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ELASTIC SCATTERING AND INTRINSIC STRUCTURE OF ELEMENTARY PARTICLES

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Abstract

Experimental data[1] on elastic scattering of π^--mesons on protons with energy E = 1,3 BeV have been analysed. It is shown that from the analysis of their angular distribution it is possible to determine the root-mean-square radius and to get the data about the distribution of matter inside the nucleon.

Root-mean-square "pion radius" is found to be equal to

$$\sqrt{\langle r^2 \rangle} = (0{,}82 \pm 0{,}06)10^{-13}\,cm.$$

In conclusion a possible experimental criterion of the existence of elementary length is considered.

I. INTRODUCTION

Particle structure is determined by studing the elastic scattering of some rays by this particle. When observing such scattering in particle ensemble we get the "mean optical image" of particle from which one may obtain the space-time picture of the intrinsic structure of particle with respect to the selected rays. We shorter waves, if we intend to obtain more detailed information about particle structure.

In § 4 we shall discuss the principle limits for the construction of such particle optical image and the limits of the applicability of the space structure of elementary particle.

Experiments on elastic scattering of fast electrons on a nucleon carried out by Hofstadter's group which, allowed to determine the form-factors of the electric charge and nucleon magnetic moment [2] [3] till present are the only example of the structure measurement of elementary particles. However, the study of the elastic scattering of fast particles of other kinds also gives the possibilities for obtaining the information about the nucleon and nucleus structure. This information contributes to the data obtained from the electron scattering experiments which, strictly speaking, in their turn give the information only about the distribution of the electric charges and currents inside the particles under investigation, i.e., about the "electric" particle structure.

Further, a detailed analysis of 1,3 BeV π^--meson scattering on protons is given [1]. This analysis, as it will be shown, makes it possible to get the information both about the "nuclear" structure of a nucleon and about its "nuclear" or "pion" radius.

II. PHASE-SHIFT ANALYSIS

As is known, the differential cross-section of the elastically scattered particles may be presented in the form:1/

$$\frac{d\sigma_{el}}{d\Omega} = \frac{\lambda^2}{4}\left|\sum_{\ell=0}^{\infty}(2\ell+1)(1-\beta_\ell)P_\ell(\cos\vartheta)\right|^2$$

Here usual notations are used, in particular, $\beta_\ell = e^{+2i\eta_\ell}$ where η_ℓ is the complex scattered wave phase.

On the basis of theoretical considerations [4] and from the direct comparison of calculations with experimental data [5]-[9] follow, that in the sufficiently high energy region $E > E^*$ ($E = 1$ BeV for π-mesons and 5 BeV for nucleons, the real part of the phase may be put to be equal to zero with the sufficient accuracy. In this case the quantity β_ℓ will be real, due to this fact the phase shift analysis is considerably simplified [5].

The values of the function

$$I(\ell) = 2\,\mathrm{Im}\,\eta_\ell = -\ln\beta_\ell \qquad (2)$$

is given in Fig. 1 in the form of a hystogramm.

1/ For simplicity we do not take into account spin dependence of the interaction and neglect the "charge-exchange" process. (compare [5]).

To calculate these values between the extreme experimental values of the differential cross-section of the diffraction scattering $d\sigma_d(\theta)/d\Omega$ from [2] the curves with the largest and the smallest curvatures were plotted. (C.f. fig. 3). The elastic scattering cross-section at zero angle in this case was normalized to the total cross-section $\sigma_t = \sigma_{el} + \sigma_{in} = 33,2 \pm 3\,mb$ according to the optical theorem. [10]

In accordance with so plotted curves two hystogramms are given in Fig. 1. Solid curves are drawn through the rectangular centres of the hystograms.

The cross-section of the nondiffraction elastic scattering $d\sigma_{nd}/d\Omega = d\sigma_{el}/d\Omega - d\sigma_d/d\Omega$ at the energy E = 1,3 BeV in ($\pi^- p$)- collisions is only some per cent of $d\sigma_d/d\Omega$ in angle region $\theta \lesssim 40^\circ$ and with good accuracy one may assume that the functions I(θ) in Fig. 1 concern only the purely diffraction scattering.

The error might appear due to big angles region, where $d\sigma_{nd}/d\Omega \gg d\sigma_d/d\Omega$. However, the approximation $d\sigma_d(\theta)/d\Omega$ in this region of rapidly falling function excludes the isotropic nondiffraction scattering[2] (see also §3).

