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ABSTRACT

Explicit expressions for all the observed effects in the reaction $p+p \to d+\pi^+$ with a consideration of the S-, p- and d- states of the π-meson are given in the paper.

I. INTRODUCTION

When performing the phenomenological analysis of the reaction

$$p + p \to d + \pi^+ \tag{I}$$

in the proton energy region $\sim$ 400-600 Mev it is necessary to express all the observed effects in terms of the S-matrix elements with a consideration of the s -, p- and d- states of the positive π^+ meson.

In spite of the fact that the analysis of the possible polarization effects in reaction /I/ was considered by many authors /I-6, I2/ in detail in neither of these papers the necessary expressions are given in full. In this paper all the ratios are presented which may be directly used in the analysis of the experimental data.

Since a pion is a spinless particle, then it is necessary to set the spin deuteron state which is determined by the mean values of the six independent matrices. The operators of spin-tensors $\hat{T}_{J\bar{m}}$ /7,8/, which have convenient transformation properties are used as independent matrices. The spin-tensor operators $\hat{T}_{J,m}$ satisfy the orthoginality and normalization relations $Sp\, T_{Jm} T^{+}_{J'm'} = (2S+1)\delta_{Jm,\,J'm'}$ and are expressed in terms of the operators of spin projections as follows :/7/

$$\begin{aligned}
&\hat{T}_{22} = \frac{\sqrt{3}}{2}(\hat{S}_x + i\,\hat{S}_y)^2; && \hat{T}_{11} = -\frac{\sqrt{3}}{2}(\hat{S}_x + i\,\hat{S}_y);\\
&\hat{T}_{21} = -\frac{\sqrt{3}}{2}\left[(\hat{S}_x + i\hat{S}_y)\hat{S}_z + \hat{S}_z(\hat{S}_x + i\hat{S}_y)\right]; && \hat{T}_{10} = \sqrt{\frac{3}{2}}\,\hat{S}_z;\\
&\hat{T}_{20} = \frac{1}{\sqrt{2}}(3\hat{S}_z^2 - 2); && \hat{T}_{00} = 1.
\end{aligned} \tag{2}$$

Using for the operators of the deuteron spin projection the matrices in the form

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}; \quad S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}; \quad S_z = \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix},$$

we obtain

$$\begin{aligned}
&T_{22} = \sqrt{3}\begin{pmatrix} 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}; \quad T_{21} = -\sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\end{pmatrix}; \quad T_{20} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & 1\end{pmatrix}\\
&T_{11} = -\sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}; \quad T_{10} = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}; \quad T_{00} = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.
\end{aligned} \tag{3}$$

For the process of particle scattering or for the reactions of the type $A+b \rightarrow C+d$ the spin -tensors are given with respect to the coordinate system, the z-axis of which is directed along the velocity of the incident particle A. This coordinate system is given in Fig. I. The system chosen is a right-hand system, the X- axis of which is directed so that the plane xOz would be a horisontal one. The mean values of the spin-tensor operators in the final state are necessary to be considered in the coordinate system from the standpoint of the experimental conditions, the z' axis of which is directed along the velocity vector of the recorded secondary particle, -in our case, of the deuteron- emitted in the direction (θ, φ).

If $\vec{k}$ is the wave vector of the incident particle, and $\vec{k'}$ is that of the secondary particle, then the Oy axis of the new coordinate system should be directed along the vector $\vec{n}=[\vec{k}\times\vec{k'}]$. Define the Eiler transition angles from the old coordinate system associated with the primary beam to the new system defined by the direction of the secondary particle and by the choice of the Oy axis . Three succesive rotations are necessary for this :

I) The rotation with respect to the Oz axis at the angle $\varphi_1=\varphi+\frac{\pi}{2}$, in the positive direction (Fig.2)

2) the rotation with respect to the Ox' axis at the angle θ in the positive direction (Fig.3) and, finally,

3) the rotation with respect to the Oz' axis at the angle $\varphi_2=-\frac{\pi}{2}$ also in the positive direction.

Thus, the Eiler angles of the transformation from the coordinate system of the beam to the coordinate system connected with the secondary particle are equal to

$$\varphi_1=\varphi+\frac{\pi}{2};\quad \theta;\quad \varphi_2=-\frac{\pi}{2}.$$

The spin- tensor operators $T_{J'm'}$ in the new coordinate system are determined by the operators T_{Jm} in the coordinate system connected with the beam by means of the relations /I3/

$$T_{Jm'}=\sum_m D^{(J)}_{mm'}(\varphi_1,\theta,\varphi_2)\cdot T_{Jm}, \qquad (4)$$

where $D^{(J)}_{mm'}$ are the elements of the threedimensional rotation group.

We have

$$D^{(J)}_{mm'}(\varphi_1,\theta,\varphi_2)=e^{-im\varphi_1}P^J_{mm'}(\cos\theta)\cdot e^{-im'\varphi_2},$$

where, in its turn, $P^J_{mm'}(\cos\theta)$ are the generalized Legandre polinomials. For our case we obtain

$$D^J_{mm'}(\varphi+\frac{\pi}{2};\theta;-\frac{\pi}{2})=e^{-im(\varphi+\frac{\pi}{2})}P^J_{mm'}(\cos\theta)e^{im'\frac{\pi}{2}}=P^J_{mm'}(\cos\theta)\cdot e^{-im\varphi}\cdot i^{m'-m} \qquad (5)$$

The matrices $P^{J}_{mm'}(\cos\theta)$ for $J = 1$ and $J = 2$ are of the form

$$P^{(1)}_{mm'}(\cos\theta) = \begin{vmatrix} \frac{1}{2}(1+\cos\theta) & \frac{i}{\sqrt{2}}\sin\theta & \frac{1}{2}(\cos-1) \\ \frac{i}{\sqrt{2}}\sin\theta & \cos\theta & \frac{i}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(\cos\theta-1) & \frac{i}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{vmatrix} \tag{6}$$

$$P^{(2)}_{mm'}(\cos\theta) = \begin{vmatrix} \frac{1}{4}(\cos\theta+1)^2 & \frac{i}{2}\sin\theta(\cos\theta+1) & -\frac{1}{2}\sqrt{\frac{3}{2}}(1-\cos^2\theta) & \frac{i}{2}\sin\theta(\cos\theta-1) & \frac{1}{4}(\cos\theta-1)^2 \\ \frac{i}{2}\sin\theta(\cos\theta+1) & \frac{1}{2}(2\cos^2\theta+\cos\theta-1) & \sqrt{\frac{3}{2}}\,i\sin\theta\cos\theta & \frac{1}{2}(2\cos^2\theta-\cos\theta-1) & \frac{i}{2}\sin\theta(\cos\theta-1) \\ -\frac{1}{2}\sqrt{\frac{3}{2}}(1-\cos^2\theta) & \sqrt{\frac{3}{2}}\,i\sin\theta\cos\theta & \frac{1}{2}(3\cos^2\theta-1) & \sqrt{\frac{3}{2}}\,i\sin\theta\cos\theta & -\frac{1}{2}\sqrt{\frac{3}{2}}(1-\cos^2\theta) \\ \frac{i}{2}\sin\theta(\cos\theta-1) & \frac{1}{2}(2\cos^2\theta-\cos\theta-1) & \sqrt{\frac{3}{2}}\,i\sin\theta\cos\theta & \frac{1}{2}(2\cos^2\theta+\cos\theta-1) & \frac{i}{2}\sin\theta(\cos\theta+1) \\ \frac{1}{4}(\cos\theta-1)^2 & \frac{i}{2}\sin\theta(\cos\theta-1) & -\frac{1}{2}\sqrt{\frac{3}{2}}(1-\cos^2\theta) & \frac{i}{2}\sin\theta(\cos\theta+1) & \frac{1}{4}(\cos\theta+1)^2 \end{vmatrix} \tag{7}$$

The lines and colomns are enumerated from $-J$ to J down-ward and from left to right.

