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# AN ALGEBRAIC SYSTEM OF FUNDAMENTAL PARTICLES

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* On leave from the Charles University, Prague, Czechoslovakia.

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A b s t r a c t:

In Section II a preliminary systematics based on correspondence between kinds of fundamental particles and pairs of irreducible representations of Dirac's or Duffin-Kemmer's algebra in space-time and in isospace is proposed. In Section III an "isoalgebra of stronglys" is defined and a theory of the unified baryon-field and meson-field is formulated. In Section IV strong and weak interactions between these two fields are investigated. An outlook on possibilities of further development of this theory is given in Section V.

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## I. Introduction

Most successful attempts to systematize the fundamental particles are based on some generalizations and refinements of Heisenberg's old notion of the isobaric spin of the nucleon |1-3, 21|. The main object of these systematizations is the group of strongly interacting particles comprising the baryons (nucleons and hyperons) and the mesons (pions and kaons). Only relatively few attempts have been made till now to understand in this way also the structure of the group of leptons ($\mu$ - mesons, electrons and neutrinos) |4-8, 21|.

In Section II of the present paper we start from today's experimental evidence that the magnitudes of spin and isospin which occur in the system of fundamental particles are $\leq 1$ and from the well known fact that particles with spin $\frac{1}{2}$ or 0 and 1 are primarily characterized by the set of four irreducible Dirac matrices $\gamma_\mu$ or Duffin-Kemmer matrices $\beta_\mu$ and not by the matrices representing the components of the spin. The spin - matrices are expressible in terms of the $\gamma$'s or $\beta$'s and appear to be of secondary origin.

We assume that similarly in the (threedimensional) isospace various kinds of particles are primarily associated not with irreducible sets of isobaric spin - matrices, but with irreducible representations of the Dirac or Duffin-Kemmer algebra. This makes a difference firstly due to the existence (in threedimensional space) of the so called twin representations of both algebras and secondly due to the existence of the irreducible representation of the Duffin-Kemmer algebra by 4 x 4 -matrices. We shall see that the correspondence works quite well and enables us to interpret purely <u>algebraically</u> also such important quantities as the "isofermion quantum number". Arguments in favour of the association of the four particles $\Sigma_{+}, \Sigma_{o}, \Sigma_{-}; \Lambda$ with the 4 x 4 -representation of the Duffin-Kemmer algebra in isospace emerge quite naturally.[1/]

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1/ The idea of the association of various kinds of elementary particles with various irreducible representations of the Dirac and the Duffin-Kemmer algebra in space-time and in isospace was at first expressed in our early papers |6-9|. At that time, however, empirical data concerning new particles were rather incomplete and so it was not possible to find use for some existing irreducible representations and to settle correctly the correspondence.

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According to the present empirical evidence there is so pregnant similarity between the family of baryons and the family of mesons that it gives rise to the conjecture that the isobaric structure of both these families of fundamental particles is basically the same and is characteristic for the whole group of stronglys. This conjecture means that besides the empirically known $\pi_o$-meson (which has I = 1 and represents the counterpart of the $\Sigma_o$-hyperon) we have to expect the existence of another neutral meson $\pi'$ (with I = 0 like the $\Lambda$-hyperon). Theoretical arguments in favour of the existence of this "fourth pion" have been found especially in some versions of the theory based on fourdimensional isobaric space |10-12, 21|. In our scheme such arguments are connected with formal advantages offered by the association of the 4 x 4 -representation of the Duffin-Kemmer algebra also with pions $\pi_+, \pi_o, \pi_-; \pi'$.[2/]

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2/ The fact that up to the present time no pion with zero isobaric spin has been observed (as a stable or semistable particle) is not necessarily in contradiction with theories assuming the existence of a $\pi'$ since the properties of this particle (caused by its interactions with baryons) can be such as to make it hardly observable in practice |12|. See also Section IV below.

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The (common) isobaric structure assigned to both the family of baryons and that of mesons can best be described by a special "isoalgebra of stronglys". The definition and properties of this algebra are briefly considered in Section III. In sequence the equations of a unified baryon-field and meson-field together with the auxiliary condition satisfied by the meson-field are formulated there.

Strong and weak interactions between these two fields (corresponding to strong and weak interactions of baryons with mesons) are considered in Section IV. A very symmetrical Lagrangian of strong interactions (containing only one strong coupling constant) is constructed which gives (already in second order) at least the proper sequence of baryon and meson masses with the mass of $\pi'$ essentially larger than the mass of the $\pi$-triplet.

The isoalgebra of stronglys contains an element R (spurion-matrix) which makes it possible to write the Lagrangian of weak interactions between baryons and mesons in a form similar to the Lagrangian of strong interactions. The charge conservation and the selection rules $|\Delta U|=1, |\Delta \underline{I}|=\frac{1}{2}, |\Delta I_3|=\frac{1}{2}$ follow automatically from this Lagrangian. The possibility of introducing in a natural way parity nonconserving (but "combined parity" conserving) terms is connected with the behaviour of individual terms under charge conjugation.

Some perspectives of further development of this theory are briefly mentioned in Section V and some useful formulas are collected in the Appendix.

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## II. Irreducible representations of the Dirac and Duffin-Kemmer algebra and their association with various kinds of fundamental particles

1. The algebra defined by the relations

$$\{\gamma_\mu , \gamma_\nu\} \equiv \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\,\delta_{\mu\nu} \tag{1}$$

( $\mu$ , $\nu$ = 1, 2, 3, 4) has only one irreducible representation, given by the usual hermitian 4 x 4 -matrices of Dirac. The (reducible) matrices $\sigma_{(\gamma)\bar\lambda}$ ( $\bar\lambda$ = 1, 2, 3) representing the components of the spin of the particle described by the Dirac wave equation

$$(\gamma_\mu \partial_\mu + M)\Psi = 0 \tag{2}$$

are given by the formula

$$\sigma_{(\gamma)\bar\lambda} = -\frac{i}{4}\,\varepsilon_{\bar\lambda\bar\mu\bar\nu}\,\gamma_{\bar\mu}\gamma_{\bar\nu} \,. \tag{3}$$

The magnitude of the spin $\sigma_{(\gamma)} = \frac{1}{2}$ .

Recent experimental results indicate that not only the electrons, neutrinos, muons and nucleons but also the hyperons have spin $\frac{1}{2}$ . Thus we can assume that all fundamental fermions are associated with the unique representation of the Dirac algebra in space-time.

2. The Duffin-Kemmer algebra defined by the relations

$$\beta_\lambda \beta_\mu \beta_\nu + \beta_\nu \beta_\mu \beta_\lambda = \delta_{\lambda\mu}\beta_\nu + \delta_{\nu\mu}\beta_\lambda \tag{4}$$

has three irreducible representations. The trivial one, $\overset{(0)}{\beta}_{\mu} = 0$, is of no use for writing first order wave equations and so we can discard it. The representations $\beta'_{\mu}$ and $\beta''_{\mu}$ are given by the known hermitian 5 x 5 -matrices and 10 x 10 -matrices respectively.

The (reducible) matrices $\sigma_{(\beta)\bar{\lambda}}$ representing the components of the spin of the particle described by the wave equation

$$(\beta_{\mu}\partial_{\mu} + m)\varphi = 0 \qquad (5)$$

are given by

$$\sigma_{(\beta)\bar{\lambda}} = -i\,\varepsilon_{\bar{\lambda}\bar{\mu}\bar{\nu}}\,\beta_{\bar{\mu}}\beta_{\bar{\nu}} \ . \qquad (6)$$

As well known, the particle described by the wave equation (5) has only spin zero states in case of $\beta_{\mu} = \beta'_{\mu}$ and only spin one states in case of $\beta_{\mu} = \beta''_{\mu}$ although we can find 0 as well as 1 among the eigenvalues of both matrices $\sigma'_{(\beta)}$ and $\sigma''_{(\beta)}$.

The set of matrices $\beta'_{\mu}$ is associated with mesons (pions and kaons). The only known particle which can be associated with $\beta''_{\mu}$ is the photon |13, 14|.

3. The Dirac algebra in isospace, defined by the relations

$$\{\rho_j , \rho_k\} = 2\,\delta_{jk} \quad (j,k = 1,2,3) \qquad (7)$$

has two unequivalent irreducible representations

$$\overset{(1)}{\rho}_j = \tau_j\,, \quad \overset{(2)}{\rho}_j = -\tau_j \tag{8}$$

where $\tau_j$ are the Pauli 2 x 2 -matrices. The isospin matrices $t_j$ can be formed from the $\rho$'s in the same manner as the $\sigma_{(\gamma)\lambda}$ from the $\gamma$'s(formula (3)). The two sets of $t_j$ arising in both unequivalent cases of $\overset{(1)}{\rho}_j$ and $\overset{(2)}{\rho}_j$ are equivalent and give the unique (irreducible) representation of the isospin $\frac{1}{2}$. We shall, therefore, not use the superscripts over $t_j$. With our special choice (8) we have

$$t_j = \frac{1}{2}\tau_j \tag{9}$$

in both cases.

Consider the matrix

$$U_{(\rho)} = \frac{2}{3} t_j \rho_j = \frac{2}{3} \rho_j t_j\ . \tag{10}$$

Using (8) and (9) we get

$$\overset{(1)}{U}_{(\rho)} = 1\ , \quad \overset{(2)}{U}_{(\rho)} = -1\ . \tag{11}$$

But the matrices $\overset{(1)}{U}_{(\rho)}$ and $\overset{(2)}{U}_{(\rho)}$, being multipla of the unit matrix, cannot be changed by any unitary transformations and so the values $\pm 1$ of $U_{(\rho)}$ characterize the two unequivalent representations of the algebra (7) invariantly.

