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METHOD OF CHARGED PARTICLE FOCUSING IN ACCELARATORS

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D u b n a

Abstract:

In this paper a variant of charged particles focusing obtained with the synchrocyclotron of the Laboratory of Nuclear Problems is considered. The particle focusing is achieved by the magnetic field formed in the gap of the deviating electromagnet with the help of iron bars. It was shown that the effect of the focusing arrangement is equivalent to that of the magnetic quadrupole lens if the definite ratio of the dimensions of these bars is maintained. By means of such lenses the density of the 600 MeV polarized proton beam intensity was increased three times in the place where the detecting arrangement was set up, the density of the ~300 MeV meson beam intensity was increased 2.7 times.

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Charged particles extracted from the accelerator's chamber pass usually rather great distance to the detecting apparatus that leads to a considerable reduction of the intensity used. To deviate the beam or to separate the flux of secondary particles of definite energy in some experiments the electromagnets are used. To use the field in the electromagnet gap not only for the deviation of the particles but also for their focusing is of practical interest. This can be achieved by forming the magnetic field on the way of the charged particles similar to that of the magnetic quadrupole lens. If a ferromagnetic body the dimension of which in the field direction is more at least than one transverse dimension then in the fields $\gg$ 9000 ersted the sample under consideration is being magnetized almost up to the saturation.

In this case the ferromagnetic body may be considered as uniformly magnetized and to calculate analytically the components of the field from the sample under consideration.* For the rectangular bar 2h high, 2a wide and ℓ long the magnetic field components H_z and H_y in the

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*The field in the electromagnet gap is supposed to be uniform.

coordinate system given in Fig. I have the form [1].

$$H_z(x,y,z)=M\Big\{\operatorname{arctg}\frac{(y-a)\,(x-e)}{(z-h)\sqrt{(x-e)^2+(y-a)^2+(z-h)^2}}-\operatorname{arctg}\frac{(y-a)\,x}{(z-h)\sqrt{x^2+(y-a)^2+(z-h)^2}}$$

$$-\operatorname{arctg}\frac{(y+a)\,(x-e)}{(z-h)\sqrt{(x-e)^2+(y+a)^2+(z-h)^2}}+\operatorname{arctg}\frac{(y+a)\,x}{(z-h)\sqrt{x^2+(y+a)^2+(z-h)^2}}+$$

$$+\operatorname{arctg}\frac{(y+a)\,(x-e)}{(z+h)\sqrt{(x-e)^2+(y+a)^2+(z+h)^2}}-\operatorname{arctg}\frac{(y+a)\,x}{(z+h)\sqrt{x^2+(y+a)^2+(z+h)^2}}-$$

$$-\operatorname{arctg}\frac{(y-a)\,(x-e)}{(z+h)\sqrt{(x-e)^2+(y-a)^2+(z+h)^2}}+\operatorname{arctg}\frac{(y-a)\,x}{(z+h)\sqrt{x^2+(y-a)^2+(z+h)^2}}\Big\} \tag{1}$$

$$H_y(x,y,z)=-M\ln\Big\{\frac{\left[x+\sqrt{x^2+(y+a)^2+(z+h)^2}\right]\cdot\left[(x-e)+\sqrt{(x-e)^2+(y-a)^2+(z+h)^2}\right]}{\left[(x-e)+\sqrt{(x-e)^2+(y+a)^2+(z+h)^2}\right]\left[x+\sqrt{x^2+(y-a)^2+(z+h)^2}\right]}\cdot$$

$$\cdot\frac{\left[(x-e)+\sqrt{(x-e)^2+(y+a)^2+(z-h)^2}\right]\cdot\left[x+\sqrt{x^2+(y-a)^2+(z-h)^2}\right]}{\left[x+\sqrt{x^2+(y+a)^2+(z-h)^2}\right]\left[(x-e)+\sqrt{(x-e)^2+(y-a)^2+(z-h)^2}\right]}\Big\} \tag{2}$$

where $M=\frac{21000 \pm 500}{4\pi}$ oersted - the magnitude of the magnization for the majority of the ferromagnetic materials.

Let us restrict the region of the magnetic field which we shall consider further by the following conditions /Fig.2/:

$$-h \le y \le h$$
$$-h \le z \le h \tag{4}$$

When choosing the geometry of the ferromagnetic bars and in the calculation of the magnetic field it is necessary to take into account the influence of the pole tips on the redistribution of the magnetic field from the saturated bars. The influence of the pole tips may be approximately taken into account by means of mirror reflection of the iron bars with respect to the plane surface of the magnet poles. /Fig. 2/.