The auxiliary factors of kind $e^{-im\varphi}\cdot i^{m'-m}$ may be presented as a general matrix so that to obtain the element D^{J} it is necessary to multiply the element $P^{J}_{mm'}$ by the corresponding elements of the matrix $e^{-im\varphi}\cdot i^{m'-m}$ which has the form

$$\begin{array}{ccccc|l} e^{i2\varphi} & ie^{i2\varphi} & -e^{i2\varphi} & -ie^{i2\varphi} & e^{i2\varphi} & \\ -ie^{i\varphi} & \boxed{e^{i\varphi}\quad ie^{i\varphi}\quad -e^{i\varphi}} & & & -ie^{i\varphi} & \text{for } J=1 \\ -1 & -i \quad\quad 1 \quad\quad i & & & -1 & \\ ie^{-i\varphi} & -e^{-i\varphi}\quad -ie^{-i\varphi}\quad e^{-i\varphi} & & & ie^{-i\varphi} & \\ e^{-i2\varphi} & ie^{-i2\varphi} & -e^{-i2\varphi} & -ie^{-i2\varphi} & e^{-i2\varphi} & \text{for } J=2 \end{array} \tag{8}$$

If $\varphi = 0$ and θ is small what occurs for reaction (I) when the deuteron having the limiting kinematic angle $\sim$ I0-I5^{0} then we obtain with the accuracy up to the terms quadratic over θ

$$D^{(1)} \approx \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}}\theta & 0 \\ \frac{1}{\sqrt{2}}\theta & 1 & -\frac{1}{\sqrt{2}}\theta \\ 0 & \frac{1}{\sqrt{2}}\theta & 1 \end{pmatrix}$$

$$D^{(2)} \approx \begin{pmatrix} 1 & -\theta & 0 & 0 & 0 \\ \theta & 1 & -\sqrt{\frac{3}{2}}\theta & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}}\theta & 1 & -\sqrt{\frac{3}{2}}\theta & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}\theta & 1 & -\theta \\ 0 & 0 & 0 & \theta & 1 \end{pmatrix}$$

2. Polarized Nucleon Beam

In this paper a method of considering the polarization phenomena suggested by V.B. Berestetsky /2/ was used.

Consider the proton beam incident on the target and polarized in the direction (ϑ, δ) with respect to their direction of motion. The spin wave function of such a pure ansemble is given by a pair of complex numbers: $q_1' \equiv q_{1/2}$ and $q_2' \equiv q_{-1/2}$

To write the explicit expressions q_1' and q_2' as the functions of the angles ϑ, δ consider the coordinate system with the z' axis directed at the spin vector, or more exactly, in which the spin function has the form: $q_1 = 1;\ q_2 = 0$. To pass into the new coordinate system it is necessary to perform the reverse spinor transformation /I4/

$$\left.\begin{aligned} q_1' &= \alpha^* q_1 + \gamma^* q_2 \\ q_2' &= \beta^* q_1 + \delta^* q_2 \end{aligned}\right\} ,$$

the matrix of which is equal to

$$\begin{vmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{vmatrix} = \begin{vmatrix} \cos\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi_1+\varphi_2)} & -i\sin\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi_2-\varphi_1)} \\ -i\sin\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi_2-\varphi_1)} & \cos\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi_1+\varphi_2)} \end{vmatrix} ,$$

where $\varphi_1, \theta, \varphi_2$ are the Eiler transformation angles. In our case

$$\varphi_1 = \delta + \frac{\pi}{2}; \quad \theta = \vartheta; \quad \varphi_2 = -\frac{\pi}{2}$$

and

$$q_1' = \cos\frac{\vartheta}{2}\, e^{-\frac{i}{2}\delta}; \qquad q_2' = \sin\frac{\vartheta}{2}\, e^{\frac{i}{2}\delta}. \tag{9}$$

Thus, the spin function of the pure state of the nucleon beam polarized in the direction $/\vartheta, \delta/$ may be written as follows:

$$q_1 \alpha(1) + q_2 \beta(1),$$

where $\alpha = \binom{1}{0}$; $\beta = \binom{0}{1}$, $q_1 = \cos\frac{\vartheta}{2} e^{-\frac{i}{2}\delta}$; $q_2 = \sin\frac{\vartheta}{2} e^{\frac{i}{2}\delta}$.

As usual in the experiments the nucleon beam is used the polarization vector of which is directed along the Oy axis . Such a kind of the polarization arises if the proton beam partly polarized is obtained as a result of the scattering of the primarily unpolarized beam at any nucleus- polarizator. Then $\vartheta = 90^\circ$, $\delta = 90^\circ$ whereas

$$q_1 = \frac{1}{\sqrt{2}} e^{-\frac{i}{4}\pi}; \quad q_2 = \frac{1}{\sqrt{2}} e^{\frac{i}{4}\pi}. \tag{IO}$$

It can be easily seen that the mean values of the operators of the spin projections on the coordinate axes determined by the relations/II/

$$\langle \sigma_z \rangle = |q_1|^2 - |q_2|^2;$$
$$\langle \sigma_x \rangle = 2 Re(q_1^* q_2);$$
$$\langle \sigma_y \rangle = 2 Im(q_1^* q_2);$$

are found in the given case to be

$$\langle \sigma_z \rangle = |q_1|^2 - |q_2|^2 = \frac{1}{2} - \frac{1}{2} = 0$$
$$\langle \sigma_y \rangle = 2 Im(q_1^* q_2) = 2 Im\left(\frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}} \frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}}\right) = 1;$$
$$\langle \sigma_x \rangle = 2 Re(q_1^* q_2) = 0.$$

3. Unpolarized Target

The unpolarized proton target cannot be described by a wave function since such a target is a mixture of two states, i.e., it is necessary to give two wave functions and the values of weights with which the latter are presented in the target. To make the caloulations more convenient one may write

$$\varphi = \varepsilon_1 \alpha(2) + \varepsilon_2' \beta(2).$$

The coefficients ε_I and ε_2 satisfy the condition $|\varepsilon_1|^2 = |\varepsilon_2|^2 = \frac{1}{2}$ (i.e.,both spin projections are equally probable) and the formal requirement $\varepsilon_1^* \varepsilon_2$ /the mixture requirement/.

4. Initial Wave Function of Two Protons

The initial wave function of two proton system, one of which is completely polarized, whereas the other- unpolarized is written as follows:

$$\Psi_n'(1,2) = e^{ikz}\left[q_1\alpha(1) + q_2\beta(1)\right]\left[\varepsilon_1\alpha(2) + \varepsilon_2\beta(2)\right]$$

For /p-p/ collisions the initial wave function must be antisymmetrized so that :

$$\Psi_{init} = \frac{1}{\sqrt{2}}\left[\Psi_n'(1,2) - \Psi_n'(2,1)\right].$$

As a result we obtain

$$\Psi_{init} = \frac{\varepsilon_1}{\sqrt{2}}\left\{\left[q_1\alpha(1)+q_2\beta(1)\right]\cdot\alpha(2)\cdot e^{ikz} - \left[q_1\alpha(2)+q_2\beta(2)\right]\cdot\alpha(1)\cdot e^{-ikz}\right\} +$$
$$+\frac{\varepsilon_2}{\sqrt{2}}\left\{\left[q_1\alpha(1)+q_2\beta(1)\right]\cdot\beta(2)\cdot e^{ikz} - \left[q_1\alpha(2)+q_2\beta(2)\right]\cdot\beta(1)\cdot e^{-ikz}\right\} =$$
$$= \frac{\varepsilon_1}{\sqrt{2}}\Psi_1 + \frac{\varepsilon_2}{\sqrt{2}}\Psi_2$$