It is now obvious that according to our programme we have to associate the representation belonging to $U_{(q)} = 1$ with the two charge-doublets $(N_+, N_0)$ and $(K_+, K_0)$ whereas the representation belonging to $U_{(q)} = -1$ with $(\Xi_0, \Xi_-)$ and with $(\varkappa_0 \equiv$ anti $-K_0$, $\varkappa_- \equiv$ anti $-K_+)$. The charge-operator can then be written generally in the form

$$q_{(p)} = t_3 + \frac{1}{2} U_{(p)} \; . \tag{12}$$

4. The Duffin-Kemmer algebra in isospace, defined by the relations

$$\xi_j \xi_k \xi_\ell + \xi_\ell \xi_k \xi_j = \delta_{jk} \xi_\ell + \delta_{\ell k} \xi_j \tag{13}$$

has four unequivalent irreducible representations. Firstly we have the trivial representation $\overset{(0)}{\xi}_j = 0$ , then two representations by 3 x 3 -matrices $\overset{(1)}{\xi}_j$ and $\overset{(2)}{\xi}_j$ which can be chosen as follows

$$\overset{(1)}{\xi}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \overset{(1)}{\xi}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \overset{(1)}{\xi}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{14a}$$

$$\overset{(2)}{\xi}_j = - \overset{(1)}{\xi}_j \tag{14b}$$

and finally one representation by 4 x 4 -matrices $\xi'_j$ of the form

$$\xi'_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}, \xi'_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}, \xi'_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} . \tag{15}$$

The isospin matrices $\theta_j$ can be expressed in terms of the $\xi$'s in the same manner as $\sigma_{(\beta)\bar{\lambda}}$ in terms of the $\beta$'s (formula (6)). With $\overset{(0)}{\xi}_j$ we have of course $\overset{(0)}{\theta}_j = 0$ and $\overset{(0)}{\theta} = 0$ . The two sets of $\theta_j$ arising in the two cases of $\overset{(1)}{\xi}_j$ and $\overset{(2)}{\xi}_j$ are again equivalent and give the unique (irreducible) representation of the isospin 1. With the special choice of (14a, b) we have $\theta_j = \overset{(1)}{\xi}_j$ in both cases. In case of $\xi'_j$ the $\theta'_j$ are reducible and with the special choice of (15) take the form

$$\theta'_j = \begin{pmatrix} \overset{(1)}{\xi}_j & 0 \\ 0 & 0 \end{pmatrix} . \tag{16}$$

In each of the four representations the matrix

$$U_{(\xi)} = \frac{2}{3}\theta_j \xi_j = \frac{2}{3}\xi_j \theta_j \tag{17}$$

is again a multiplum of the unit matrix, viz

$$\overset{(0)}{U}_{(\xi)} = 0, \quad \overset{(1)}{U}_{(\xi)} = -\overset{(2)}{U}_{(\xi)} = \frac{4}{3} , \quad U'_{(\xi)} = 0 . \tag{18}$$

It is now clear that according to our programme we have to associate the 3 + 1 hyperons $\Sigma_+, \Sigma_0, \Sigma_-; \Lambda$ with the irreducible representation $\xi'_j$ , since only in this case it is possible to define the charge operator by the formula

$$q'_{(\xi)} = \theta'_3 + \frac{1}{2}U'_{(\xi)} \tag{19}$$

(formally identical with (12)) and so to maintain the same definition of the charge in the whole family of baryons. The matrices $\xi'_j$ find their use also in constructing the Lagrangian of strong interactions where they produce the term responsible for the processes $\Sigma \rightleftharpoons \Lambda + \pi$.

The arguments just mentioned cannot be used similarly in favour of the existence of $\pi'$ and of the association of $\xi'_j$ also with $\pi_+, \pi_0, \pi_-; \pi'$.[3)]

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3/ Even if there would exist only the triplet $\pi_+, \pi_0, \pi_-$ associated with $\overset{(1)}{\xi}_j$ (or $\overset{(2)}{\xi}_j$ ), the same definition of the charge could still be used for the whole group of stronglys. The term $\frac{1}{2}\overset{(1)}{U}_\xi$ (or $\frac{1}{2}\overset{(2)}{U}_\xi$ ) namely would give no contribution to the pion charge and current inasmuch as $\pi_-$ is an antiparticle of $\pi_+$. Indeed, the identification $\pi_\pm \equiv$ anti- $\pi_\mp$, $\pi_0 \equiv$ anti- $\pi_0$ means an auxiliary condition for the pion wave function $\varphi_\pi$ in virtue of which $\bar{\varphi}_\pi \beta_\mu \varphi_\pi = -\bar{\varphi}_\pi \beta_\mu \varphi_\pi = 0$.
(See e.g. |8| or |15|).

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There are, however, other formal arguments in favour of the conjecture that both families of stronglys have the same isobaric structure and that the representations $\overset{(1)}{\rho}_j$ , $\overset{(2)}{\rho}_j$ and $\xi'_j$ are characteristic for the whole group (of stronglys). We shall meet with some of such arguments below.

From the remaining representations of $\xi_j$ the trivial one, $\overset{(0)}{\xi}_j$, can be associated with the photon and the representations $\overset{(1)}{\xi}_j$ and $\overset{(2)}{\xi}_j$ with leptons. In case of leptons we have two possibilities: Either we can try to associate the representation $\overset{(1)}{\xi}_j$, say, with the triplet ( $e_+$, $\nu \equiv$ anti-$\nu$, $e_-$ ) and then $\overset{(2)}{\xi}_j$ with ( $\mu_+$, $\mu_o \equiv$ anti-$\mu_o$, $\mu_-$ ) or to associate $\overset{(1)}{\xi}_j$ with the triplet ( $\mu_+$ , $\nu$ , $e_-$ ) and then $\overset{(2)}{\xi}_j$ with ( $e_+$ , anti-$\nu$ , $\mu_-$ ) |16,17|. Note that in the first case no change in the definition of the charge operator $q_{(\xi)} = \theta_3 + \frac{1}{2} U_{(\xi)}$ is necessary inasmuch as $e_+$ ( $\mu_+$ ) is an antiparticle of $e_-$ ( $\mu_-$ ). See also footnote 3/. In the second case the charge operator must be redefined for the leptons: $q_{(\xi)} = \theta_3$ . See however Section V for another possibility.

In the next two Sections we shall investigate more thoroughly the group of strongly interacting particles.

## III. Algebra of stronglys in isospace

## Field of baryons and field of mesons

1. According to our assumption concerning the isobaric structure of the group of stronglys, both the family of baryons and that of mesons are associated in isospace with the same reducible set of matrices $\omega_j$ ($j = 1,2,3$) which can be written in the form of the direct sum

$$\omega_j = \begin{pmatrix} \overset{(1)}{\rho}_j & 0 & 0 \\ 0 & \overset{(2)}{\rho}_j & 0 \\ 0 & 0 & \xi'_j \end{pmatrix} \tag{20}$$

Let us try to characterize algebraically the matrices $\omega_j$. We remark, first of all, that these matrices fulfil the following relations:

$$\sum_P (\omega_j \omega_k \omega_\ell - \delta_{jk} \omega_\ell) = 0 , \tag{21}$$

$$[\omega_j , \lambda_k] = i \varepsilon_{jk\ell} \omega_\ell , \tag{22}$$

$$[\lambda_j , \lambda_k] = i \varepsilon_{jk\ell} \lambda_\ell , \tag{23}$$

$$\lambda_j \omega_k + \lambda_k \omega_j = \omega_j \lambda_k + \omega_k \lambda_j = \delta_{jk} U \tag{24}$$

where $\sum_P$ denotes the sum over all six permutations of the indices $j, k, \ell$ From (24) we get immediately

$$U = \tfrac{2}{3}\lambda_j\omega_j = \tfrac{2}{3}\omega_j\lambda_j \; . \tag{24a}$$

Let us consider, conversely, the algebra defined by the relations (21) - (24). As we are interested only in such its representations in which the matrices $\lambda_j$ are algebraically expressible in terms of the $\omega$'s, we need not consider those (pathological) irreducible representations in which the $\lambda$'s are irreducible and at the same time the $\omega$'s are reducible (zero matrices). All representations of the kind we are interested in can be formed as direct sums of representations with irreducible $\omega$'s. But it is possible to show that the only irreducible sets of $\omega_j$ which belong to the algebra defined by the relations (21) - (24) are the following:

$$\overset{(0)}{\omega}_j = \overset{(0)}{\xi}_j = 0, \quad \overset{(1)}{\omega}_j = \overset{(1)}{\rho}_j, \quad \overset{(2)}{\omega}_j = \overset{(2)}{\rho}_j, \quad \omega'_j = \xi'_j \; .$$ 4)

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4/ The proof is contained in the equations

$$[\omega_j, U] = 0,$$
$$U^3 = U,$$
$$(\{\omega_j, \omega_k\} - 2\delta_{jk})\cdot U = 0,$$
$$(\omega_j\omega_k\omega_\ell + \omega_\ell\omega_k\omega_j - \delta_{jk}\omega_\ell - \delta_{\ell k}\omega_j)(1 - U^2) = 0$$

indices $j, k, \ell$ From (24) we get immediately

$$U = \frac{2}{3} \lambda_j \omega_j = \frac{2}{3} \omega_j \lambda_j \ . \tag{24a}$$