The system of the mirror reflected bars for case of plane boundary is calculated on the basis of the image method developed for currents.|2| In this case one may write for the magnitization of the mirror reflected bars that

$$M_{ref} = M \cdot \left(\frac{\mu - 1}{\mu + 1}\right)^{K} \qquad (5)$$

where M_{ref} is the magnitization of the mirror reflected bars,

M is the magnitization of the bars in the electromagnet gap,

μ is the magnetic penetrability of the medium,

K is the number of reflection.

It is assumed in Eq.|5| that μ = const. in all the half space and is independent of the induction. For the ferromagnetic bodies this condition is not generally performed, however, one can easily see that $M_{refl.}$ stops to be dependent upon the magnetic penetrability of the medium at sufficiently high values of the latter. If one restricts oneself by the induction region when $\mu > 10$, then one may approximately consider that the bars mirror reflected have the same magnitude of the magnitization, ie.,

$$M_{refl.} \simeq M \qquad (6)$$

Such an approximation is quite allowed since the contribution to the magnitude and the distribution of the magnetic field component from the reflections is not relatively great and a small addition to the field of main bars. It is possible to show that the deviation which the charged particle undergo in the bar field of the final length ℓ is equal to the deviation on the length ℓ in the bar field of the infinite length with the same transverse dimensions. So, for the bar of the final length ℓ, height 2h and the wideness 2a the following relations take place

$$\int_{-\infty}^{\infty} H_y(x,y,x)\,dx = H_{oy}(y,z)\cdot \ell \qquad (7)$$

$$\int_{-\infty}^{\infty} H_z(x,y,z)\,dx = H_{oz}(y,z)\cdot \ell \qquad (8)$$

where $H_y(x,y,z)$ and $H_z(x,y,z)$ may be determined from (1) and (2) whereas $H_{oy}(y,z)$ and $H_{oz}(y,z)$ are the components of the field strength from the infinitely -long bar with the same cross section 2h x 2a. Such a substitution means that the effect of the magnetic field with the edge falling sections may be substituted by the same effect of the homogeneous field the magnitude of which is determined for the bar of the infinite length. The field has the sharp geometric boundaries and is concentrated in the region of the length ℓ.

With the help of Eqs. |7| and |8| one may make the calculation of the necessary field gradients for the particle focusing for the infinitely -long bars.* So, for H_z and H_y the components of the magnetic field, the mirror reflection is being taken into account, one may obtain the following expressions in the coordinate system given in Fig. 2, in the plane $x = 0$.

$$H_z(y,0) = 4M\left\{\sum_n (-1)^n \operatorname{arctg}\frac{\left(\frac{y}{h}+1\right)}{(2n+1)} - \sum_n (-1)^n \operatorname{arctg}\frac{\left(\frac{y}{h}+1+\frac{2a}{h}\right)}{(2n+1)} + \right.$$

$$\left. + \sum_n (-1)^n \operatorname{arctg}\frac{\left(\frac{y}{h}-1\right)}{(2n+1)} - \sum_n (-1)^n \operatorname{arctg}\frac{\left(\frac{y}{h}-1-\frac{2a}{h}\right)}{(2n+1)}\right\} \tag{9}$$

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* The curvature of the charged particle trajectory in the main field of the magnet is neglected.

$$H_y(z,0) = 2M\left\{ \sum_n (-1)^n \ln \frac{\left[1+\left(\frac{z}{h}+2n+1\right)^2\right]}{\left[1+\left(\frac{z}{h}-2n+1\right)^2\right]} + \right.$$

$$\left. + \sum_n (-1)^n \ln \frac{\left[1+\theta+\left(\frac{z}{h}-2n-1\right)^2\right]}{\left[1+\theta+\left(\frac{z}{h}+2n+1\right)^2\right]} \right\} \qquad (10)$$

where $\theta = \frac{4a}{h}\left(1+\frac{a}{h}\right)$, $\qquad n = 0, 1, 2 \ldots$

Expanding expressions |9| and |10| in a series and summing up the coefficients before the identical degrees y and z we obtain that

$$H_z(y,0) = 8M\theta\left\{ \frac{y}{h}\left[\sum_n (-1)^n \frac{(2n+1)}{m_n(m_n+\theta)}\right] - \right.$$

$$\left. - \frac{y^3}{3h^3}\left[\sum_n (-1)^n \frac{(2n+1)\left[(4-m_n)(3m_n^2+3m_n\theta+\theta^2)-3m_n^3\right]}{m_n^3(m_n+\theta)^3}\right] + \ldots \right\} \qquad (11)$$

$$H_y(z,0) = 8M\theta\left\{ \frac{z}{h}\left[\sum_n (-1)^n \frac{(2n+1)}{m_n(m_n+\theta)}\right] - \right.$$

$$\left. - \frac{z^3}{3h^3}\left[\sum_n (-1)^n \frac{(2n+1)\left[(4-m_n)(3m_n^2+3m_n\theta+\theta^2)-3m_n^3\right]}{m_n^3(m_n+\theta)^3}\right] + \ldots \right\} \qquad (12)$$

where $m_n = (2n+1)^2+1$, $\qquad n = 0, 1, 2 \ldots$

The estimations show that the coefficient of the third degree

Y and Z is less than 1% of the coefficient of the linear term.