If denote $\frac{\varepsilon_1}{\sqrt{2}}\Psi_1 = \varphi_1$; $\frac{\varepsilon_2}{\sqrt{2}}\Psi_2 = \varphi_2$ and make some transformations we get

$$\varphi_1 = \frac{\varepsilon_1}{\sqrt{2}}\left\{\left(q_1\chi_{1,1} + \frac{q_2}{\sqrt{2}}\chi_{1,0}\right)\left(e^{ikz} - e^{-ikz}\right) + \frac{q_2}{\sqrt{2}}\chi_{0,0}\left(e^{ikz} + e^{-ikz}\right)\right\}$$
$$\varphi_2 = \frac{\varepsilon_2}{\sqrt{2}}\left\{\left(q_2\chi_{1,-1} + \frac{q_1}{\sqrt{2}}\chi_{1,0}\right)\left(e^{ikz} - e^{-ikz}\right) - \frac{q_1}{\sqrt{2}}\chi_{0,0}\left(e^{ikz} + e^{-ikz}\right)\right\};$$

where $\chi_{1,1} = \alpha(1)\cdot\alpha(2)$;

$$\chi_{1,0} = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) + \beta(1)\cdot\alpha(2)\right];\quad \chi_{0,0} = \frac{1}{\sqrt{2}}\left[\beta(1)\alpha(2) - \alpha(1)\beta(2)\right];$$
$$\chi_{1,-1} = \beta(1)\cdot\beta(2);$$

Expanding the plane wave in spherical harmonics and making use of the asymptotic representation for the functions $j_\ell(kr) = \sqrt{\frac{\pi}{2kr}}\,J_{\ell+1/2}(kr)$ at $kr \gg \ell$ we get /9/

$$e^{ikz} \approx \sum_{\ell=0}^{\infty} i^\ell\sqrt{4\pi}\cdot\sqrt{2\ell+1}\cdot\frac{1}{2ikr}\left[e^{i(kr-\frac{\ell\pi}{2})} - e^{-i(kr-\frac{\ell\pi}{2})}\right]Y_{\ell,0}(\cos\theta);$$

and

$$e^{ikz} \pm e^{-ikz} = \frac{i}{kr}\sqrt{\pi}\sum_{\ell=0}^{\infty} i^\ell\sqrt{2\ell+1}\cdot\left[1\pm(-1)^\ell\right]\left[e^{-i(kr-\frac{\ell\pi}{2})} + e^{i(kr-\frac{\ell\pi}{2})}\right]\cdot Y_{\ell,0}(\cos\theta).$$

Omitting the factor $\frac{i}{kr}$, independent of the angles / θ, φ / and of the index ℓ , and taking into account only the ingoing wave we find

$$\varphi_1 = \varepsilon_1\sqrt{\frac{\pi}{2}}\sum_{\ell=0}^{\infty} i^\ell\sqrt{2\ell+1}\cdot\left\{\left[1-(-1)^\ell\right]\left[q_1\chi_{1,1}Y_{\ell,0}(\cos\theta) + \frac{q_2}{\sqrt{2}}\chi_{1,0}Y_{\ell,0}(\cos\theta)\right] +\right.$$
$$\left. + \left[1+(-1)^\ell\right]\cdot\frac{q_2}{\sqrt{2}}\chi_{0,0}Y_{\ell,0}(\cos\theta)\right\}$$

$$\varphi_2 = \varepsilon_2 \sqrt{\frac{\pi}{2}} \sum_{\ell=0}^{\infty} i^\ell \sqrt{2\ell+1} \Big\{ [1-(-1)^\ell][q_2 \chi_{1,-1} Y_{\ell,0}(\cos\theta) + \frac{q_1}{\sqrt{2}} \chi_{1,0} Y_{\ell,0}(\cos\theta)] - $$
$$ - [1+(-1)^\ell] \cdot \frac{q_1}{\sqrt{2}} \chi_{0,0} \cdot Y_{\ell,0}(\cos\theta) \Big\}.$$

Express further the products $\chi_{S,M} Y_{\ell,0}$ in terms of the eigenfunctions of the total momentum J and its projection M , making use of the relations

$$Y_{\ell,0}(\cos\theta) \cdot \chi_{1,M} = \sum_{J=\begin{cases}\ell-1\\ \ell \\ \ell+1\end{cases}} (\ell 1 0 M | \ell 1 J M) \mathcal{Y}^M_{J(\ell 1)}(\cos\theta, \varphi);$$

$$Y_{\ell,0}(\cos\theta)\chi_{0,0} = \mathcal{Y}^0_{\ell(\ell 0)},$$

where $(\ell 1 0 M|\ell 1 J M)$ are the Clebsh- Gordon coefficients, while $\mathcal{Y}^M_{J(\ell 1)}$ is the eigenfunction of the particle with spin $S = 1$, with the total momentum J and the projection of the total momentum M. In our particular case of the plane wave expanding along the OZ axis the projection of the total momentum is M equal to the spin projection S_z . Then

$$\varphi_1 = \varepsilon_1 \sqrt{\frac{\pi}{2}} \sum_{\ell=0}^{\infty} i^\ell \sqrt{2\ell+1} \Big\{ [1-(-1)^\ell][q_1 \sum_J (\ell 1 0 1/\ell 1 J 1) \mathcal{Y}^1_{J(\ell 1)} + $$
$$ + \frac{q_2}{\sqrt{2}} \sum_J (\ell 1 0 0/\ell 1 J 0) \mathcal{Y}^0_{J(\ell 1)}] + [1+(-1)^\ell] \cdot \frac{q_2}{\sqrt{2}} \mathcal{Y}^0_{\ell(\ell 0)} \Big\};$$

$$\varphi_2 = \varepsilon_2 \sqrt{\frac{\pi}{2}} \sum_{\ell=0}^{\infty} i^\ell \sqrt{2\ell+1} \Big\{ [1-(-1)^\ell][q_2 \sum_J (\ell 1 0 -1|\ell 1 J -1) \mathcal{Y}^{-1}_{J(\ell 1)} + $$
$$ + \frac{q_1}{\sqrt{2}} \sum_J (\ell 1 0 0|\ell 1 J 0) \mathcal{Y}^0_{J(\ell 1)}] + [1+(-1)^\ell] \cdot \frac{q_1}{\sqrt{2}} \mathcal{Y}^0_{\ell(\ell 0)} \Big\}.$$

5. Possible Transitions in the Reaction $p+p \to d+\pi^+$

At the proton energy $\sim$ 600 Mev it is necessary to take into account S-, p- and d- states of a positive pion in reaction /I/. This leads to the following possible transitions:

Initial state	$^3P_1$		$^1S_0$	$^1D_2$	$^3P_2$	$^3F_2$	$^3F_3$
Final state	$^3S_1\ s_1$	$^3S_1\ d_1$	$^3S_1\ p_0$	$^3S_1\ p_2$	$^3S_1\ d_2'$	$^3S_1\ d_2$	$^3S_1\ d_3$
Transition amplitude	C'_{s1}	C'_{d1}	C_{p0}	C_{p2}	Cd'_2	Cd'_3	Cd'_4

The deuteron state was necessary to denote as $(^3S_1 + {}^3D_1)$, however, as in the previous papers we write everywhere $^3S_1$.

The transition amplitudes denoted in Table /I/ as it is seen from the following possess the properties of the S-matrix elements formulated in /9/.