Let us consider, conversely, the algebra defined by the relations (21) - (24). As we are interested only in such its representations in which the matrices $\lambda_j$ are algebraically expressible in terms of the $\omega$'s, we need not consider those (pathological) irreducible representations in which the $\lambda$'s are irreducible and at the same time the $\omega$'s are reducible (zero matrices). All representations of the kind we are interested in can be formed as direct sums of representations with irreducible $\omega$'s. But it is possible to show that the only irreducible sets of $\omega_j$ which belong to the algebra defined by the relations (21) - (24) are the following:

$$\overset{(0)}{\omega}_j = \overset{(0)}{\xi}_j = 0, \quad \overset{(1)}{\omega}_j = \overset{(1)}{\rho}_j, \quad \overset{(2)}{\omega}_j = \overset{(2)}{\rho}_j, \quad \omega'_j = \xi'_j \ . \quad ^{4)}$$

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4/ The proof is contained in the equations

$$[\omega_j, U] = 0,$$

$$U^3 = U,$$

$$(\{\omega_j, \omega_k\} - 2\delta_{jk}) \cdot U = 0,$$

$$(\omega_j \omega_k \omega_\ell + \omega_\ell \omega_k \omega_j - \delta_{jk} \omega_\ell - \delta_{\ell k} \omega_j)(1 - U^2) = 0$$

which, besides many others, can be deduced purely algebraically from (21) - (24). The deduction of these equations and a more detailed investigation of the properties of the algebra will be given in a separate paper by one of the authors (M.L.).

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To characterize formally the peculiar composition of our reducible set of the $\omega$-matrices (20), we must state some more conditions which are satisfied only by these $\omega$-matrices as a whole (not by all their irreducible parts individually). In this respect we can mention that for our $\omega$-matrices (20) there exist a <u>unitary</u> and <u>symmetrical</u> matrix $\Omega$ with the property

$$\Omega \omega_j^T = \omega_j \Omega . \tag{25}$$

Using (22) - (25) it is then possible to see that $\Omega$ fulfils also the relations

$$\Omega \lambda_j^T = - \lambda_j \Omega , \quad \Omega U^T = - U \Omega . \tag{26a,b}$$

With our special representation (8), (15) and (20) the matrix $\Omega$ takes the form

$$\Omega = \begin{pmatrix} 0 & -i\tau_2 & 0 \\ i\tau_2 & 0 & 0 \\ 0 & 0 & (1-2\theta')(2\xi_2'^2 - 1) \end{pmatrix} . \tag{27}$$

if two arbitrary phase constants are fixed conveniently.

We can see that the existence of $\Omega$ requires that in the direct sum of irreducible $\omega$'s the matrices $\overset{(1)}{\omega}_j$ and $\overset{(2)}{\omega}_j$ occur in pairs (if they should occur at all). Yet a possible presence of $\overset{(0)}{\omega}_j$ cannot be excluded thereby. However, for our $\omega$-matrices (20) there exists further a unitary and hermitian matrix $R$ which fulfils the relations

$$\{R, U^2\} = R \tag{28}$$

$$[R, q] = 0 \tag{29}$$

$$\Omega R^T = -R\Omega \tag{30}$$

where

$$q = \lambda_3 + \tfrac{1}{2} U . \tag{31}$$

It is easily found that such an $R$ can exist only if $\overset{(0)}{\omega}_j$ does not occur in $\omega_j$ and if to each $\omega_j'$ a pair $\overset{(1)}{\omega}_j$, $\overset{(2)}{\omega}_j$ is present (and conversely). If again two arbitrary phase constants are chosen conveniently, the matrix $R$ belonging to our $\omega$-matrices (20) can be written in the form

$$R=\begin{pmatrix} 0 & \tau^+ \\ \tau & 0 \end{pmatrix}, \quad \tau = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \end{pmatrix} . \tag{32}$$

There exist finally two sets of unitary and hermitian matrices $Y_j$ and $Z_j$ (reflection operators) with the properties

$$Y_j \omega_k = (2\delta_{jk} - 1)\omega_k Y_j ,$$

(no summation) (33a,b)

$$Z_j \omega_k = (1 - 2\delta_{jk})\omega_k Z_j ,$$

$$\Omega Y_j^T = Y_j \Omega, \quad \Omega Z_j^T = Z_j \Omega . \tag{34a,b}$$

From (22)-(25) and (33a,b) then

$$Y_j \lambda_k = (2\delta_{jk} - 1)\lambda_k Y_j ,$$

(no summation) (35a,b)

$$Z_j \lambda_k = (2\delta_{jk} - 1)\lambda_k Z_j$$

and

$$[Y_j, U] = 0, \quad \{Z_j, U\} = 0 . \tag{36a,b}$$

In our special representation these matrices can be expressed in the form (no summation)

$$Y_j = \begin{pmatrix} -\tau_j & 0 & 0 \\ 0 & \tau_j & 0 \\ 0 & 0 & 2\xi_j'^2 - 1 \end{pmatrix}, \quad Z_j = \begin{pmatrix} 0 & -i\tau_j & 0 \\ i\tau_j & 0 & 0 \\ 0 & 0 & (1-2\theta')(2\xi_j'^2 - 1) \end{pmatrix} . \tag{37}$$

Notice that $Y_3 = e^{i\pi q}$ so that from (29)

$$[Y_3, R] = 0 . \tag{36c}$$

All the matrices $\Omega$ , $Y_j$ , $Z_j$ and R will prove useful in the theory of the baryon-field and meson-field, the matrix R (spurion-matrix) especially for formulating the laws of weak interactions between baryons and mesons. Before passing on to these points let us just remark that the matrices $\lambda_j$ (belonging to the $\omega$ -matrices (20)) can be expressed in the form

$$\lambda_j = -i\varepsilon_{jkl}\lambda^2\omega_k\omega_l = \begin{pmatrix} t_j & 0 & 0 \\ 0 & t_j & 0 \\ 0 & 0 & \theta_j' \end{pmatrix}, \tag{38}$$

(38a)

$$\lambda = \frac{1}{2}(3-\omega_j^2) + \frac{1}{4}\left(\frac{1}{3}\omega_k\omega_j^2\omega_k - 1\right) = \begin{pmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & \theta' \end{pmatrix}.$$

This $\lambda$ also fulfils the equations $\lambda_j^2 = \lambda(\lambda+1) = 2\lambda - \frac{1}{4}U^2$, $\lambda U = U\lambda = \frac{1}{2}U$, $\{\lambda, \omega_j\} = \omega_j$ and $\Omega\lambda^T = \lambda\Omega$.

2. Now, we can introduce a unified baryon-field $\psi(x)$ and meson-field $\varphi(x)$ , each with eight components in iso-space (and four or five components in space-time respectively). The wave functions

$$\Psi_{N_+}, \Psi_{N_o}, \Psi_{\Xi_o}, \Psi_{\Xi_-}, \Psi_{\Sigma_+}, \Psi_{\Sigma_o}, \Psi_{\Sigma_-}, \Psi_{\Lambda} \tag{39a}$$

and

$$\varphi_{k_+}, \varphi_{k_o}, \varphi_{\varkappa_o}, \varphi_{\varkappa_-}, \varphi_{\pi_+}, \varphi_{\pi_o}, \varphi_{\pi_-}, \varphi_{\pi'} \tag{39b}$$

are to be <u>defined</u> as simultaneous <u>eigenfuctions</u> of the mutually commuting operators $U$, $\lambda$ and $\lambda_3$ (or $q = \lambda_3 + \frac{1}{2}U$). These three operators form a <u>complete set</u> and are all diagonal in our special representation given by (20), (8) and (15). Consequently, in this representation, each of the eigenfuctions (39a,b) has only one nonzero isocomponent, e.g. $\Psi_{N_+}$ and $\varphi_{k_+}$ only the first one, $\Psi_{N_o}$ and $\varphi_{k_o}$ only the second one, etc. The general $\Psi(x)$ and $\varphi(x)$ can then be written as follows:

$$\Psi^T \equiv (N_+^T, N_o^T, \Xi_o^T, \Xi_-^T, \Sigma_+^T; \Sigma_o^T, \Sigma_-^T, \Lambda^T), \tag{40a}$$

$$\varphi^T \equiv ((k_+)^T, (k_o)^T, (\varkappa_o)^T, (\varkappa_-)^T, (\pi_+)^T, (\pi_o)^T, (\pi_-)^T, (\pi')^T). \tag{40b}$$

The symbols $N_+$, ... or $(K_+)$, ... denote the usual Dirac or Duffin-Kemmer wave functions. The simpler symbols $K_+$, ... are not used in (40b) (instead of $(K_+)$,... ) since they are reserved for other purpose (see (47a) below).