In accordance with the values I(θ) (see Fig. 1) the cross-sections were calculated:

$$\sigma_t = \sigma_{in} + \sigma_d\,, \quad \sigma_{in} = \sum_{\theta=0}^{10} \sigma_{in}(\theta); \quad \sigma_d = \sum_{\theta=0}^{10} \sigma_d(\theta) \qquad (3)$$

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[2] The nondiffraction cross-section σ_{nd} is ~ 15% of σ_{in} due to great angles.

which are in good agreement with the experimental values8

$$\sigma_{in} = (24,6 - 29,0)mb\,; \quad \sigma_{in}^{exp} = 26\,mb$$

$$\sigma_{d} = (7,7 \div 7,9)mb\,; \quad \sigma_{d}^{exp} = 7,5 \pm 1,2\,mb$$

$$\sigma_{t} = (32,3 \div 36,9)mb\,; \quad \sigma_{t}^{exp} = 33,2 \pm 3\,mb.$$

The calculated values

$$\Delta_{in}(\ell) = \sigma_{in}(\ell)/\sigma_t \qquad \text{and} \qquad \Delta_d(\ell) = \sigma_d(\ell)/d_t \qquad \text{(in per cent)}$$

are given in Fig. 2.

It is seen from this figure that with $\ell > 6\text{-}7$ partial cross-sections rapidly decrease with the increase of ℓ.

The angular distribution of elastically scattered particles reconstructed in consistence with (1) by the first ten values are in good agreement with the initial curves. An insignificant contribution of terms with great values ℓ which were not taken into account is due to the fact that pion nucleon interaction is a short-range one.

III. QUASI-CLASSICAL APPROXIMATION AND PROTON STRUCTURE

At high energies of the scattering particles when the wave length λ becomes considerably smaller in comparison with the dimensions of the scattering system and the relative change of the absorption coefficient in the nuclear matter in the interval $\lambda \Delta K/K \ll 1$ the quasi-classical approximation is applicable with good accuracy. In our case $\lambda = 0,28 \cdot 10^{-13}$cm and some times less than the nucleon dimensions. Using the values $I(e)$ according to the well-known formulas [11] and assuming that the nucleon is purely absorbing and $\tau = \lambda\sqrt{e(\ell+1)} = \lambda e$ the cross-sections $\sigma_{in} = (25,5 \pm 1,5)mb$, $\sigma_d = (7,4 \pm 0,1)mb$ were calculated. The angular distribution of elastically scattered particles is represented, in Fig. 3 by the dotted lines calculated in accordance with [11]. Solid curves designate the extreme values of experimental angular distribution from [1] with the largest and the smallest curvatures.

Godd agreement of the calculated magnitudes with the corresponding ones calculated in the previous section and with their experimental values may be considered as one of justifications of the further application of the quasi-classical approximation. Using this approximation from the integral equation determining the imaginary part of the phase

$$I(\rho) = \int_0^{\sqrt{L^2-\rho^2}} K(\sqrt{\rho^2+s^2})\,ds\,, \quad \rho = \lambda\sqrt{\ell(\ell+1)} = \lambda e \tag{4}$$

(here $L = e_{max}$) we can calculate the pion absorption coefficient in nucleon as a function of the distance τ from the nucleon centre using the known values of $I(e)$. For this purpose we rewrite equation (4) in the form:

$$I(\rho) = \int_0^L K(\tau)\,Q(\tau,\rho)\,d\tau \tag{5}$$

where

$$Q(\tau,\rho) = \begin{cases} \tau/\sqrt{\tau^2-\rho^2} & \text{for} \quad \tau > \rho \\ 0 & \text{for} \quad \tau \le \rho \end{cases} \tag{6}$$

For the numerical solution (5) is suitable to present in such a form:

$$I_j = \sum_{i,j=1}^{n} K_i P_{ij} \tag{7}$$

where

$K_i = K(r_i)$, $P_{ij} = Q(r_i, \rho_j)\frac{L}{n}$, $I_j = I[\varrho(\rho_j)]$, $\rho_j = (j - 1/2)\frac{L}{n}$ - the mean point of the j -interval.

This linear equation system has "triangular form" in virtue of (6) and the solution may be easily found by the successive substitutions. The function K(r) thus calculated is represented in Figure 4. This function determines the "pion structure" of a proton averaged over the space interval $\Delta r \sim \lambda$.

For the root-mean-square "pion radius" of a proton

$$\langle r^2 \rangle = \int_0^L r^4 K(r)\,dr \Big/ \int_0^L r^2 K(r)\,dr \tag{8}$$

the following values were obtained

$$\sqrt{\langle r^2 \rangle} = (0{,}82 \pm 0{,}06)\cdot 10^{-13}\,cm.$$

that coincides with the "electromagnetic radius" of a proton obtained from Hofstadter's group experiments |2|,|3|. As is seen from Fig. 4, the absorption coefficient essentially increases in central region of the proton. However, the values in this region are determined not quite precisely, as they are dependent on the approximation of $d\sigma_d(\theta)/d\Omega$ in the large angle interval.