Taking into account only those initial states which are presented in Table I, we get the expression for the ingoing wave described by ψ_1 and ψ_2:

$$\psi_1 = \varepsilon_1\sqrt{2\pi}\cdot\Big\{iq_1\Big[-\sqrt{\tfrac{3}{2}}\,Y^1_{1(11)}+\sqrt{\tfrac{3}{2}}\,Y^1_{2(11)}-Y^1_{2(31)}+\sqrt{\tfrac{7}{2}}\,Y^1_{3(31)}\Big]+$$

$$+iq_2\Big[Y^0_{2(11)}+\sqrt{\tfrac{3}{2}}\,Y^0_{2(31)}\Big]+q_2\Big[\sqrt{\tfrac{1}{2}}\,Y^0_{0(00)}-\sqrt{\tfrac{5}{2}}\,Y^0_{2(20)}\Big]\Big\};$$

$$\psi_2 = \varepsilon_2\sqrt{2\pi}\Big\{iq_2\Big[\sqrt{\tfrac{3}{2}}\,Y^{-1}_{1(11)}+\sqrt{\tfrac{3}{2}}\,Y^{-1}_{2(11)}-Y^{-1}_{2(31)}-\sqrt{\tfrac{7}{2}}\,Y^{-1}_{3(31)}\Big]+$$

$$+iq_1\Big[Y^0_{2(11)}+\sqrt{\tfrac{3}{2}}\,Y^0_{2(31)}\Big]-q_1\Big[\sqrt{\tfrac{1}{2}}\,Y^0_{0(000)}-\sqrt{\tfrac{5}{2}}\,Y^0_{2(20)}\Big]\Big\}. \tag{12}$$

Let us construct the final wave functions. Since in the final state the particle with spin S = I is emitted then the three component wave functions - spherical-vectors $\vec{Y}_{J\ell M}$ are necessary to be used. Where

$$\vec{Y}_{J\ell M} = \sum_\mu (\ell 1 M-\mu,\mu|\ell 1 J M)\,Y_{\ell,M-\mu}(\theta,\varphi)\cdot\vec{\chi}_{1,\mu}$$

The explicit expressions for those spherical vectors which we shall be in need of further have the following form /15/

$$\vec{Y}_{321}=\sqrt{\tfrac{1}{4\pi}}\Big[\tfrac{1}{\sqrt{8}}\sin^2\theta\cdot e^{i2\varphi}\cdot\vec{\chi}_{1,-1}-2\sin\theta\cos\theta\cdot e^{i\varphi}\vec{\chi}_{1,0}+\tfrac{1}{\sqrt{2}}(3\cos^2\theta-1)\cdot\vec{\chi}_{1,1}\Big];$$

$$\vec{Y}_{32,-1}=\sqrt{\tfrac{1}{4\pi}}\Big[\sqrt{\tfrac{1}{8}}\sin^2\theta\cdot e^{-i2\varphi}\vec{\chi}_{1,1}+2\sin\theta\cdot\cos\theta\cdot e^{-i\varphi}\vec{\chi}_{1,0}+\tfrac{1}{\sqrt{2}}(3\cos^2\theta-1)\cdot\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{221}=\sqrt{\tfrac{1}{4\pi}}\Big[-\tfrac{1}{2}\sqrt{\tfrac{5}{2}}(3\cos^2\theta-1)\vec{\chi}_{1,1}-\sqrt{\tfrac{5}{4}}\sin\theta\cos\theta\, e^{i\varphi}\vec{\chi}_{1,0}+\sqrt{\tfrac{5}{8}}\sin^2\theta\cdot e^{i2\varphi}\cdot\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{22,-1}=\sqrt{\tfrac{1}{4\pi}}\Big[\tfrac{1}{2}\sqrt{\tfrac{5}{2}}(3\cos^2\theta-1)\vec{\chi}_{1,-1}-\sqrt{\tfrac{5}{4}}\sin\theta\cos\theta\cdot e^{-i\varphi}\cdot\vec{\chi}_{1,0}-\sqrt{\tfrac{5}{8}}\sin^2\theta\cdot e^{-i2\varphi}\cdot\vec{\chi}_{1,1}\Big];$$

$$\vec{Y}_{121}=\sqrt{\tfrac{1}{4\pi}}\Big[\tfrac{1}{2\sqrt{2}}(3\cos^2\theta-1)\vec{\chi}_{1,1}+\tfrac{3}{2}\sin\theta\cos\theta\cdot e^{i\varphi}\cdot\vec{\chi}_{1,0}+\tfrac{3}{2\sqrt{2}}\sin^2\theta\cdot e^{i2\varphi}\cdot\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{12,-1}=\sqrt{\tfrac{1}{4\pi}}\Big[\tfrac{3}{2\sqrt{2}}\sin^2\theta\cdot e^{-i2\varphi}\vec{\chi}_{1,1}-\tfrac{3}{2}\sin\theta\cdot\cos\theta\cdot e^{-i\varphi}\vec{\chi}_{1,0}+\tfrac{1}{2\sqrt{2}}(3\cos^2\theta-1)\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{220}=\sqrt{\tfrac{1}{4\pi}}\sqrt{\tfrac{15}{4}}\sin\theta\cdot\cos\theta\cdot\Big[-e^{-i\varphi}\vec{\chi}_{1,1}-e^{i\varphi}\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{210}=\sqrt{\tfrac{1}{4\pi}}\Big[\tfrac{1}{2}\sin\theta\cdot e^{-i\varphi}\vec{\chi}_{1,1}+\sqrt{2}\cdot\cos\theta\cdot\vec{\chi}_{1,0}-\tfrac{1}{2}\sin\theta\cdot e^{i\varphi}\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{010}=\sqrt{\tfrac{1}{4\pi}}\Big[\sqrt{\tfrac{1}{2}}\sin\theta\cdot e^{-i\varphi}\vec{\chi}_{1,1}-\cos\theta\cdot\vec{\chi}_{1,0}-\sqrt{\tfrac{1}{2}}\sin\theta\, e^{i\varphi}\vec{\chi}_{1,-1}\Big];$$

$$\vec{Y}_{10,\pm1}=\sqrt{\tfrac{1}{4\pi}}\;\vec{\chi}_{1,\pm1}. \tag{13}$$

There is only outgoing wave in the final state, the coefficients B of which are connected with the coefficients A of the initial ingoing wave by the scattering matrix S /9/ as follows

$$B^{J\ell' M}=\sum_\ell S^J_{\ell'\ell}\,A^{J\ell M}. \tag{14}$$

Table II.

