)\, T^{JM}(j_\beta, j_a)$$

$$\tag{6}$$

From invariance of M under rotations it follows that the
functions $\Phi^{JM}_{j_\beta j_a}$ can be represented in the form

$$\Phi^{JM}_{j_\beta j_a}(\vec{n}_\beta, \vec{n}_a) = \sum_\lambda a^\lambda_{j_\beta j_a J}\, \Psi^{*\lambda}_{JM}(\vec{n}_\beta, \vec{n}_a) \tag{7}$$

where the $a^\lambda_{j_a j_\beta J}$ are arbitrary complex scalar functions
which depend on the energy and $(\vec{n}_a \cdot \vec{n}_\beta)$ and

$$\Psi^\lambda_{JM}(\vec{n}_\beta, \vec{n}_a) = \sum_{\mu,\mu'} (z-\lambda, z+\lambda\, \mu \mu' | z-\lambda, z+\lambda\, JM)\, \mathcal{Y}_{z-\lambda,\mu}(\vec{n}_a)\, \mathcal{Y}_{z+\lambda,\mu'}(\vec{n}_\beta) \tag{8}$$

are independent functions of two unit vectors $\vec{n}_a$ and $\vec{n}_\beta$ which
transform according to the irreducible representation of the
rotation group of weight $(2J +I)$.

If the relative intrinsic parity does not change, one gets

$2 \, \nu = J$ for even J

$2 \, \nu = J+1$ for odd J

If the relative parity changes

$2 \, \nu = J+1$ for even J

$2 \, \nu = J$ for odd J

The number of values of λ depends on the properties of the Clebsch-Gordan coefficients in (8) and equals

$N(J) = J + I$ for even J

$N(J) = J$ for odd J , if the intrinsic parity does not change and

$N(J) = J$ for even J

and $N(J) = J+1$ for odd J

if the intrinsic parity in (I) changes.

In order to determine the total number of experiments required for reconstruction of the reaction amplitude, we present the number $N_0(i_1, i_2, I_1, I_2)$ of $a^{\lambda}_{j_a j_b J}$ functions. This number is

$$N_0(i_1, i_2, I_1, I_2) = \sum_{|I_1-I_2|}^{I_1+I_2} \sum_{|i_1-i_2|}^{i_1+i_2} \sum_{|j_a-j_b|}^{j_a+j_b} N(J)$$

If the spins of all the particles participating in the reactions are integers then

$$N_0(i_1, i_2, I_1, I_2) = \frac{1}{2}(2i_1+1)(2i_2+1)(2I_1+1)(2I_2+1) \pm \frac{1}{2}(-1)^{i_1+i_2+I_1+I_2} \tag{9}$$

In all other cases

$$N_0(i_1, i_2, I_1, I_2) = \frac{1}{2}(2i_1+1)(2i_2+1)(2I_1+1)(2I_2+1) \tag{10}$$

The upper sign in (9) refers to the case when the intrinsic

parity is constant and the lower to the case when it changes.

3. Time Reversal Invariance

It can be seen that the condition of invariance under rotation reduces the number of terms in matrix from

$$(2i_1+1)(2i_2+1)(2I_1+1)(2I_2+1) \quad to \quad N_0(i_1, i_2, I_1, I_2)$$

If in (I) β equals α^* and β' equals α' (elastic scattering of particles with arbitrary spins) further reduction of the number of terms in the expansion of M and hence the number of required experiments, can be attained[2] by taking into account invariance under time reversal.

In the general case reaction (I) time reversal symmetry yields a relation between the matrices M of the direct and inverse reactions. If $M_{\alpha\beta}(\vec{n}_\alpha, \vec{n}_\beta)$ denoted the matrix of the reaction which is the reciprocal of (I) then

$$k_\alpha M^+_{\beta\alpha}(\vec{n}_\beta, \vec{n}_\alpha) = k_\beta T M_{\alpha\beta}(-\vec{n}_\alpha, -\vec{n}_\beta) T^{-1} \tag{II}$$

where k_α and k_β are the momenta of relative motion in the initial and final states and T is a time reversal operator. The action of the latter on a function with definite values of the square of the angular momentum and of its projection is defined by the equation[7].