Thus, in the region determined by the conditions |4| the magnetic field has practically the constant gradient the magnitude of which may be calculated from the expression:

$$\frac{\partial H_z}{\partial y} = \frac{\partial H_y}{\partial z} = \frac{8M\theta}{h}\left[\sum_n (-1)^n \frac{(2n+1)}{m_n(m_n+\theta)}\right] \tag{13}$$

When making the numerical calculations it is quite sufficient to calculate the first four terms of this series since the contribution from the subsequent terms is small and may be neglected. It follows from the analysis of formula |13| that the chosen geometry of the rectangular bars acts as a magnetic quadrupole lens with the aperture 2h, focusing the charged particles in one plane and defocusing them in the other perpendicular plane. To obtain the focusing effect in the two mutually perpendicular planes it is necessary to place successively two or some quadrupole lenses with the gradients alternating in sign. In the case under consideration the change of the sign of the magnetic field intensity gradient is achieved by the symmetrical permutation of the iron bars with respect to the plane ZX /Fig.2/. The calculation of the focusing effect of the lens may be made by the following formulas |3|. For the particles moving in the plane XZ , the optical strength of two-section lens is equal to

$$\frac{1}{F_{xz}} = K^{\frac{1}{2}}(\sin\alpha\, ch\alpha - \cos\alpha\, sh\alpha + CK^{\frac{1}{2}}\sin\alpha\, sh\alpha) \qquad (14)$$

where $K = \frac{\partial H_y}{\partial z} \Big/ HR\ cm^{-2}$, $HR = \frac{pc}{300}$ oersted/cm

pc is the momentum of the charged particle in ev,

$\frac{\partial H_y}{\partial z}$ is the field gradient expressed in oersted/cm,

$\alpha = K^{\frac{1}{2}}\ell$, ℓ the length of the lens section in cm,

c the distance between the lens sections.

The main plane of the lens is situated at the distance

$$X_{xz} = -F_{xz}\left[1 - \cos\alpha\, ch\alpha + \sin\alpha\, sh\alpha + CK^{\frac{1}{2}}\sin\alpha\, ch\alpha\right] \qquad (15)$$

from the back edge of the lens. Expanding |14| and |15| in a series and restricting the essential terms we obtain that

$$\frac{1}{F_{xz}} \simeq K^2\ell^2\left[\frac{2}{3}\ell + c\right] \qquad (16)$$

$$x_{xz} \simeq -\frac{1}{\frac{2}{3}K\ell} \qquad (17)$$

One may obtain analogous expressions for the particles focused in the plane yx if in expressions |14| and |15| K is substituted for $-K$. Then if $K^{1/2}$ for $iK^{1/2}$ and α for $i\alpha$ are substituted the optical strength and the position of the main plane may be written in the form

$$\frac{1}{F_{xy}} = K^{1/2}\left[\sin\alpha\, ch\alpha - \cos\alpha\, sh\alpha + cK^{1/2} sh\alpha \sin\alpha\right] \simeq K^2\ell^2\left(\frac{2}{3}\ell + c\right) \tag{18}$$

$$X_{xy} = -F_{xy}\left[1 - ch\alpha\cos\alpha - sh\alpha\sin\alpha - cK^{1/2} sh\alpha\cos\alpha\right] \simeq \frac{1}{\frac{2}{3}K\ell} \tag{19}$$

The distances L_1 from the front edge of the lense to the source of the charged particles |Fig.3| and L_2 -from the back edge of the lense to the point in which the particles must be focused are connected with the parameters of the magnetic lense by the following expressions:

$$L_{2xz} = \frac{\left[(\cos\alpha\, ch\beta - \sin\alpha\, sh\beta) + (L_1K^{1/2})^{-1}(\sin\alpha\, ch\beta + \cos\alpha\, sh\beta) - cK^{1/2} ch\beta(\sin\alpha - (L_1K^{1/2})^{-1}\cos\alpha)\right]}{K^{\frac{1}{2}}\left[(\sin\alpha\, ch\beta - \cos\alpha\, sh\beta) - (L_1K^{1/2})^{-1}(\sin\alpha\, sh\beta + \cos\alpha\, ch\beta) + cK^{1/2} sh\beta(\sin\alpha - (L_1K^{1/2})^{-1}\cos\alpha)\right]} \tag{20}$$