Transition	The coefficients A of the initial wave function		The coefficients B of the final wave function	
	$A_{\uparrow}^{JeM} \times \frac{1}{\varepsilon_1\sqrt{2\pi}}$	$A_{\downarrow}^{JeM} \times \frac{1}{\varepsilon_2\sqrt{2\pi}}$	$B_{\uparrow}^{Je'M} = \sum_{e} S_{ll'}^{J} A^{JeM}$	$B_{\downarrow}^{Je'M} = \sum_{e} S_{ll'}^{J} A^{JeM}$
$^3P_1 \nearrow {}^3S_1\, s_1$	$-iq_1\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{1}_{1(11)})$	$iq_2\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{-1}_{1(11)})$	$-iq_1\sqrt{\frac{3}{2}}\,c_{s_1}' \quad (Y_{101})$	$iq_2\sqrt{\frac{3}{2}}\,c_{s_1}' \quad (Y_{1,0,-1})$
$\searrow {}^3S_1\, d_1$	$-iq\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{1}_{1(11)})$	$iq_2\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{-1}_{1(11)})$	$-iq_1\sqrt{\frac{3}{2}}\,Cd_1' \quad (Y_{121})$	$iq_2\sqrt{\frac{3}{2}}\,Cd_1' \quad (Y_{1,2,-1})$
$^1S_0 \longrightarrow {}^3S_1\, p_0$	$\sqrt{\frac{1}{2}}\,q_2 \quad (\mathcal{Y}^{0}_{0(00)})$	$-\sqrt{\frac{1}{2}}\,q_1 \quad (\mathcal{Y}^{0}_{0(00)})$	$\sqrt{\frac{1}{2}}\,q_2 C_{p_0} \quad (Y_{010})$	$-\sqrt{\frac{1}{2}}\,q_1 C_{p_0} \quad (Y_{010})$
$^1D_2 \longrightarrow {}^3S_1\, p_2$	$-\sqrt{\frac{5}{2}}\,q_2 \quad (\mathcal{Y}^{0}_{2(20)})$	$\sqrt{\frac{5}{2}}\,q_1 \quad (\mathcal{Y}^{0}_{2(20)})$	$-\sqrt{\frac{5}{2}}\,q_2 C_{p_2} \quad (Y_{210})$	$\sqrt{\frac{5}{2}}\,C_1 C_{p_2} \quad (Y_{210})$
$^3P_2 \longrightarrow {}^3S_1\, d_2$	$iq_1\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{1}_{2(11)})$	$iq_2\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{-1}_{2(11)})$	$iq_1\sqrt{\frac{3}{2}}\,Cd_2' \quad (Y_{221})$	$iq_2\sqrt{\frac{3}{2}}\,Cd_2' \quad (Y_{22,-1})$
	$iq_2 \quad (\mathcal{Y}^{0}_{2(11)})$	$iq_1 \quad (\mathcal{Y}^{0}_{2(11)})$	$iq_2\,Cd_2' \quad (Y_{220})$	$iq_1\,Cd_2' \quad (Y_{220})$
$^3F_2 \longrightarrow {}^3S_1\, d_2$	$-iq_1 \quad (\mathcal{Y}^{0}_{2(31)})$	$-iq_2 \quad (\mathcal{Y}^{-1}_{2(31)})$	$-iq_1\,Cd_3' \quad (Y_{221})$	$-iq_2\,Cd_3' \quad (Y_{22,-1})$
	$iq_2\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{0}_{2(31)})$	$iq_1\sqrt{\frac{3}{2}} \quad (\mathcal{Y}^{0}_{2(31)})$	$iq_2\sqrt{\frac{3}{2}}\,C'd_3 \quad (Y_{220})$	$iq_1\sqrt{\frac{3}{2}}\,Cd_3' \quad (Y_{220})$
$^3F_3 \longrightarrow {}^3S_1\, d_3$	$iq_1\sqrt{\frac{7}{2}} \quad (\mathcal{Y}^{1}_{3(31)})$	$-iq_2\sqrt{\frac{7}{2}} \quad (\mathcal{Y}^{-1}_{3(31)})$	$iq_1\sqrt{\frac{7}{2}}\,C'd_4 \quad (Y_{321})$	$-iq_2\sqrt{\frac{7}{2}}\,Cd_4' \quad (Y_{32,-1})$

Here it is denoted $C_s' = iC_s$, and also $C_d' = -iC_d$

Such relation is necessary to use twice, for the initial wave φ_1, describing the target protons polarized "upward", and also for φ_2 to which the target protons polarized "downward".

The results of the calculations of the coefficients $B^{J\ell' M}$, as well as the values $A^{J\ell M}$ used are given in detail in Table 2. In brackets there indicated eigenfunctions $\mathcal{Y}, Y$ of the total momentum J and M at the beginning for the two proton system, at the end -for the pion and deuteron system.

The final wave functions are written as follows

$$\left.\begin{aligned} \vec{F}_1 &= \alpha_1 \vec{\chi}_{1,1} + \beta_1 \vec{\chi}_{1,0} + \gamma_1 \vec{\chi}_{1,-1}; \\ \vec{F}_2 &= \alpha_2 \vec{\chi}_{1,1} + \beta_2 \vec{\chi}_{1,0} + \gamma_2 \vec{\chi}_{1,-1}. \end{aligned}\right\} \tag{15}$$

The mean values of the spin-tensors are expressed in terms of the components of the wave functions /15/ as follows:

$$\begin{aligned} \langle T_{00}\rangle &= \sum_{i=1,2}(|\alpha_i|^2+|\beta_i|^2+|\gamma_i|^2); & \langle T_{11}\rangle &= -\sqrt{\tfrac{3}{2}}\sum_{i=1,2}(\alpha_i^*\beta_i+\beta_i^*\gamma_i); \\ \langle T_{10}\rangle &= \sqrt{\tfrac{3}{2}}\sum_{i=1,2}(|\alpha_i|^2-|\gamma_i|^2); & \langle T_{21}\rangle &= -\sqrt{\tfrac{3}{2}}\sum_{i=1,2}(\alpha_i^*\beta_i-\beta_i\gamma_i^*); \\ \langle T_{20}\rangle &= \sqrt{\tfrac{1}{2}}\sum_{i=1,2}(|\alpha_i|^2-2(|\beta_i|^2+|\gamma_i|^2) & \langle T_{22}\rangle &= \sqrt{3}\sum_{i=1,2}\alpha_i^*\gamma_i. \end{aligned} \tag{16}$$

6. Polarization Effects in the Reaction $p+p\rightarrow d+\pi^+$.

If one makes all the transformations using Table 2, and the expressions for the spherical vectors $\vec{Y}_{J\ell M}$ /13/ then for the components of the wave functions of the deuteron and pion system we get

$$\begin{aligned} \alpha_1 &= \frac{q_1}{2}\left[\sqrt{\tfrac{3}{2}}\,C_s + D_2(3\cos^2\theta-1)\right] + \frac{q_2}{4}\sin\theta\cdot e^{-i\varphi}(C_- - \sqrt{15}\cos\theta\cdot d_+); \\ \beta_1 &= -\frac{q_1}{2}\sin\theta\cdot\cos\theta\cdot e^{i\varphi}\cdot D_0 - \frac{q_2}{2\sqrt{2}}\cos\theta\cdot C_+; \\ \gamma_1 &= \frac{q_1}{2}\sin^2\theta\cdot e^{i2\varphi}D_1 - \frac{q_2}{4}\sin\theta\cdot e^{i\varphi}(C_- + \sqrt{15}\cos\theta\cdot d_+); \\ \\ \alpha_2 &= -\frac{q_1}{4}\sin\theta\cdot e^{-i\varphi}(C_- + \sqrt{15}\cos\theta\cdot d_+) - \frac{q_2}{2}\sin^2\theta\cdot e^{-i2\varphi}\cdot D_1; \\ \beta_2 &= \frac{q_1}{2\sqrt{2}}\cos\theta\cdot C_+ - \frac{q_2}{2}\sin\theta\cdot\cos\theta\cdot e^{-i\varphi}\cdot D_0; \\ \gamma_2 &= \frac{q_1}{4}\sin\theta\cdot e^{i\varphi}(C_- - \sqrt{15}\cos\theta\cdot d_+) - \frac{q_2}{2}\left[\sqrt{\tfrac{3}{2}}\,C_s + D_2(3\cos^2\theta-1)\right]. \end{aligned} \tag{17}$$

where (θ, φ) is the direction of the deuteron emission and

$$\left.\begin{aligned} C_+ &= Cp_0 + \sqrt{10}\, Cp_2; & D_0 &= \tfrac{3}{2}\sqrt{\tfrac{3}{2}}\, Cd_1 + \sqrt{\tfrac{5}{4}}\, d_- + \sqrt{14}\, Cd_4; \\ C_- &= Cp_0 - \sqrt{\tfrac{5}{2}}\, Cp_2; & D_1 &= \tfrac{\sqrt{7}}{4}\, Cd_4 + \sqrt{\tfrac{5}{8}}\, d_- - \tfrac{3\sqrt{3}}{4}\, Cd_1; \\ d_+ &= Cd_2 + \sqrt{\tfrac{3}{2}}\, Cd_3; & D_2 &= \tfrac{\sqrt{7}}{2}\, Cd_4 - \sqrt{\tfrac{5}{8}}\, d_- - \tfrac{\sqrt{3}}{4}\, Cd_1 \\ d_- &= \sqrt{\tfrac{3}{2}}\, Cd_2 - Cd_3; & & \end{aligned}\right\} \qquad (17a)$$