$$T \psi_{JM} = (-1)^{J-M} \psi_{J-M} \tag{I2}$$

The law of transformation of $T^{JM}(j_\alpha, j_\beta)$ under time reversal follows from (I2):

$$T T^{JM}(j_\alpha, j_\beta) T^{-1} = (-1)^{J-M} T^{J-M} \tag{I3}$$

Employing the expansion of $M_{\alpha\beta}(\vec{n}_a, \vec{n}_\beta)$

$$M_{\alpha\beta}(\vec{n}_a, \vec{n}_\beta) = \sum_{JM\, j_a j_\beta} \sum_\lambda \beta^\lambda_{j_a j_\beta J} \psi^{*\lambda}_{JM}(\vec{n}_a, \vec{n}_\beta)\, T^{JM}(j_a, j_\beta) \tag{14}$$

and using relations (13), (5) and also the relations

$$\psi^\lambda_{JM}(\vec{n}_\beta, \vec{n}_a) = (-1)^{2l-J}\, \psi^{-\lambda}_{JM}(\vec{n}_a, \vec{n}_\beta) \tag{15}$$

and

$$\psi^\lambda_{JM}(-\vec{n}_a, -\vec{n}_\beta) = (-1)^{2l}\, \psi^\lambda_{JM}(\vec{n}_a, \vec{n}_\beta) \tag{16}$$

one obtains from (11) the relation between the scalar expansion coefficients of matrices M for the direct and inverse reactions

$$k_a \cdot a^\lambda_{j_\beta j_a J} = k_\beta\, \beta^{-\lambda}_{j_a j_\beta J} \tag{17}$$

Formulae (11) or (17) permit one to construct the matrix of the inverse reaction knowing M of the direct reaction and vice versa.

Experimentally measurable mean values of the spin operators in the direct and inverse reactions can be directly related on basis of condition (11). Under real conditions either the mean values of the spin operators of separate particles (unpolarized cross section, vector and tensor polarizations in beam) or else the mean values of their direct products (polarization correlation) are measured. It is therefore convenient to expand the density matrix, referring to the polarization state of the particles, into direct products of irreducible tensor operators of separate particles.

One gets

$$\rho_a = \sum_{J=0}^{2i_1} \sum_{J'=0}^{2i_2} \sum_{M=-J}^{J} \sum_{M'=-J'}^{J'} \left\langle T^{JM} T^{J'M'} \right\rangle\, \overset{+}{T}{}^{JM} \overset{+}{T}{}^{J'M'} \tag{18}$$

for the density matrix of particles a and a' and

$$\rho_\beta = \sum_{k=0}^{2I_1} \sum_{k'=0}^{2I_2} \sum_{N=-k}^{k} \sum_{N'=-k'}^{k'} \left\langle \theta^{kN} \theta^{k'N'} \right\rangle\, \overset{+}{\theta}{}^{kN} \overset{+}{\theta}{}^{k'N'} \tag{19}$$

for β and β' .

Operators T^{JM} and $T^{J'M'}$ act on the spin variables of particles a and a' and Θ^{KN} and $\Theta^{K'N'}$ on spin variables of β and β' respectively.

The expressions for T^{JM} can be obtained from (3) by putting $j_a = j_\beta = i_1$, and the value of $A(i_1, i_1, J)$ is defined by the condition

$$Sp\ T^{+JM} T^{JM} = 2i_1 + 1 \qquad . \tag{20}$$

Under similar conditions for $T^{J'M'}$, Θ^{KN} and $\Theta^{K'N'}$, i_1 must be replaced by i_2, I_1 and I_2 respectively. The quantities $\langle T^{JM} T^{J'M'} \rangle$ for the direct reaction are prescribed by the conditions of the experiment. The quantities $\langle \Theta^{KN} \Theta^{K'N'} \rangle$ can then be expressed through $\langle T^{JM} T^{J'M'} \rangle$ and the matrix of the direct reaction, $M_{\beta a} (\vec{n}_\beta, \vec{n}_a)$

$$\sigma_{\beta a} \langle \Theta^{KN} \Theta^{K'N'} \rangle = \sum_{JJ'MM'} \langle T^{JM} T^{J'M'} \rangle\ Sp\ \Theta^{KN} \Theta^{K'N'} \times \tag{21}$$

$$M_{\beta a} (\vec{n}_\beta, \vec{n}_a)\ T^{+JM}\ T^{+J'M'}\ \dot{M}_{\beta a} (\vec{n}_\beta, \vec{n}_a)$$

and

$$\sigma_{\beta a} = Sp\ M_{\beta a} \cdot \rho_a \cdot M^{+}_{\beta a} \tag{22}$$