$$L_{2xy} = \frac{\left[(ch\alpha\cos\beta + sh\alpha\sin\beta) + (L_1K^{1/2})^{-1}(sh\alpha\cos\beta + ch\alpha\sin\beta) + cK^{\frac{1}{2}}\cos\beta(sh\alpha + (L_1K^{\frac{1}{2}})^{-1}ch\alpha)\right]}{K^{\frac{1}{2}}\left[(ch\alpha\sin\beta - sh\alpha\cos\beta) - (L_1K^{\frac{1}{2}})^{-1}(ch\alpha\cos\beta - sh\alpha\sin\beta) + cK^{\frac{1}{2}}\sin\beta(sh\alpha + (L_1K^{\frac{1}{2}})^{-1}ch\alpha\right.}$$

where $\alpha = K^{1/2}\ell_1$, $\beta = K^{1/2}\ell_2$,

ℓ_1 is the length of the first section of the lens,

ℓ_2 is the length of the second section of the lens./Fig.3/

Such a lens has a strong astigmatism for the particles focusing in the perpendicular planes. It is due to the fact that the main planes of such a lens lie on the different sides from the back edge of the lens, whereas the focus distances F_{xz} and F_{xy} are identical. To eliminate the astigmatism in two-section magnetic lens the length of one section of the lens may be somewhat changed, so that L_{2xy} and L_{2xz} determined by the expression |20| would coincide. The lens which has less astigmatism is shown in Fig. 4.

The optical strength of such a lens for the particles focused in the plane xy is determined by the expression

$$\frac{1}{F_{xy}} = K^{\frac{1}{2}}\Big\{\sin 2\alpha\, ch 2\alpha - sh 2\alpha + cK^{\frac{1}{2}}(\sin 2\alpha\, sh 2\alpha - 2\sin^2\alpha\, ch 2\alpha) - c^2 K \sin^2\alpha\, sh 2\alpha\Big\} \qquad (21)$$

and the distance of the main plane from the back edge of the lens is equal to

$$X_{xy} = -F_{xy}\Big\{1 - \cos 2\alpha\, ch 2\alpha - cK^{\frac{1}{2}}(\cos 2\alpha\, sh 2\alpha - \sin 2\alpha\, ch 2\alpha) + c^2 K \sin 2\alpha\, sh 2\alpha\Big\} \qquad (22)$$

where $\alpha = K^{1/2}\ell$.

Expanding |21| and |22| in a series and restricting by the first terms we have

$$\frac{1}{F_{xy}} = 2K^2\ell^2\Big[\frac{2}{3}\ell + c\Big] \qquad (23)$$

$$X_{xy} = -(2\ell + c) \tag{24}$$

For the particles focusing in the plane ZX, the expressions for $\frac{1}{F_{zx}}$ and X_{zx} are obtained by the substitution of K for -K in |21|, |22|, |23| and |24|. Since "K" enters into the expression |23| quadratically, and the position of the main plane is generally independent of K, then in the first approximation such focusing system does not possess astigmatism. The main plane of the lens is situated exactly in the middle and L_1 and L_2 is measured from the middle of the lens up to the source of the charged particles and the point of their focusing in the calculations of the focusing distance. It should be noted that the three-section lens at the same values of the general length and the field gradient has, approximately, the optical strength two times less if compared with the two section lens.

By means of the obtained formulae 2 variants of the focusing lenses were calculated. Their parameters and purpose are given in the following Table:

The purpose of the lens	the form of the lens	a см	h см	L_1 м	L_2 м	K см$^{-2}$	F м	ℓ_1 см	ℓ_2 см	C см
Focus.polar. protons E = 600 MeV	2 section lens	1,5	3,25	4,2	5,7	$2{,}56.10^{-4}$	2,5	45	44	6
Focusing π -meson E = 300 MeV	3 section lens	1,5	3,25	2,3	5,4	$7{,}16.10^{-4}$	1,6	17	34	10

The general form of one section of the lens is given in Fig. 5. The chosen construction of the lenses makes it possible to place them in the electromagnet gap quite easily and quickly.

The whole arrangement was kept in the electromagnet gap by pressing one of the ends of the iron bars moving slightly in vertical to the electromagnet poles.

The curvature of the iron bars was the same as that of the trajectory of particle motion, whereas the front and back edges of the lens were made perpendicular to the direction of the beam. Separate sections of the lens were hold on the guide runners that makes it possible to change the optical strength of the lenses and to make the final tuning of all the system for the maximum increase of the intensity.

For the ~600 MeV polarized proton beam the intensity was increased three times in the place where the apparatus was set up and for mesons with the energy ~ 300 MeV -2.7 times.

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