We shall make the most natural assumption that the mass of the free baryon (meson) is the same, viz M (m), in all its isobaric states and that the observed splitting of mass-values in the baryon-family as well as in the meson-family arises from strong and electromagnetic interactions. The free fields $\psi$ and $\varphi$ then fulfil the wave equations (2) and (5) respectively) (with $\beta_\mu = \beta'_\mu$ ) and the commutation relations

$$\{\psi_{a\alpha}(x), \bar{\psi}_{b\beta}(x')\} = \frac{\delta_{ab}}{i} S^{(\psi)}_{\alpha\beta}(x-x'), \tag{41}$$

$$[\varphi_{a\alpha}(x), \bar{\varphi}_{b\beta}(x')] = \frac{\delta_{ab}}{i} S^{(\varphi)}_{\alpha\beta}(x-x') \tag{42}$$

where $\bar{\psi} = \psi^+\gamma_4$ , $\bar{\varphi} = \varphi^+(2\beta_4^2-1)$ and

$$S^{(\psi)}_{\alpha\beta}(x) = (\gamma_\mu \partial_\mu - M)_{\alpha\beta}\, \Delta(M;x),$$

$$S^{(\varphi)}_{\alpha\beta}(x) = \left(\beta_\mu \partial_\mu \left(1 - \frac{1}{m}\beta_\nu \partial_\nu\right)\right)_{\alpha\beta} \Delta(m;x), \tag{43}$$

$$\Delta(M;x) = \frac{1}{(2\pi)^3 i}\int d^4p\, e^{ip_\mu x_\mu}\, \delta(p_\mu p_\mu + M^2)\,\varepsilon(p),$$

$$\varepsilon(p) = \frac{p_0}{|p_0|} .$$

The indices a,b = 1, ..., 8. The indices $\alpha, \beta$ = 1, ..., 4 or 1, ..., 5. We have also $\{\psi_{a\alpha}(x), \psi_{b\beta}(x')\}=0$, $[\psi_{a\alpha}(x), \varphi_{b\beta}(x')]=0$,

but $[\varphi_{a\alpha}(x), \varphi_{b\beta}(x')] \neq 0$ because of the auxiliary condition (46). From (42) and (46) we obtain namely

$$[\varphi_{a\alpha}(x), \varphi_{b\beta}(x')] = \frac{1}{i}\Omega_{ab}\left(S^{(\varphi)}(x-x')B\right)_{\alpha\beta} .$$

Introduce new field functions

$$\psi^{(a)} = \Omega C \bar{\psi}^T, \quad \varphi^{(a)} = \Omega B \bar{\varphi}^T \tag{44}$$

and

$$\psi^{(c)} = Z_2 \psi^{(a)}, \quad \varphi^{(c)} = Z_2 \varphi^{(a)} \tag{45}$$

where C and B are the well known unitary matrices with the properties $C\gamma_\mu^T = -\gamma_\mu C, C^T = -C, \; B\beta_\mu^T = -\beta_\mu B, \; B^T = B$ . These functions fulfil the same equations (2), (41), (5), (42) as $\psi$ and $\varphi$ . Further, if the electric current four-vectors are defined by the usual formulas

$$J_\mu^{(\psi)} = ie : \bar{\psi}\gamma_\mu q \psi : , \quad J_\mu^{(\varphi)} = ie : \bar{\varphi}\beta_\mu q \varphi :$$

where : ...... : denotes the normal product, we easily find that the substitutions $\psi \to \psi^{(a)}, \varphi \to \varphi^{(a)}$ leave the $J$ 's unchanged, whereas $\psi \to \psi^{(c)}, \varphi \to \varphi^{(c)}$ change their signs. From (27) and (37) we see that in our special representation $Z_2 \Omega = 1$

so that $\psi^{(c)} = C\bar{\psi}^T$, $\varphi^{(c)} = \beta\bar{\varphi}^T$. Clearly $\psi^{(c)}$ and $\varphi^{(c)}$ are the charge-conjugate fields of $\psi$ and $\varphi$ .

The physical identity of $\mathcal{K}_{\varrho}$ with anti- $K_{\varrho}$ , of $\pi_{\pm,0}$ with anti-$\pi_{\mp,0}$ and of $\pi'$ with anti- $\pi'$ can be expressed by imposing on $\varphi$ the subsidiary condition

$$\varphi^{(a)} = \varphi \tag{46}$$

and by adding a factor $\frac{1}{2}$ in the above formula for $\mathcal{J}_{\mu}^{(\varphi)}$ and similar (quadratic) quantities.

We shall assume, as usual, that all isocomponents of $\psi$ transform in the same way in space-time. Similar assumption will be made as concerns the isocomponents of $\varphi$ . This means that we shall suppose that the (iso) components of

$$\Phi = \frac{1}{4\sqrt{m}} \eta_{\mu}^{+} \beta_{\mu} \varphi \tag{47}$$

are all pseudoscalars.[5/]

---

5/ These (simplest) assumptions are not necessary for the applicability of our formalism. Generally, the operators of ordinary-space reflections of $\varphi$ or $\psi$ could contain various isoinvariant factors, different from unity, like e.g. $(2U^2 - 1)$ or $(1 - 2\lambda + U^2)$.

---

In virtue of (5) and (42) $\Phi$ satisfies the equations

$$(\partial_\mu \partial_\mu - m^2)\phi = 0\,,\ [\phi_a(x), \phi^*_\beta(x')] = i\,\delta_{a\beta}\,\Delta(m; x-x').$$

In our representation (20) the components of $\phi$ will be denoted as

$$K_+,\ K_o,\ \mathcal{K}_o,\ \mathcal{K}_-,\ \pi_+,\ \pi_-,\ \pi,\ \pi'\ . \tag{47a}$$

Only the quantity $\phi$ (not $\psi$) will occur directly in the Lagrangian of strong and weak interactions. The condition (46) yields $\phi^{(a)} = \Omega\phi^{+T} = \phi$ which means the following conditions for the components (47a):

$$\pi^*_{\pm,o} = \pi_{\mp,o}\ ,\quad \pi'^* = \pi',$$

$$\mathcal{K} = \begin{pmatrix}\mathcal{K}_o\\ \mathcal{K}_-\end{pmatrix} = \begin{pmatrix}K^*_o\\ -K^*_+\end{pmatrix} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}K_+\\ K_o\end{pmatrix}^* = i\tau_2 K^*\ . \tag{46a}$$

We introduce further the Harrisch-Chandra operators <u>in isospace</u> by the defining relations

$$\omega_j\,\omega_k\,L_\ell = L_j\,\delta_{k\ell}\,,\ L^+_j L_k = \delta_{jk}\,,\ L^+_j\,\Omega = L^T_j\ . \tag{48}$$

By these relations the $L_j$ are determined except for a sign which is chosen conveniently in the following. Denoting

$$L^+_j\,\phi = \pi_j\ ,\quad L^+_j\,\psi = \Sigma_j\ , \tag{49}$$

we then find that

$$\pi_1 = \frac{1}{\sqrt{2}}(\pi_+ + \pi_-),\ \pi_2 = \frac{i}{\sqrt{2}}(\pi_+ - \pi_-),\ \pi_3 = \pi_o$$

and similarly for $\Sigma_j$ . We find also that

$$\tfrac{1}{3} L_j^+ \omega_j \phi = \pi', \quad \tfrac{1}{3} L_j^+ \omega_j \psi = \Lambda \tag{50}$$

Finally we see that $\pi_j^* = \pi_j$, $\pi'^* = \pi'$ from (46a).

The operators $L_j$ fulfil (besides the defining relations (48)) a series of other useful relations which are collected in the Appendix together with the above used properties of the original Harrish-Chandra operators $\eta_\mu$.

## IV. Strong and weak interactions of baryons with mesons

1. Consider the following eight expressions of the Yukawa type:

$$
\begin{aligned}
a_1 &= :\bar{\psi} i \gamma_5 \underset{\sim}{\omega} \psi \underset{\sim}{\pi}: ; & b_1 &= :\bar{\psi} i \gamma_5 \underset{\sim}{\lambda} \psi \underset{\sim}{\pi}: , \\
a_1' &= :\bar{\psi} i \gamma_5 \psi \pi': , & b_1' &= :\bar{\psi} i \gamma_5 \frac{U}{2} \psi \pi': , \\
a_2 &= :\bar{\psi} i \gamma_5 \underset{\sim}{\omega} \phi \underset{\sim}{\Sigma}: , & b_2 &= :\bar{\psi} i \gamma_5 \underset{\sim}{\lambda} \phi \underset{\sim}{\Sigma}: , \\
a_2' &= :\bar{\psi} i \gamma_5 \phi \Lambda: , & b_2' &= :\bar{\psi} i \gamma_5 \frac{U}{2} \phi \Lambda: .
\end{aligned}
\tag{51}
$$

With our assumptions concerning the transformation properties of $\psi$ and $\varphi$, all these eight expressions (51) are invariant under proper Lorentz transformations and ordinary space reflection

$$\psi \to \gamma_4 \psi , \quad \phi \to -\phi . \tag{P}$$

Under the charge conjugation

$$\psi \to \psi^{(c)} = \tau_2 \Omega C \bar{\psi}^T , \quad \phi \to \phi^{(c)} = \tau_2 \Omega \phi^{+T} \tag{C}$$

they go over into their hermitian conjugates. Inasmuch as the quantities $a_1$, $b_1$, $a_1'$, $b'_1$ are already hermitian, they are invariant under (C). All the eight expressions (51) are further invariant under rotations and $Y$-reflections in isospace (see Appendix (A 16, 17)). The four a's are also invariant under the Z-reflections (A 18) in isospace whereas the four b's are in this case pseudoinvariant (change their signs).

From the point of view of our formalism the quantities (51) must be considered as the simplest invariants and isoinvariants, quadratical in $\psi$ and linear in $\phi$. We therefore expect that the Lagrangian of strong interactions will be a certain linear (more or less symmetrical) combination of just these quantities (plus its hermitian conjugate).