The mean values of the spin- tensors in the general case of the partly polarized beam may be written in the form

$$\langle T \rangle = \langle T_{unpol} \rangle + P \cdot \langle T \rangle_{pol} ,$$

where P is the degree of the proton beam polarization at the direction of the O_y axis. Then

$$\langle T_{00} \rangle_{unpol} = \gamma_0 + \gamma_2 \cos^2\theta + \gamma_4 \cdot \cos^4\theta ,$$

where

$$\left.\begin{aligned} \gamma_0 &= \tfrac{3}{8}|C_S|^2 + \tfrac{1}{8}|C_-|^2 + \tfrac{1}{4}\left(|D_1|^2 + |D_2|^2\right) - \tfrac{1}{2}\sqrt{\tfrac{3}{2}}\, \mathrm{Re}\, C C_S^* D_1); \\ \gamma_2 &= \tfrac{1}{8}\left(|C_+|^2 - |C_-|^2\right) + \tfrac{15}{8}|d_+|^2 + \tfrac{3}{2}\sqrt{\tfrac{3}{2}}\, \mathrm{Re}\,(C_S^* D_1) + \\ &\quad + \tfrac{1}{4}\left(|D_0|^2 - 2|D_1|^2 - 6|D_2|^2\right); \\ \gamma_4 &= \tfrac{1}{4}\left(9|D_2|^2 + |D_1|^2 - |D_0|^2\right) - \tfrac{15}{8}|d_+|^2 \end{aligned}\right\} \qquad (18)$$

If the main transition in reaction /I/ at energies $\sim$ 600 MeV is assumed, as it is done in /I0/, to be the transition $^1D_2 \rightarrow {}^3S_1 P_2$ while all the rest transitions are very unlikely if compared with this main one and if neglect the products and squares of the small amplitudes and also put the transition amplitudes $^3F_2 \rightarrow {}^3S_1 d_2$ and $^3F_3 \rightarrow {}^3S_1 d_3$ to be equal to zero then

$$\begin{aligned} \gamma_0' &= \tfrac{5}{16}|Cp_2|^2 - \tfrac{\sqrt{10}}{8}\, \mathrm{Re}\,(Cp_2^* Cp_0); \\ \gamma_2' &= \tfrac{15}{16}|Cp_2|^2 + \tfrac{3}{8}\sqrt{10}\, \mathrm{Re}\,(Cp_2^* Cp_0); \\ \gamma_4' &= 0 \end{aligned} \qquad (18a)$$

Further

$$\left.\begin{aligned} &\langle T_{00} \rangle_{pol} = \sin\theta \cdot \cos\varphi \cdot (\lambda_0 + \lambda_1 \cos\theta + \lambda_2 \cos^2\theta + \lambda_3 \cos^3\theta), \\ &\text{where} \\ &\lambda_0 = \tfrac{1}{4}\sqrt{\tfrac{3}{2}}\, \mathrm{Im}\,(C_-^* C_S) + \tfrac{1}{4}\, \mathrm{Im}\left[(D_1 + D_2)^* C_-\right]; \\ &\lambda_1 = \tfrac{1}{4}\sqrt{\tfrac{45}{2}}\, \mathrm{Im}\,(C_S^* d_+) + \tfrac{\sqrt{15}}{4}\, \mathrm{Im}\left[d_+^* (D_2 - D_1)\right]; \end{aligned}\right\}$$

$$\left.\begin{aligned}\lambda_2 &= \frac{1}{4}\, Im\left[c_-^*\,(3D_2 + D_1)\right] + \frac{1}{2\sqrt{2}}\, Im\,(c_+^*\, D_o),\\ \lambda_3 &= \frac{\sqrt{15}}{4}\, Im\left[d_+^*\,(D_1 - 3D_2)\right.\end{aligned}\right\}\qquad (19)$$

and, also like (I8a)

$$\left.\begin{aligned}\lambda_o' &= \frac{\sqrt{15}}{8}\, Im\,(c_{p_2}^*\, c_5) - \frac{\sqrt{15}}{4\sqrt{2}}\, Im\,(c_{p_2}^*\, c\, d_1);\\ \lambda_1' &= 0;\\ \lambda_2' &= \frac{9}{8}\sqrt{\frac{15}{2}}\, Im\,(c_{p_2}^*\, c\, d_1) + \frac{15}{8}\sqrt{\frac{3}{2}}\, Im\,(c_{p_2}^*\, c\, d_2);\\ \lambda_3' &= 0.\end{aligned}\right\}\qquad (19a)$$

The probability of the deuteron emission with the given polarization and quadrupolarization is defined by the mean values of the spin- tensors of higher rank

$$\langle T_{11}\rangle_{unpol} = -\frac{i}{4}\sqrt{\frac{3}{2}}\,\sin\theta\cos\theta\cdot e^{i\varphi}(\nu_o + \nu_2\cdot\cos^2\theta)$$

where

$$\nu_o = \sqrt{\frac{3}{2}}\, Im\,(D_o^*\, c_5) + Im\left[(D_2 + D_1)^* D_o\right] + \frac{1}{2\sqrt{2}}\, Im\,(c_+^*\, c_-);\qquad (20)$$

$$\nu_2 = Im\left[D_o^*\,(3D_2 + D_1)\right];$$

and

$$\nu_o' = \frac{3}{2}\sqrt{5}\, Im\,(c_{p_2}^*\, c_{p_o});$$

$$\nu_2' = 0.\qquad (20a)$$

$$\langle T_{11}\rangle_{pol} = \frac{i}{8}\sqrt{\frac{3}{2}}\,\cos\theta\{\mu_o + \mu_1\cos\theta + \mu_2\cdot\cos^2\theta + \mu_3\cos^3\theta + \sin^2\theta\cdot e^{i2\varphi}(\mu_4 + \mu_5\cdot\cos\theta),$$

where

$$\begin{aligned}\mu_o &= \sqrt{3}\, Re\,(c_+^*\, c_5) - \sqrt{2}\, Re\,(c_+^*\, D_2) - Re\,(D_o^*\, c_-);\\ \mu_1 &= -\sqrt{15}\, Re\,(D_o^*\, d_+);\\ \mu_2 &= 3\sqrt{2}\, Re\,(c_+^*\, D_1) + Re\,(D_o^*\, c_-);\\ \mu_3 &= \sqrt{15}\, Re\,(D_o^*\, d_+);\\ \mu_4 &= -Re\,(D_o^*\, c_-) - \sqrt{2}\, Re\,(c_+^*\, D_1);\\ \mu_5 &= \sqrt{15}\, Re\,(D_o^*\, d_+);\end{aligned}\qquad (21)$$