For the inverse reaction

$$\sigma_{a\beta} \langle T^{JM} T^{J'M'} \rangle = \sum_{KK'NN'} \langle \Theta^{KN} \Theta^{K'N'} \rangle \times \tag{23}$$

$$Sp\ T^{JM}\ T^{J'M'}\ M_{a\beta} (\vec{n}_a, \vec{n}_\beta)\ \Theta^{+KN}\ \Theta^{+K'N'}\ M^{+}_{a\beta} (\vec{n}_a, \vec{n}_\beta)$$

and

$$\sigma_{a\beta} = Sp\ M_{a\beta} \cdot \rho_\beta\ M^{+}_{a\beta} \tag{24}$$

From (II) and (I3) we obtain[x)]

$$k_a^2 \, Sp \, \theta^{KN} \theta^{K'N'} M_{\beta a} (\vec{n}_\beta, \vec{n}_a) \overset{+}{T}{}^{JM} \overset{+}{T}{}^{J'M'} \overset{+}{M}_{\beta a} (\vec{n}_\beta, \vec{n}_a) =$$

$$= (-1)^{J+J'+K+K'+M+M'+N+N'} k_\beta^2 \, Sp \, T^{J-M} T^{J'-M'} M_{a\beta} (-\vec{n}_a, -\vec{n}_\beta) \overset{+}{\theta}{}^{K-N} \overset{+}{\theta}{}^{K'-N'} \overset{+}{M}_{a\beta} (-\vec{n}_a, -\vec{n}_\beta) \quad (25)$$

For $J = J' = K = K' = 0$, that is, when all θ and T reduce to unit matrices, the well known relation between the unpolarized cross sections $\sigma_{a\beta}^{\circ}$ and $\sigma_{\beta a}^{\circ}$ follows from (25) if account is made of the fact that $\sigma_{\beta a}^{\circ}$ and $\sigma_{a\beta}^{\circ}$ can be expressed through M as follows:

$$\sigma_{\beta a}^{\circ} = \frac{1}{(2i_1+1)(2i_2+1)} \, Sp \, M_{\beta a} \overset{+}{M}_{\beta a}$$

and

$$\sigma_{a\beta}^{\circ} = \frac{1}{(2I_1+1)(2I_2+1)} \, Sp \, M_{a\beta} \overset{+}{M}_{a\beta}$$

In the general case (25) yields the relation between the contribution to the mean value of operator $\theta^{KN} \theta^{K'N'}$ in the direct reaction (the contribution arising from the fact that the mean value of $T^{JM} T^{J'M'}$ is not zero in the initial state) and the contribution to the mean value of $T^{J-M} T^{J'-M'}$ (which is due to the fact that in the initial state of the inverse reaction the mean value of $\theta^{KN} \theta^{K'N'}$ does not vanish).

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4. Determination of Reaction Matrix

We shall now proceed to analyze the conditions underlying the determination of the reaction matrix from the experimental data. If not all simultaneously operative reactions are taken into account, $M_{\beta a}$ can be reconstructed only with an accuracy to a common phase factor. Since the functions $a^{\lambda}_{J_6 J_a J}$ are complex, the number of necessary experiments is $2 N_o (i_1, i_2, I_1, I_2) - 1$ where N_o is defined in (9) and (I0).

If reactions proceeding along parallel channels are investigated simultaneously with reaction (I), then for determination of the total number of required experiments, not only the symmetry properties considered above but the unitarity condition for the S-matrix should also be used[I,I2]; also see[I3].

If only two reaction channels are involved, the unitarity condition can be represented as follows

$$2\pi i \left[\overset{+}{M}_{aa} (\vec{n_a}, \vec{n_a}') - M_{aa} (\vec{n_a}', \vec{n_a}) \right] =$$

$$= k_a \int d\omega(\vec{n}) \left\{ \overset{+}{M}_{aa} (\vec{n}, \vec{n_a}') M_{aa} (\vec{n}, \vec{n_a}) + \overset{+}{M}_{\beta a} (\vec{n}, \vec{n_a}') M_{\beta a} (\vec{n}, \vec{n_a}) \right\} \qquad (26)$$

$$2\pi i \left[\overset{+}{M}_{a\beta} (\vec{n_a}, \vec{n_\beta}) - M_{\beta a} (\vec{n_\beta}, \vec{n_a}) \right] =$$