For simplicity as well as physical reasons we assume that there is only one "strong" coupling constant, i.e. that the coefficients with which the quantities (51) enter the Lagrangian can be only $\pm$ G or 0. It is, however, easy to show that we can assign zero coefficients neither to all a's nor to all b's. This can be seen as follows: The most general self-mass operator of the baryon which can arise from any charge independent interaction of the baryon field $\psi$ with the meson field $\varphi$ has, independently of any perturbation theory, obviously the form

$$\delta M = M_{00} + M_{10}\lambda + M_{01}U + M_{02}U^2.$$

If all the a's (or all the b's) were absent from our Lagrangian, it would be invariant (or change only its sign) under the "reflections" $\psi \to Z_j\psi$, $\varphi \to Z_j\varphi$. Then, of course, the equation

$$Z_j^+ \delta M Z_j = \delta M \qquad \text{(no summation)}$$

would be valid for the corresponding self-mass operator and therefore the term linear in $U$ (which anticommutes with $Z_j$)

would be absent from $\delta M$. This term, however, expresses the mass-difference between N and $\Xi$ which is just the largest in the baryon family. Thus neither the a's nor the b's can be dispended completely.

A convenient Lagrangian of strong interactions seems to be the following rather symmetrical combination of the invariants (51):[6/]

$$\begin{aligned} \mathcal{L}^{(1)} &= G[(a_1 - b_1 + a_1' - b_1') - (a_2 - b_2 + a_2' - b_2')] + h.c. = \\ &= G\{\bar{\Psi} i\gamma_5[\underset{\sim}{\pi}(\underset{\sim}{\omega} - \underset{\sim}{\lambda}) + \pi'(1 - \tfrac{U}{2})]\Psi - \\ &\quad - \bar{\Psi} i\gamma_5[\underset{\sim}{\Sigma}(\underset{\sim}{\omega} - \underset{\sim}{\lambda}) + \Lambda(1 - \tfrac{U}{2})]\Phi\} + h.c. \end{aligned} \tag{52}$$

6/ Another possible special form of the Lagrangian $\mathcal{L}^{(1)}$ is considered in the Appendix.

This Lagrangian leads to the following expressions for the second order self-mass operators (which can be computed easily using the formulas collected in the Appendix):

$$\begin{aligned} \delta M^{(2)} &= \bar{M}(5 + 32\lambda - 20U + 4U^2), \\ \delta m^{(2)} &= \bar{m}(20 + 32\lambda - 26U^2). \end{aligned} \tag{53a,b}$$

The factors $\bar{M}$ and $\bar{m}$ are the usual second order self-masses of a Dirac particle and a pseudoscalar neutral particle in pseudoscalar interaction. As well known |18, 19| $\bar{M}$ is positive (logarithmically divergent without cut-off) whereas $\bar{m}$ is negative (quadratically divergent without cut-off). Thus inserting into (53a,b) the appropriate eigenvalues of $\lambda$ and U we obtain the following second order mass-spectra of baryons and mesons:

| $\underline{M+\delta M^{(2)}=}$ | | $\underline{m+\delta m^{(2)}=}$ | |
|---|---|---|---|
| $=M+5\bar{M}$ | for N and $\Lambda$, | $=m-10\lvert\bar{m}\rvert$ | for K, |
| $=M+37\bar{M}$ | for $\Sigma$, | $=m-20\lvert\bar{m}\rvert$ | for $\pi'$, |
| $=M+45\bar{M}$ | for $\Xi$, | $=m-52\lvert\bar{m}\rvert$ | for $\pi$. |

The degenration of the N- and $\Lambda$-mass level is obviously an accidental feature of the second order perturbation calculation. Of course, also the other (rather favourable) features of these mass-spectra could be more or less accidental in the same sense. Nevertheless we see that a Lagrangian which is a very simple and very symmetrical combination of the invariants (51) can in principle cause the distinctly unsymmetrical distribution of the experimental mass values in the baryon and meson family.

Especially it does not seem impossible that the actual mass of $\pi'$ is greater than the sum of the masses of $\pi_+$, $\pi_-$ and $\pi_0$ In this case $\pi'$ could never be observed because it would decay practically instantaneously into $\pi_+ + \pi_- + \pi_0$ There would be no trouble with "too much gammas" even if the mass of $\pi'$ would be greater than the mass of $3\pi_0$ , since $\pi'$ cannot decay into $3\pi_0$ because in this state $I \neq 0$.

Notice that using the formulas (A 7, 8, 9) we can immediately write down $\mathcal{L}^{(s)}$ in the customary notation as follows:

$$\mathcal{L}^{(s)} = iG\Big\{\tfrac{1}{2}(\bar{N}\gamma_5\underset{\sim}{\tau}N)\underset{\sim}{\pi} - \tfrac{3}{2}(\bar{\Xi}\gamma_5\underset{\sim}{\tau}\Xi)\underset{\sim}{\pi} + 2i(\bar{\underset{\sim}{\Sigma}}\gamma_5 \times \underset{\sim}{\Sigma})\underset{\sim}{\pi} +$$
$$+ \tfrac{1}{2}(\bar{N}\gamma_5 N)\pi' + \tfrac{3}{2}(\bar{\Xi}\gamma_5\Xi)\pi' -$$
$$- \tfrac{1}{2}(\bar{N}\gamma_5 K)\Lambda - \tfrac{3}{2}(\bar{\Xi}\gamma_5 i\tau_2 K^*)\Lambda -$$
$$- \tfrac{1}{2}(\bar{N}\gamma_5\underset{\sim}{\tau}K)\underset{\sim}{\Sigma} + \tfrac{3}{2}(\bar{\Xi}\gamma_5\underset{\sim}{\tau}\, i\tau_2 K^*)\underset{\sim}{\Sigma}\Big\} + h.c. \tag{52a}$$

We see that neither the usual term $(\bar{\Lambda}i\gamma_5\underset{\sim}{\Sigma})\underset{\sim}{\pi}$ nor the possible terms $(\bar{\Lambda}i\gamma_5\Lambda)\pi'$ and $(\bar{\underset{\sim}{\Sigma}}i\gamma_5\underset{\sim}{\Sigma})\pi'$ are contained in (52a), because the contributions to these terms coming from various a-invariants just cancel each other. However the presence of these terms in the Lagrangian (i.e. as primary interactions) is not necessary because the processes $\Sigma \rightleftarrows \Lambda + \pi$, $\Lambda \rightleftarrows \Lambda + \pi'$, $\Sigma \rightleftarrows \Sigma + \pi'$ result from combinations of the remaining primary interactions like e.g. $\Sigma \rightleftarrows N +$ anti-K $\rightleftarrows \pi + N +$ anti-K $\rightleftarrows \pi + N \Lambda$ , etc.

2. To construct the Lagrangian $\mathcal{L}^{(w)}$ of weak interactions between baryons and mesons we introduce first of all the quantities

$$\underset{\sim}{K} = \underset{\sim}{L}^{+} R \Phi \cong \left( \frac{\mathcal{K}_{-} + K_{+}}{\sqrt{2}}, \ \frac{\mathcal{K}_{-} - K_{+}}{i\sqrt{2}}, \ \frac{\mathcal{K}_{0} - K_{0}}{\sqrt{2}} \right),$$

$$K_{(s)} = \frac{1}{3i} \underset{\sim}{L}^{+} \underset{\sim}{\omega} R \Phi = \frac{\mathcal{K}_{0} + K_{0}}{\sqrt{2}},$$

$$\underset{\sim}{B} = \underset{\sim}{L}^{+} R \Psi = \left( \frac{\Xi_{-} + N_{+}}{\sqrt{2}}, \ \frac{\Xi_{-} - N_{+}}{i\sqrt{2}}, \ \frac{\Xi_{0} - N_{0}}{\sqrt{2}} \right), \tag{54}$$

$$B_{(s)} = \frac{1}{3i} \underset{\sim}{L}^{+} \underset{\sim}{\omega} R \Psi = \frac{\Xi_{0} + N_{0}}{\sqrt{2}}.$$

which are identical with the likewise denoted quantities of d'Espagnat, Prentki and Salam (5).

Secondly we define the eight quantities $a_1^{w}, \ldots, b_2'^{w}$ simply by replacing in the eight expressions (51) the quantities $\underset{\sim}{\pi}, \pi', \underset{\sim}{\Sigma}$ and $\Lambda$ by $\underset{\sim}{K}$, $K_{(s)}$, $\underset{\sim}{B}$ and $B_{(s)}$ respectively. Finally we construct the Lagrangian $\mathcal{L}^{(w)}$ by replacing in the Lagrangian $\mathcal{L}^{(s)}$ the quantities $a_1, \ldots, b_2'$ by the new quantities $a_1^{w}, \ldots, b_2'^{w}$ (and $G$ by $g$), so that $L^{(w)}$ becomes

$$\mathcal{L}^{(w)} = g\left[(a_1^{w} - b_1^{w} + a_1'^{w} - b_1'^{w}) - (a_2^{w} - b_2^{w} + a_2'^{w} - b_2'^{w})\right] + h.c. =$$

$$= g\Big\{\bar{\Psi} i \gamma_5 \Big[\underset{\sim}{K}(\underset{\sim}{\omega} - \underset{\sim}{\lambda}) + K_{(s)}\Big(1 - \frac{U}{2}\Big)\Big]\Psi -$$

$$- \bar{\Psi} i \gamma_5 \Big[\underset{\sim}{B}(\underset{\sim}{\omega} - \underset{\sim}{\lambda}) + B_{(s)}\Big(1 - \frac{U}{2}\Big)\Big]\Phi\Big\} + h.c. \tag{55}$$

Let us now see what are the formal properties of this $\mathcal{L}^{(w)}$. The new expressions $a_1^w$, ..., $b_2'^w$ and therefore $L^{(w)}$ too are, of course, again invariant under proper Lorentz transformations and under the space reflection (P). However they are no more isoinvariant because $\underset{\sim}{K}$ and $\underset{\sim}{B}$ are no iso(pseudo) vectors and $K_{(s)}$ and $B_{(s)}$ are no isoscalars. Nevertheless, in virtue of (36c) the quantities $a_1^w$, ... remain invariant at least under the reflection $\psi \to \gamma_3 \psi$, $\phi \to \gamma_3 \phi$. This means that we have automatically the charge conservation in weak interactions. We shall see below that the additional operator R in front of $\phi$ and $\psi$ in (54) plays a role similar to a charge conserving spurion with isobaric spin $\frac{1}{2}$.