and

$$\left.\begin{aligned}
\mu_0' &= \sqrt{30}\cdot Re\,(c_{p_2}^* c_S) + \frac{5}{4}\sqrt{15}\,Re\,(c_{p_2}^* c d_1) + \frac{15}{4}\sqrt{3}\,Re\,(c_{p_2}^* c d_2)\\
\mu_1' &= 0\\
\mu_2' &= -\frac{9}{4}\sqrt{15}\cdot Re\,(c_{p_2}^* c d_1) - \frac{35}{4}\sqrt{3}\,Re\,(c_{p_2}^* c d_2);\\
\mu_3' &= 0;\\
\mu_4' &= \frac{5}{4}\sqrt{15}\,Re\,(c_{p_2}^* c d_1) + \frac{5}{2}\sqrt{3}\,Re\,(c_{p_2}^* c d_2).
\end{aligned}\right\} \quad (21a)$$

$$\langle T_{10}\rangle_{\text{unpol}} \equiv 0.$$

$$\langle T_{10}\rangle_{\text{pol}} = \frac{1}{4}\sqrt{\frac{3}{2}}\,\sin\theta\cdot\sin\varphi\left\{\xi_0 + \xi_1\cos\theta + \xi_2\cos^2\theta + \xi_3\cos^3\theta\right\};$$

where

$$\left.\begin{aligned}
\xi_0 &= \sqrt{\frac{3}{2}}\,Re\,(c_S^* c_-) + Re\left[(D_1 - D_2)^* c_-\right];\\
\xi_1 &= \sqrt{15}\,Re\left[(D_1 + D_2)^* d_+\right] - \sqrt{\frac{15}{2}}\,Re\,(c_S^* d_+);\\
\xi_2 &= Re\left[(3D_2 - D_1)^* c_-\right]\\
\xi_3 &= -\sqrt{15}\,Re\left[(3D_2 + D_1)^* d_+\right].
\end{aligned}\right\} \quad (22)$$

and

$$\left.\begin{aligned}
\xi_0' &= -\frac{\sqrt{15}}{2}\,Re\,(c_{p_2}^* c_S) - \frac{1}{2}\sqrt{\frac{15}{2}}\,Re\,(c_{p_2}^* c d_1) + \frac{5}{2}\sqrt{\frac{3}{2}}\,Re\,(c_{p_2}^* c d_2);\\
\xi_1' &= 0;\\
\xi_2' &= 5\sqrt{\frac{3}{2}}\,Re\,(c_{p_2}^* c d_2);\\
\xi_3' &= 0.
\end{aligned}\right\} \quad (22a)$$

where

$$\left.\begin{aligned}
\langle T_{20}\rangle_{\text{unpol}} &= \sqrt{\frac{1}{2}}\left\{\eta_0 + \eta_2\cdot\cos^2\theta + \eta_4\cdot\cos^4\theta\right\};\\
\eta_0 &\equiv \gamma_0;\\
\eta_2 &= \frac{15}{8}|d_+|^2 - \frac{1}{2}\left(3|D_2|^2 + |D_1|^2 + |D_0|^2\right) -\\
&\quad - \frac{1}{8}|c_-|^2 - \frac{1}{4}|c_+|^2 + \frac{3}{2}\sqrt{\frac{3}{2}}\,Re\,(c_S^* D_2);\\
\eta_4 &= \frac{1}{4}\left(|D_1|^2 + 2|D_0|^2 + 9|D_2|^2\right) - \frac{15}{8}|d_+|^2;
\end{aligned}\right\} \quad (23)$$

and

$$\eta_0' \equiv \gamma_0' = \frac{5}{16}|c_{p_2}|^2 - \frac{\sqrt{10}}{8} Re\,(c_{p_2}^* c_{p_0});$$
$$\eta_2' = -\frac{45}{16}|c_{p_2}|^2 - \frac{3}{8}\sqrt{10}\cdot Re\,(c_{p_2}^* c_{p_0});$$
$$\eta_4' = 0 \qquad (23a)$$

$$\langle T_{20}\rangle_{pol} = \frac{\sin\theta\cdot\cos\theta}{4\sqrt{2}}\left\{\chi_0 + \chi_1\cdot\cos\theta + \chi_2\cdot\cos^2\theta + \chi_3\cdot\cos^3\theta\right\};$$

where

$$\chi_0 = \sqrt{\tfrac{3}{2}}\,Im(c_-^* c_5) + Im\left[(D_2 - D_1)^* c_-\right];$$
$$\chi_1 = \sqrt{\tfrac{45}{2}}\,Im(c_5^* d_+) + \sqrt{15}\,Im\left[(D_1 - D_2)^* d_+\right];$$
$$\chi_2 = Im\left[c_-^*(3D_2 + D_1)\right] - \frac{4}{\sqrt{2}}\,Im\,(c_+^* D_0);$$
$$\chi_3 = \sqrt{15}\,Im\left[(3D_2 - D_1)^* d_+\right]. \qquad (24)$$

and

$$\chi_0' = -\frac{\sqrt{15}}{2}\,Im\,(c_{p_2}^* c_5) + \frac{1}{2}\sqrt{\tfrac{15}{2}}\,Im\,(c_{p_2}^* c d_1) - \frac{5}{2}\sqrt{\tfrac{3}{2}}\,Im\,(c_{p_2}^* c d_2);$$
$$\chi_1' = 0;$$
$$\chi_2' = -\frac{9}{2}\sqrt{\tfrac{15}{2}}\,Im\,(c_{p_2}^* c d_1) - \frac{15}{2}\sqrt{\tfrac{3}{2}}\,Im\,(c_{p_2}^* c d_2);$$
$$\chi_2' = -4\chi_2';$$
$$\chi_3' = 0 \qquad (24a)$$

$$\langle T_{21}\rangle_{unpol} = \frac{1}{8}\sqrt{\tfrac{3}{2}}\,\sin\theta\cdot\cos\theta\cdot e^{i\varphi}(\rho_0 + \rho_2\cdot\cos^2\theta);$$

where

$$\rho_0 = \sqrt{2}\,Re\,(c_-^* c_+) + \sqrt{6}\,Re\,(D_0^* c_5) - 2Re\left[D_0^*(D_1 + D_2)\right];$$
$$\rho_2 = 2Re\left[D_0^*(3D_2 + D_1)\right]; \qquad (25)$$

and

$$\rho_0' = -5|c_{p_2}|^2 + \sqrt{\tfrac{5}{2}}\,Re\,(c_{p_2}^* c_{p_0});$$
$$\rho_2' = 0. \qquad (25a)$$

$$\langle T_{21}\rangle_{unpol} = \frac{1}{4}\sqrt{\tfrac{3}{2}}\cos\theta\left\{\tau_0 + \tau_1\cos\theta + \tau_2\cdot\cos^2\theta + \tau_3\cdot\cos^3\theta + \right.$$
$$\left. + \sin^2\theta\cdot e^{i2\varphi}(\tau_4 + \tau_5\cdot\cos\theta)\right\};$$

where

$$
\left.\begin{aligned}
\tau_0 &= \frac{1}{\sqrt{2}} \operatorname{Im}(D_2^* C_+) - \frac{\sqrt{3}}{2} \operatorname{Im}(C_S^* C_+) + \frac{1}{2} \operatorname{Im}(C_-^* D_0);\\
\tau_1 &= \frac{\sqrt{15}}{2} \operatorname{Im}(d_+^* D_0);\\
\tau_2 &= \frac{3}{\sqrt{2}} \operatorname{Im}(D_2^* C_+) - \frac{1}{2} \operatorname{Im}(C_-^* D_0);\\
\tau_3 &= -\frac{\sqrt{15}}{2} \operatorname{Im}(d_+^* D_0);\\
\tau_4 &= \frac{1}{\sqrt{2}} \operatorname{Im}(D_1^* C_+) + \frac{1}{2} \operatorname{Im}(C_-^* D_0);\\
\tau_5 &= -\frac{\sqrt{15}}{2} \operatorname{Im}(d_+^* D_0)
\end{aligned}\right\} \quad (26)
$$