$$= \int d\omega(\vec{n}) \left\{ k_a \overset{+}{M}_{a\beta} (\vec{n}, \vec{n_\beta}) M_{aa} (\vec{n}, \vec{n_a}) + k_\beta \overset{+}{M}_{\beta\beta} (\vec{n}, \vec{n_\beta}) M_{\beta a} (\vec{n}, \vec{n_a}) \right\}$$

$$2\pi i \left[\overset{+}{M}_{\beta\beta} (\vec{n_\beta}, \vec{n_\beta}') - M_{\beta\beta} (\vec{n_\beta}', \vec{n_\beta}) \right] =$$

$$= k_\beta \int d\omega(\vec{n}) \left\{ \overset{+}{M}_{\beta\beta} (\vec{n}, \vec{n_\beta}') M_{\beta\beta} (\vec{n}, \vec{n_\beta}) + \overset{+}{M}_{a\beta} (\vec{n}, \vec{n_\beta}') M_{a\beta} (\vec{n}, \vec{n_\beta}) \right\}$$

In accord with (9) and 5 (I0) the number of scalar functions in matrix $M_{\beta a}$ is $N_o (i_1, i_2, I_1, I_2)$. The number of independent scalar functions reduces to

$$N (i_1, i_2) = \frac{1}{2} N_o(i_1, i_2, i_1, i_2) + \frac{1}{2} (2i_1+1)(2i_2+1) \qquad (27)$$

for matrix M_{aa} and to

$$N(I_1, I_2) = \frac{1}{2} N_0 (I_1, I_2, I_1, I_2) + \frac{1}{2} (2I_1+1)(2I_2+1) \tag{28}$$

for matrix $M_{\beta\beta}$, if time reversal invariance is taken into account. Thus the total number of scalar functions entering the reaction matrices of the processes under consideration is

$$N(i_1, i_2) + N(I_1, I_2) + N_0 (i_1, i_2, I_1, I_2)$$

These functions are complex and hence all values cited above should be doubled. However, equations (26) lead to integral relations between the scalar functions in the expansion of M . The number of these relations coincides with the total number of coefficients in matrices M_{aa} , $M_{\beta\beta}$ and $M_{a\beta}$ and this precisely equals the necessary number of experiments (it being assumed that all experiments are performed at a one prescribed energy).

The considerations presented here for the case of two channels can be directly extended to the case of an arbitrary number of channels.

In conclusion the authors consider it a pleasant duty to express their appreciation to Professor Ya.A.Smorodinsky for interesting discussions of the problems considered here.

APPENDIX

It is of interest to investigate the azimuthal dependence of the angular distribution of the products of reaction (I) when the incident beam (particle a) is polarized and the target is not polarized. In this case the cross section for the reaction has the following form

$$\sigma_{\beta a}(\vec{n}_6,\vec{n}_a) = Sp\, \rho_a \overset{+}{M}_{\beta a} M_{\beta a} = \frac{1}{(2i_1+1)(2i_2+1)} \sum_{JM} \langle T^{JM}\rangle\, Sp\,\overset{+}{T}{}^{JM}\overset{+}{M}_{\beta a} M_{\beta a} \quad (A.I)$$

Since $\overset{+}{M}_{\beta a} M_{\beta a}$ is a quadratic matrix of the order $(2i_1+1)(2i_2+1)$ it can be expanded in orthogonal products $T^{JM}\, T^{J'M'}$ Making use of the condition of invariance of $\overset{+}{M}_{\beta a} M_{\beta a}$ under space rotations and reflections we get

$$\overset{+}{M}_{\beta a} M_{\beta a} = \sum_{J\lambda} A^{\lambda}_{J} \sum_{M} \psi^{\lambda*}_{JM} T^{JM} + Q \qquad (A.2)$$

where, analogously to (7), A^{λ}_{J} - are arbitrary scalar functions and Q contain as multipliers the operators $T^{J'M'}$ for $J' \neq 0$. From (A,I) and (A.2) it follows

$$\sigma_{\beta a}(\vec{n}_6,\vec{n}_a) = \sum_{J\lambda} A^{\lambda}_{J} \sum_{M=-J}^{J} \langle T^{JM}\rangle\, \psi^{\lambda*}_{JM}(\vec{n}_6,\vec{n}_a) \qquad (A.3)$$