Now let us consider the charge conjugation. Using the various formulas collected in the text and in the Appendix (see especially (30), (A5b) and (A6a,b)) we find that all the eight expressions $a_1^w$, ..., $b_2'^w$ go over into their hermitian conjugates so that $\mathcal{L}^{(w)}$ is invariant under (C). Notice however that the components of $\underset{\sim}{K}$ are <u>pseudo</u>hermitian whereas $K_{(s)}$ is hermitian so that also the quantities $a_1^w$, $b_1^w$ are <u>pseudo</u>hermitian whereas $a_1'^w$, $b_1'^w$ are hermitian. Thus, effectively, the terms $a_1^w$, $b_1^w$ are absent from $L^{(w)}$ because they just cancel with their hermitian conjugates.

As well known, the Lagrangian $L^{(w)}$ need not be invariant under (C). In such a case we expect that it will be invariant at least under (P.C). It is easy to construct such parity non-conserving (but "combined parity" conserving) terms to be added to the Lagrangian (66). We see that under (C) the quantities $ia_1^w$, etc. go over into minus hermitian conjugates so that the hermitian expressions ($ia_1^w$ + h.c.), etc. are pseudoinvariant under (C). They are, of course, still invariant under (P) and thus pseudoinvariant under (P.C). Therefore we must further replace $i\gamma_5$ by 1 to obtain hermitian expressions ($i\alpha_1^{w}$ + h.c.), ..., ($i\beta_2'^{w}$ + h.c.), invariant under (P.C). (Notice that in this case ($i\alpha_1'^{w}$ + h.c.) = 0.) From the quantities $i\alpha_1^{w}$, etc. an expression $L'^{(w)}$ like (55) can be constructed. As we are not about to consider in this paper the nonconservation of parity in weak baryon-meson interactions (but only the possible isobaric structure of the Lagrangian) we shall born ourselves to the Lagrangian (55) which is invariant under both (P) and (C) separately.

We shall not consider in detail the physical consequences of the Lagrangian (55) but shall be satisfied by showing that it contains the selection rules $|\Delta U|=1, |\Delta \underset{\sim}{I}|=\frac{1}{2}, |\Delta I_3|=\frac{1}{2}$ for the decay of hyperons. Using the formulas (A7,8,9) we can immediately write down $L^{(w)}$ in the customary notation as follows:

$$
\begin{aligned}
\mathcal{L}^{(W)} = ig\Big\{ & (\bar{\underset{\sim}{\Sigma}}\gamma_5 \underset{\sim}{\Sigma})K_{(s)} + (\bar{\Lambda}\gamma_5 \Lambda)K_{(s)} + \\
& + \tfrac{1}{2}(\bar{N}\gamma_5 N)K_{(s)} + \tfrac{3}{2}(\bar{\Xi}\gamma_5 \Xi)K_{(s)} \\
& - \tfrac{1}{2}(\bar{N}\gamma_5 K)B_{(s)} - \tfrac{3}{2}(\bar{\Xi}\gamma_5 i\tau_2 K^*)B_{(s)} - \\
& - \tfrac{1}{2}(\bar{N}\gamma_5 \underset{\sim}{\tau} K)\underset{\sim}{B} + \tfrac{3}{2}(\bar{\Xi}\gamma_5 \underset{\sim}{\tau} i\tau_2 K^*)\underset{\sim}{B} - \\
& -(\bar{\Lambda}\gamma_5 \underset{\sim}{B})\underset{\sim}{\pi} + i(\bar{\underset{\sim}{\Sigma}}\gamma_5 \times \underset{\sim}{B})\underset{\sim}{\pi} - (\bar{\underset{\sim}{\Sigma}}\gamma_5 B_{(s)})\underset{\sim}{\pi} - \\
& -(\bar{\underset{\sim}{\Sigma}}\gamma_5 \underset{\sim}{B})\pi' - (\bar{\Lambda}\gamma_5 B_{(s)})\pi'\Big\} + h.c. \qquad (55a)
\end{aligned}
$$

The terms which contain $\underset{\sim}{\pi}$ can be arranged as follows:

$$
\begin{aligned}
ig\Big\{ & -\bar{\Sigma}_+\gamma_5(N_+\pi_0 + \sqrt{2}N_0\pi_+) + \frac{1}{\sqrt{2}}\bar{\Sigma}_0\gamma_5(\sqrt{2}N_+\pi_- - N_0\pi_0) - \\
& - \frac{1}{\sqrt{2}}\bar{\Sigma}_0\gamma_5(\Xi_0\pi_0 + \sqrt{2}\Xi_-\pi_+) - \bar{\Sigma}_-\gamma_5(\sqrt{2}\Xi_0\pi_- - \Xi_-\pi_0) - \\
& - \frac{1}{\sqrt{2}}\bar{\Lambda}\gamma_5(\sqrt{2}\Xi_0\pi_0 + \sqrt{2}\Xi_-\pi_+) - \frac{1}{\sqrt{2}}\bar{\Lambda}\gamma_5(\sqrt{2}N_+\pi_- - N_0\pi_0)\Big\} + h.c.
\end{aligned}
$$

This expression is just of the same form as the Lagrangian of weak interactions between baryons and pions considered recently by Ning Hu |20|. We see that the states of the decay products, into which the particles $\Sigma_+, \Sigma_0, \Sigma_-$ and $\Lambda$ can decay (at least virtually) by our primary weak interactions, have all $I = \frac{1}{2}$ and $I_3 = \frac{1}{2}, \pm\frac{1}{2}, -\frac{1}{2}$ and $\pm\frac{1}{2}$ respectively. No term is present among our primary weak interactions which would cause the decay $\Sigma_- \rightarrow N_0 + \pi_-$(with $I = \frac{3}{2}$, $I_3 = -\frac{3}{2}$ in final state). The contributions to such a term just cancel each other in our Lagrangian. As shown by Ning Hu, this is not a defect since the decay of $\Sigma_-$ can be accounted for by cooperation of the strong and weak primary interactions which are already present in our Lagrangians $L^{(w)}$ and $L^{(s)}$, for instance

$$\Sigma_- \rightarrow \Sigma_0 + \pi_- \rightarrow N_+ + \pi_- + \pi_- \rightarrow N_0 + \pi_- .$$

To obtain the expression (55) we assumed that all terms of the "strong" Lagrangian $L^{(s)}$ have their counterparts in $L^{(w)}$. If we abandon this conjecture and assume instead that e.g. the terms with $K_{(s)}$and $B_{(s)}$ are not effective in weak interactions,we obtain much simpler Lagrangian of weak interactions in which the term $(i g / \sqrt{2}) \bar{\Sigma}_- \gamma_5 N_0 \pi_-$ is present.

The terms in the first four rows in (55a) lead to the decay of K-mesons into pions via baryon-antibaryon pairs and strong pion-baryon interactions. Note that in our formalism, in contradistinction to the scheme of weak interactions proposed by d'Espagnat, Prentki and Salam |5|, we obtain also weak interactions between ($N$, $\Xi$ ) and K (see 2nd to 4th row of (55a)) which follow the selection rules $|\Delta U| = 1$, $|\Delta I| = \frac{1}{2}$,

$|\Delta I_3| = \frac{1}{2}$ too. On the other hand we do not obtain the non-convenient term like $(\bar{\underset{\sim}{B}} i\gamma_5 \times \underset{\sim}{B})\underset{\sim}{\pi}$ which would induce the anomalous decay $\Xi_- \rightarrow N_0 + \pi_-$ .

Finally let us remark that the poor symmetry properties of our "weak" Lagrangian (55) are in full accordance with the principle that the strength of the interaction is a descending function of the degree of symmetry of the corresponding Lagrangian. The "strong" Lagrangian $L^{(s)}$ is invariant under all rotations and reflections in isospace, the Lagrangian of electromagnetic interactions is invariant under rotations around the third axis in the isospace and under the reflection through the 1,2 - plane (and therefore also under the reflection through the origin) and finally $L^{(w)}$ is invariant only under the reflection through the 1,2-plane in isospace. Cf. also |4|.