and

$$
\left.\begin{aligned}
\tau_0' &= \frac{\sqrt{30}}{2} \operatorname{Im}(C_{p_2}^* C_S) - \frac{1}{8}\sqrt{15} \operatorname{Im}(C_{p_2}^* C d_1) + \frac{5}{8}\sqrt{3} \operatorname{Im}(C_{p_2}^* C d_2);\\
\tau_1' &= 0;\\
\tau_2' &= \frac{9}{8}\sqrt{15} \operatorname{Im}(C_{p_2}^* C d_1) + \frac{35}{8}\sqrt{3} \operatorname{Im}(C_{p_2}^* C d_2);\\
\tau_3' &= 0;\\
\tau_4' &= -\frac{\sqrt{15}}{8} \operatorname{Im}(C_{p_2}^* C d_1) - \frac{15}{8}\sqrt{3} \operatorname{Im}(C_{p_2}^* C d_2);\\
\tau_5' &= 0 .
\end{aligned}\right\} \quad (26a)
$$

$$
\left.\begin{aligned}
&\langle T_{22} \rangle_{\text{unpol}} = \frac{\sqrt{3}}{8} \sin^2\theta \cdot e^{i2\varphi} (\omega_0 + \omega_2 \cdot \cos^2\theta);\\
&\text{where}\\
&\omega_0 = \sqrt{6} \operatorname{Re}(C_S^* D_1) - 2 \operatorname{Re}(D_2^* D_1) - \frac{1}{2} |C_-|^2;\\
&\omega_2 = 6 \operatorname{Re}(D_2^* D_1) + \frac{15}{2} |d_+|^2;
\end{aligned}\right\} \quad (27)
$$

and

$$
\left.\begin{aligned}
\omega_0' &= -4\gamma_0' = -4\eta_0';\\
\omega_2' &= 0
\end{aligned}\right\} \quad (27a)
$$

$$\langle T_{22}\rangle_{pol} = \frac{\sqrt{3}}{8}\sin\theta\, e^{i\varphi}\Big\{\Psi_0 + \Psi_1\cdot\cos\theta + \Psi_2\cos^2\theta + \Psi_3\cdot\cos^3\theta + $$
$$+ \sin^2\theta\cdot e^{i2\varphi}(\Psi_4 + \Psi_5\cdot\cos\theta)\Big\};$$

where

$$\Psi_0 = \operatorname{Im}(C_-^* D_2) - \sqrt{\tfrac{3}{2}}\operatorname{Im}(C_-^* C_S);$$
$$\Psi_1 = \sqrt{15}\operatorname{Im}(d_+^* D_2) - \sqrt{\tfrac{45}{2}}\operatorname{Im}(d_+^* C_S);$$
$$\Psi_2 = -3\operatorname{Im}(C_-^* D_1);$$
$$\Psi_3 = -3\sqrt{15}\operatorname{Im}(d_+^* D_2);$$
$$\Psi_4 = \operatorname{Im}(C_-^* D_1);$$
$$\Psi_5 = \sqrt{15}\operatorname{Im}(D_1^* d_+). \qquad (28)$$

and

$$\Psi_0' = \frac{\sqrt{15}}{2}\operatorname{Im}(C_{p_2}^* C_S) + \frac{1}{4}\sqrt{\tfrac{15}{2}}\operatorname{Im}(C_{p_2}^* C d_1) + \frac{5}{4}\sqrt{\tfrac{3}{2}}\operatorname{Im}(C_{p_2}^* C d_2);$$
$$\Psi_1' = 0;$$
$$\Psi_2' = -\frac{3}{4}\sqrt{\tfrac{15}{2}}\operatorname{Im}(C_{p_2}^* C d_1) - \frac{15}{4}\sqrt{\tfrac{3}{2}}\operatorname{Im}(C_{p_2}^* C d_2);$$
$$\Psi_3' = 0;$$
$$\Psi_4' = \frac{1}{4}\sqrt{\tfrac{15}{2}}\operatorname{Im}(C_{p_2}^* C d_1) + \frac{5}{4}\sqrt{\tfrac{3}{2}}\operatorname{Im}(C_{p_2}^* C d_2);$$
$$\Psi_5' = 0. \qquad (28a)$$

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REFERENCES

I. K.Watson, C.Richman, Phys.Rev., 83, I256, I95I.

2. V.B. Berestetsky Dokl.Akad.Nauk SSSR, 94, 42I, I954.

3. R.Marshak and A.Messiah, Nuov. Cim., II, 337, I954.

4. S.M.Bilenky, L.I. Lapidus. Report of the Joint Institute for Nuclear Researoh, I955.

5. L.Wolfenstein, Phys.Rev., 98, 766, I955.

6. F.Mandl and T.Regge, Phys. Rev., 99 (I955), I478.

7. W.Lakin, Phys. Rev., 98, No I, I39-I44, I955.

8. A.Simon, Phys. Rev., 92, I050, I953.

9. J.Blatt and L.Biedenharn, Rev.Mod.Phys., 24, 258, I952.

I0. Yu.K.Akimov, O.V.Savchenko, L.M.Soroko
JETP (in print), I958.

II. L.Wolfenstein, Phys. Rev., 75, I664, I949.

I2. L.I.Lapidus, JETP, I, (7) 204, I957.

I3. I.M.Gelfand, Z.Ia. Shapiro. Uspekhi mat. Nauk.VII, I; 47, I952.

I4. L.Landau, E.Lifshitz, Quantum Mechanics, 232, I948.

I5. E.Condon, G.Shortly. Theory of Atomic Spectra, page 57-58, I949.

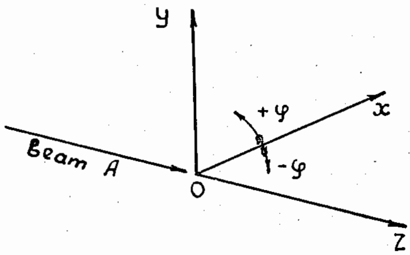

Fig. 1

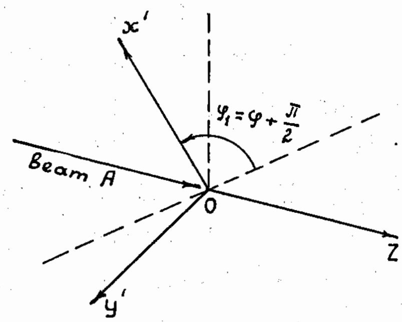

Fig. 2.

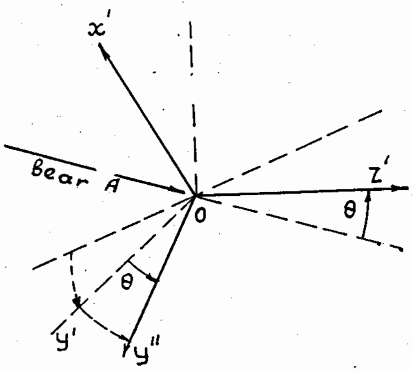

Fig. 3.

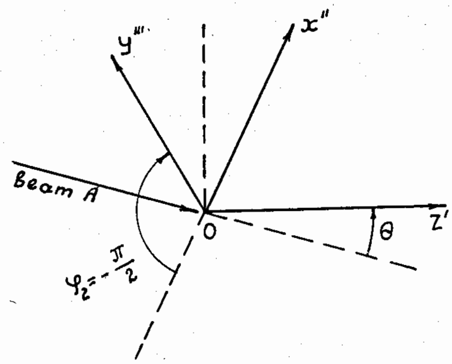

Fig. 4.