The azimuthal dependence of the cross section is due to terms with $M \neq 0$. Therefore the sum over M in (A.3) can be represented as follows

$$\langle T^{J0}\rangle\, \psi^{\lambda*}_{J0} + \sum_{M=1}^{J}\left\{ \langle T^{JM}\rangle\, \psi^{\lambda*}_{JM} + \langle T^{J-M}\rangle\, \psi^{\lambda*}_{J-M}\right\} =$$
$$= \langle T^{J0}\rangle\, \psi^{\lambda*}_{J0} + \sum_{M=1}^{J}\left\{ \langle T^{JM}\rangle\, \psi^{\lambda*}_{JM} + (-1)^{J+M}\langle T^{J-M}\rangle\, \psi^{\lambda}_{JM}\right\} \qquad (A.4)$$

To obtain the latter expression use was made of the relation

$$\psi^{\lambda*}_{J-M} = (-1)^{J+M} \psi^{\lambda}_{JM} \tag{A.5}$$

which follows from the definition of the functions ψ^{λ}_{JM} , if it be taken into account that, in accord with (I2), the phases of the spherical functions are chosen so that

$$Y^{*}_{\ell m} = (-1)^{\ell-m} Y_{\ell-m}$$

In the following it will be considered that polarization of particle a is the result of a reaction with unpolarized particles. In this case, as will be shown below,

$$\langle T^{JM} \rangle = (-1)^{J+M} \langle T^{J-M} \rangle \tag{A.6}$$

(A.4) then reduced to

$$\langle T^{J0} \rangle \psi^{\lambda*}_{J0} + \sum_{M=1}^{J} \langle T^{JM} \rangle \left\{ \psi^{\lambda*}_{JM} + \psi^{\lambda}_{JM} \right\} =$$
$$= \langle T^{J0} \rangle \psi^{\lambda*}_{J0} + \sum_{M=1}^{J} \langle T^{JM} \rangle B^{\lambda}_{JM} 2\cos M\varphi \tag{A.7}$$

as   in the chosen system

$$\psi^{\lambda}_{JM} = e^{iM\varphi} B^{\lambda}_{JM}(\theta) \;\; ; \;\; B^{\lambda*}_{JM}(\theta) = B^{\lambda}_{JM}(\theta)$$

From (A.7), (A.3) and (A.A) it follows

$$\sigma_{ba}(\vec{n}_b, \vec{n}_a) = \sum_{J=0}^{2i_1} \sum_{\lambda} A^{\lambda}_{J} \left\{ \langle T^{J0} \rangle \psi^{\lambda*}_{J0} + 2\sum_{M=1}^{J} \langle T^{JM} \rangle B^{\lambda}_{JM} \cos M\varphi \right\}, \tag{A.8}$$

and this can be transformed to

$$\sigma_{ba}(\vec{n}_b, \vec{n}_a) = \sigma^{o}_{ba}(\vec{n}_b, \vec{n}_a) + \sum_{q=1}^{i_1} \langle T^{2q,0} \rangle A_{2q}(\theta) +$$
$$+ \sum_{k=1}^{2i_1} \cos k\varphi \cdot \sin^k\theta \sum_{J=k}^{2i_1} B_J(\theta) \langle T^{Jk} \rangle \tag{A.9}$$

In virtue of property (A.6) only terms with even J remain in the first sum in (A.9).

We now return to the proof of relations (A.6). For $\langle T^{JM} \rangle$ we have

$$\sigma_0 \langle T^{JM} \rangle = \frac{1}{n} Sp M \overset{+}{M} T^{JM}$$

where the quantities σ_0 and MM^+ refer to a reaction in which particle a is produced; n is the number of initial spin states of this reaction. Employing for $M\overset{+}{M}$ an expansion of the same type as (A.2)

$$MM^+ = \sum_{J\lambda} C_J^\lambda \sum_M \psi_{JM}^\lambda \overset{+}{T}^{JM} + R \qquad (A.10)$$

we get

$$\sigma_0 \langle T^{JM} \rangle = \frac{1}{n} \sum_\lambda C_J^\lambda \psi_{JM}^\lambda \qquad (A.11)$$

In the chosen coordinate system the ψ_{JM}^λ are real ($2z$ is an even number as MM^+ is always a scalar). Making use of this circumstance we immediately obtain (A.6) from (A.11) and (A.3).

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