## V. Conclusion

We have seen in Section III that a matrix-algebra in the threedimensional space can be defined which characterizes the isobaric structure of both the family of baryons and that of mesons and enables us to describe all baryons (mesons) by one universal baryon field $\psi$ (meson field $\varphi$ ).7/

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7/ The relations defining this matrix-algebra in isospace can easily be generalized to relations defining a quite analogous algebra in the fourdimensional space-time. As will be shown in a separate paper by one of the authors (M.L.), the elements $\alpha_\mu$ of this "fourdimensional" algebra (counterparts of the elements $\omega_j$ of our isoalgebra) admit as their irreducible representations just only either the Dirac matrices $\gamma_\mu$ or the Duffin-Kemmer 5 x 5 -matrices $\beta'_\mu$ (not the 10 x 10-matrices $\beta''_\mu$ ). An element $\mathcal{N}$ exists in this fourdimensional algebra which is an exact counterpart of the element U of the isoalgebra and "takes the value" 1(0) for $\alpha_\mu = \gamma_\mu (\beta'_\mu)$. Possible physical implications of this new algebra are studied with the obvious aim of further unification of the description of the whole group of stronglys - by introducing one multicomponent field $\chi$ of stronglys which separates into the baryon field $\psi$ and meson field $\varphi$. One can hope that also a more definite form of the Lagrangian $\mathcal{L}^{(s)}$ will emerge from this theory of the universal "strong field" $\chi$ .

---

The Lagrangian of strong interactions can be expressed in terms of the whole fields $\psi$ and $\varphi$ in the form of the very symmetrical linear combination (52) of the simplest Yukawa-type invariants (51). This symmetry, however, becomes hidden or disappears in the customary notation (52a) using the wave functions of individual baryons and mesons. Thus, although the Lagrangian contains only one coupling constant G, the strength of interaction of the baryon with the meson is different in their different isobaric states. The splitting of the mass values in the baryon family is caused by the interaction with pions as well as with kaons. Similar conclusion has been drawn also by Ning Hu |22| from another point of view. The fourth pseudoscalar pion $\pi'$ (with I = 0), as introduced in this paper, may well be unobservable in practice due to its large mass and its ability to decay rapidly into

$$\pi_+ + \pi_- + \pi_0 \; .$$

The Lagrangian of weak interactions of baryons with mesons can be written in a closed form quite similar to the Lagrangian of strong interactions. A special operator belonging to the isoalgebra provides automatically for the charge conservation and at the same time for the selection rule $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ . The group of symmetry operations in isospace admitted by this weak Lagrangian is just the minimum admissible.

Now, as concerns the other particles, not belonging to the group of stronglys: The first question is, if the algebra defined solely by the relations (21) - (24) (i.e. without the additional conditions requiring the existence of $\Omega$ and R which can be regarded as characteristic only for the stronglys) is competent for the whole system of fundamental particles. We believe that the answer is affirmative. As mentioned at the beginning of Section III, besides the three irreducible representations which find their use in the description of the group of stronglys, there exists in the first place the trivial representation in which all matrices $\omega_j$ , $\lambda_j$ and U are zero. This representation could correspond to the photon. In the second place there exist also "pathological" irreducible representations in which $\omega_j$ (and U) are zero whereas $\lambda_j$ are given by any nontrivial irreducible set of spin matrices satisfying the relations (23).

If we add the requirement that the charge operator should always be given by the formula $q = \lambda_3 + \frac{1}{2}U$ and that its eigenvalues can be only 0 and $\pm$ 1, we see that from amongst the pathological representations only that with $\lambda$ =1 remains. This representation can now be associated with the lepton triplet ( $\mu_+$ , $\nu$ , $e_-$ ) without any necessity of changing the definition of the charge operator in case of leptons.[8/] Provisions for the mass-difference between $\mu$ and $e$ as well as for the vanishing of the mass of $\nu$ can be made in the same manner as

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8/ It can be also associated with the Schwinger boson-triplet consisting of the photon and two heavy charged vector-mesons. The above mentioned trivial representation then remains free to be associated with the graviton |21|.

in the papers |5| and/or |21| .

Finally let us note that also the "fourdimensional" algebra (mentioned in footnote 7)) admits "pathological" representations in which $\alpha_\mu$ are zero matrices whereas the spin matrices $\sigma_{\mu\nu}$ (corresponding to $\lambda_j$ ) are nonzero. Such representations can be used to write the wave equations of non-strongly interacting particles in the form recently proposed by Feynman and Gell-Mann |23|.

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## Appendix

Besides (48) the $L_j$ fulfil the relations

$$L_j L_j^+ = (L_j L_j^+)^2 = \lambda - \frac{1}{2}U^2,$$
$$\{L_j L_j^+, \omega_k\} = (1-U^2)\omega_k, \ [L_j L_j^+, \lambda_k] = 0, \qquad \text{(A1a,b,c)}$$

$$\lambda_j \lambda_k L_\ell = \delta_{jk} L_\ell - \delta_{j\ell} L_k, \qquad \text{(A2)}$$

$$\omega_j L_k = \frac{1}{3}\delta_{jk}\omega_\ell L_\ell, \quad \lambda_j L_k = i\varepsilon_{jk\ell} L_\ell, \qquad \text{(A3a,b)}$$

$$\lambda_j \omega_k L_\ell = 0, \ L_j^+ \omega_k L_\ell = 0, \ L_j^+ U = 0, \qquad \text{(A4a,b,c)}$$

$$\left.\begin{aligned} L_j^+ Y_k &= (2\delta_{jk}-1)L_j^+ \\ L_j^+ Z_k &= (1-2\delta_{jk})L_j^+ \end{aligned}\right\} \text{(no summations)} \qquad \text{(A5a,b)}$$

$$L_j^+\{R, Z_2\} = 0, \ L_j^+ \omega_j [R, Z_2] = 0. \qquad \text{(A6a,b)}$$

Further we have the decompositions

$$\omega_j = \lambda_j(U+U^2) + \lambda_j(U-U^2) + \frac{1}{2}(\omega_k L_k L_j^+ + L_j L_k^+ \omega_k), \qquad \text{(A7)}$$

$$\lambda_j = \frac{1}{2}\lambda_j(U^2+U) + \frac{1}{2}\lambda_j(U^2-U) - i\varepsilon_{jk\ell} L_k L_\ell^+, \qquad \text{(A8)}$$

$$1 = \frac{1}{2}(U^2+U) + \frac{1}{2}(U^2-U) + L_k L_k^+ + \frac{1}{9}\omega_j L_j L_k^+ \omega_k \qquad \text{(A9)}$$

and finally the useful formulas

$$\omega_j^2 = 3 + U^2 - 2\lambda\,, \quad \lambda_j^2 = 2\lambda - \tfrac{1}{4}U^2 = \lambda(\lambda+1)\,,$$

$$\tfrac{1}{3}\omega_j L_k L_j^+ \omega_k = \tfrac{1}{9}\omega_j L_j L_k^+ \omega_k = 1 - \lambda - \tfrac{1}{2}U^2. \qquad \text{(A10a,b,c)}$$

All these relations and the fundamental equations of the algebra and of the field theory in Section III are invariant under arbitrary unitary transformation

$$O \to u\,O\,u^+ \qquad \text{with } O = \omega_j, \lambda_j, U, R, Y_j, Z_j\,,$$

$$\Omega \to u\,\Omega\,u^T,\ L_j \to u L_j\,,\ \psi \to u\psi,\ \varphi \to u\varphi.$$

Notice that especially the definitions of the quantities $\pi_j, \pi', \Sigma_j, \Lambda, K_j, K_{(s)}, B_j, B_{(s)}$ in terms of $L_j, \omega_j, R, \psi$ and $\Phi$ are invariant under $(u)$, as necessary. Thus the whole theory is independent of any special choice of the representation of the isoalgebra. Of course, the representation in which the matrices U, $\lambda$ and $\lambda_3$ are all diagonal is physically distinguished as most convenient because the eigenvalues of these matrices characterize the observable isobaric states of the baryon and meson.

The Harrish-Chandra matrices $\eta_\mu$ have the following properties

$$\eta_\mu^+ \eta_\nu = \delta_{\mu\nu}\,, \quad \eta_\nu \eta_\nu^+ = \beta\,,$$

$$\beta^2 = \beta\,; \quad \{\beta_\nu, \beta\} = \beta_\nu\,, \qquad \text{(A11a,b,c,d)}$$

$$\beta_\lambda \beta_\mu \eta_\nu = \delta_{\mu\nu} \eta_\lambda , \quad \eta_\lambda^+ \beta_\mu \eta_\nu = 0, \qquad \text{(A12a,b)}$$

$$\beta_\lambda \eta_\mu = \tfrac{1}{4} \delta_{\lambda\mu} \beta_\nu \eta_\nu , \quad \eta_\mu^+ B = - \eta_\mu^T$$

(A13a,b)

These relations are similar to those between $\omega_j$ and $L_k$. Using the definition (44) of $\varphi^{(a)}$ and (A13b) we find that $\phi^{(a)} = \frac{1}{4\sqrt{m}} \eta_\mu^+ \beta_\mu \varphi^{(a)} = \frac{1}{4\sqrt{m}} (\varphi^+ \beta_\mu \eta_\mu \Omega)^T = \Omega \phi^{+T}$.

The transformation of $\varphi$ corresponding to an infinitesimal Lorentz transformation is

$$\varphi \to (1 + \tfrac{i}{2} \eta_{\mu\nu} \sigma_{(\beta)\mu\nu}) \varphi , \quad (\sigma_{(\beta)\mu\nu} = -i [\beta_\mu , \beta_\nu]) \qquad \text{(A 14)}$$

and according to our assumption the transformation corresponding to the total ordinary space reflection is

$$\varphi \to (1 - 2 \beta_4^2) \varphi . \qquad \text{(A15)}$$

Then we easily find that $\bar{\varphi} \beta_\mu \varphi$ is a vector, $\phi_\mu = \eta_\mu^+ \varphi$ a pseudovector, $\bar{\varphi} \varphi$ a scalar and $\phi$ a pseudoscalar in space-time. We also find that $\varphi^{(a)}$ ( $\phi^{(a)}$ ) transforms in the same manner as $\varphi$ ( $\phi$ ). The condition $\varphi = \varphi^{(a)}$ is therefore invariant under Lorentz transformation and space reflection.

The transformation of $\psi$ (or $\varphi$ ) corresponding to infinitesimal rotation in isospace is

$$\psi \ (\text{or } \varphi) \to \left(1+\frac{i}{2}\eta_{jk}\lambda_{jk}\right)\psi\ (\text{or } \varphi) \tag{A16}$$

where

$$\lambda_{jk} = \varepsilon_{jkl}\lambda_l = -i[\lambda_j, \lambda_k] = -i\lambda^2[\omega_j, \omega_k] .$$

If we further take as the transformation corresponding to reflection of the $j$-th axis in isospace the (d'Espagnat-Prentki) reflection

$$\psi \ (\text{or } \varphi) \to Y_j \psi \ (\text{or } \varphi), \tag{A17}$$

we find that

$$\bar{\psi}\omega_j\psi, \bar{\psi}\lambda_j\psi, \bar{\psi}\omega_j\phi, \bar{\psi}\lambda_j\phi, \bar{\varphi}\omega_j\varphi, \bar{\varphi}\lambda_j\varphi, \Sigma_j, \pi_j$$

are all isopseudovectors and

$$\bar{\psi}\psi, \bar{\psi}U\psi, \bar{\psi}\phi, \bar{\psi}U\phi, \bar{\varphi}\varphi, \bar{\varphi}U\varphi, \Lambda, \pi'$$

are isoscalars (cf. the formulas (A5a,b) and (33a,b) - (36a,b)).

If however we take instead of (A17)

$$\psi \ (\text{or } \varphi) \to Z_j \psi \ (\text{or } \varphi), \tag{A18}$$

we find that

$$\bar{\psi}\omega_j\psi, \bar{\psi}\omega_j\phi, \bar{\varphi}\omega_j\varphi, \Sigma_j, \pi_j$$

are isovectors,

$$\bar{\psi}\lambda_j\psi, \bar{\psi}\lambda_j\phi, \bar{\varphi}\lambda_j\varphi$$

are isopseudovectors,

$$\bar{\psi}\psi, \bar{\psi}\phi, \bar{\varphi}\varphi, \Lambda, \pi'$$

are isoscalars and

$$\psi U\psi, \bar{\psi}U\phi, \bar{\varphi}U\varphi$$

are isopseudoscalars.

We find also that in both cases (A17) and (A18) the functions $\psi^{(a)}$ and $\varphi^{(a)}$ transform in the same manner as $\psi$ and $\varphi$ so that the condition $\varphi = \psi^{(a)}$ is also isoinvariant.

With a Lagrangian $\mathcal{L}^{(3)}$ of the general form

$$\mathcal{L}^{(3)} = \sum (G \cdot a + F \cdot b) + h.c.$$

we obtain the following expressions for the second order self-mass operators:

$$\begin{aligned}
\delta M^{(2)} = \bar{\bar{M}} \Big\{ & 12G_1(G_1 + G_2 + G_2') + 4G_1'(G_1' + 2G_2') + 3G_2(G_2 + 2G_2') + 11G_2'^2 + F_2'^2 + \\
& + \lambda\big[-8G_1(G_1 + G_2 + G_2') + 8G_1'(G_2 - G_2') + 2G_2(3G_2 - 2G_2') + 8F_1(F_1 - 2F_2) - \\
& - 10G_2'^2 + 9F_2^2 - F_2'^2\big] + U\big[12G_1F_1 + 4G_1'F_1' + 3G_2F_2 + G_2'F_2'\big] \cdot \\
& + U^2\big[4G_1(G_1 - 2G_2 - 2G_2') - 4G_1'(G_2 + G_2') - G_2(3G_2 + 4G_2') - F_1(F_1 - 8F_2) + \\
& + F_1'^2 - 5G_2'^2 - \tfrac{15}{4}F_2^2 - \tfrac{1}{4}F_2'^2\big]\Big\},
\end{aligned}$$

$$\begin{aligned}
\delta m^{(2)} = \bar{\bar{m}} \Big\{ & 8G_1'(4G_1' + 3G_2 + G_2') + 4(3G_2^2 + G_2'^2 + F_1'^2) + \\
& + \lambda\big[8G_1(3G_1 + G_2 + G_2') - 2G_2(5G_2 - 2G_2') + 8F_2(F_2 - 2F_1) - \\
& - 8G_1'(4G_1' + 3G_2 + G_2') + 2(6F_1^2 - 2F_1'^2 - G_2'^2)\big] + \\
& + U^2\big[-4G_1(3G_1 + G_2 + G_2') - 4G_1'(4G_1' + 3G_2 + G_2') - G_2(G_2 + 2G_2') + \\
& + 2F_1(4F_2 - 3F_1) - G_2'^2 - \tfrac{5}{2}F_2^2 - 2F_1'^2 + \tfrac{1}{2}F_2'^2\big]\Big\}.
\end{aligned}$$

Inserting

$$G_1 = G_1' = -G_2 = -G_2' = -F_1 = -F_1' = F_2 = F_2' = G\,, \quad \bar{\bar{M}}G^2 = \bar{M}\,,$$

we get the special formulas (53a,b).

Another interesting special Lagrangian $\mathcal{L}^{(3)}$ is obtained with

$$G_1 = -G_1' = G_2 = -G_2' = -F_1 = \frac{1}{2}F_1' = F_2 = -\frac{1}{2}F_2' = G$$

namely

$$\mathcal{L}^{(3)} = G\left[(a_1 - b_1) - (a_1' - 2b_1') + (a_2 + b_2) - (a_2' + 2b_2')\right] + h.c. \tag{A19}$$

This Lagrangian yields the self-mass operators

$$\delta M^{(2)} = \bar{M}\left(36 + 5\lambda - 15U - \frac{39}{4}U^2\right)$$

$$\delta m^{(2)} = \bar{m}\left(48 + 12\lambda - \frac{85}{2}U^2\right)$$

and the mass-spectra

| $M + \delta M^{(2)} =$ | | $m + \delta m^{(2)} =$ | |
|---|---|---|---|
| $= M + 13\frac{3}{4}\bar{M}$ | for N, | $= m - 11\frac{1}{2}\lvert\bar{m}\rvert$ | for K, |
| $= M + 36\bar{M}$ | for $\Lambda$, | $= m - 48\lvert\bar{m}\rvert$ | for $\pi'$, |
| $= M + 41\bar{M}$ | for $\Sigma$, | $= m - 60\lvert\bar{m}\rvert$ | for $\pi$. |
| $= M + 43\frac{3}{4}\bar{M}$ | for $\Xi$. | | |

Now the $\Lambda$-N mass-difference becomes very large whereas the $\Xi - \Sigma$ and $\pi' - \pi$ mass-differences are rather small.

This could possibly be amended by a better mothod of calculation.

Written in the customary notation the Lagrangian (A19) reads

$$\mathcal{L}^{(5)}=iG\Big\{\tfrac{1}{2}(\bar{N}\gamma_5\underset{\sim}{\tau}N)\underset{\sim}{\pi}-\tfrac{3}{2}(\bar{\Xi}\gamma_5\underset{\sim}{\tau}\cdot\Xi)\underset{\sim}{\pi}+2(\bar{\Lambda}\gamma_5\underset{\sim}{\Sigma})\underset{\sim}{\pi}+2i(\bar{\underset{\sim}{\Sigma}}\gamma_5\times\underset{\sim}{\Sigma})\underset{\sim}{\pi}+$$
$$+\tfrac{3}{2}(\bar{N}\gamma_5\underset{\sim}{\tau}K)\underset{\sim}{\Sigma}-\tfrac{1}{2}(\bar{\Xi}\gamma_5\underset{\sim}{\tau}\,i\tau_2K^*)\underset{\sim}{\Sigma}-2(\bar{N}\gamma_5K)\Lambda-$$
$$2(\bar{\Xi}\gamma_5\Xi)\pi'-2(\bar{\Lambda}\gamma_5\Lambda)\pi'\Big\}+h.c.$$

We see that the meson $\pi'$ is primarily coupled only with $\Xi$ and $\Lambda$, further that the coupling ($\bar{\Xi}K^*\Sigma$ ) is weaker than ($\bar{N}\ K\ \Sigma$ ) whereas ( $\bar{\Xi}\ K^*\ \Lambda$ ) is lacking entirely. This could inhibit the production of $\pi'$ and of $\Xi$+ 2K. On the other hand the production of $\Lambda$ + K and $\Sigma$ + K is enhanced.

By the method explained in Section IV/2 a "weak" Lagrangian corresponding to (A19) can also be constructed. As concerns the terms containing $\underset{\sim}{\pi}$ , it differs from (55a) only in the sign of the term $\bar{\Lambda}\gamma_5\underset{\sim}{B}\,\underset{\sim}{\pi}$.

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