This inaccuracy in the separation of the diffraction scattering decreases rapidly with the energy increase, as the fraction of the nondiffraction elastic processes becomes negligible.

So with $E = 5\,Bev$ $\sigma_{nd} = 0{,}06\,\sigma_{in}$; with

$E = 7\,Bev$ $\sigma_{nd} = 0{,}014\,\sigma_{in}$.

(Calculation according to the statistical theory [17]).

If in the peripheral regions of the proton π-mesons are mainly present one may assume that

$$K(\tau) = K \, \rho(\tau) \tag{10}$$

where K is the energy dependent coefficient of the meson absorption by the peripheral ~~π-meson~~ field, and $\rho(\tau)$ is the mean density of the π-meson cloud near the point τ. Within the accuracy of the experimental data the analytic form of $\rho(\tau)$ may be approximated by different curves of type described in [2].

In the central regions of the nucleon K(r) is very likely to be determined by the other kinds of the particles (nucleons, hyperons, K-mesons) and the formula (9) is not applicable.

IV. A POSSIBLE EXPERIMENTAL CRITERION OF ELEMENTARY LENGTH EXISTENCE

During the last years the idea that there may really exist the limit of the applicability of the conventional space-time discription of the particle structure connected with the existence of a certain "elementary length" has been frequently suggested. This idea was expressed in different versions of the theory of "nonlocal field" or "nonlocal interaction", (see, e.g., [14]). Such theoretical schemes lead to the form-factors which weaken the interaction for the short waves. In this way one might hope to eliminate the divergences from the modern quantum theory arising just due to the ultrashort waves.

It is possible, however, that this point is the weakest one in the "nonlocal" theories 3/. G.V. Vataghin and E.Fermi were the first to notice in their statistical theory of multiple production [16] that at high energies the interaction become not weak, but, on the contrary, a strong one. The calculations show that the weak (in the sense of the generally accepted classification) interaction of Fermi type (ν, e, μ) also becomes strong at high energies (the cross section $> \lambda^2$) [17].

Now we should like to put a question: under what conditions from the purely empirical standpoint would it be possible to speak about the nonlocality? Evidently, these conditions will occur when it would become impossible to use the elastic scattering of particles as a mesons of studying their structure. Thus, the matter depends upon the assymptotic behaviour of the cross-sections at high energies.

If with $\lambda \to 0$ all the elastic scattering for a certain internal region R will tend to the diffraction scattering on a "black sphere" of the radius R the elastic scattering will give no information about the intrinsic structure of this region any longer and the maximum information will be limited by the data about the outer dimensions of the "sphere".

The scattering cross-section which is due to the process in this region will be equal to $\pi R^2 (\gg \pi \lambda^2)$ and equals the corresponding inelastic scattering.

3/ This circumstance was also noted by M.A. Markov. [15]

In this case instead of the discription of the space-time structure the problem about possible ways of particle transformation will become important.

The magnitude of R from this point of view is the same length scale which determines the real nonlocality, i.e., the limit of the applicability of space-time description of the particle structure.

It can be seen from the analysis of the pion scattering on the proton that there is a tend of the appearing of the "blackness" in the central nucleon region.

From the point of view given here the further study of the energetic dependence of elastic pion scattering may be of principal importance.

It can be said that the dimension of the nonlocality R must not be a universal length S_0 it may depend upon the kind of the interaction. The minimum scale of the space-time discription R determined by the dimensions of the "black sphere" may be introduced into the theory in a relativistic invariant way. Indeed, the scattered wave phase ζ_e is an invariant. We intend to consider it as a function of two invariants |19|:

$$D=\frac{\Gamma_\mu \Gamma^\mu}{\mathcal{P}_\mu \mathcal{P}^\mu} \qquad \text{and} \qquad F=P_\mu P^\mu+\frac{(P_\mu \mathcal{P}^\mu)^2}{\mathcal{P}^\mu \mathcal{P}_\mu} \tag{11}$$

Here $\mathcal{P}_\mu$ is the four-dimensional energy-memGentum vector of all the system on the whole, whereas P_μ is the same for the relative motion of the incident particle and the particle of the scatterer, and finally,

$$\Gamma_\mu = \varepsilon_{\mu\nu\alpha\beta} M_{\alpha\beta} P_\nu . \tag{12}$$

Here $\varepsilon_{\mu\nu\alpha\beta}$ is the fully antisymmetical unit tensor of the forth rank, $M_{\alpha\beta}$ is the momentum antisymmetrical tensor. Using there invariants the "black sphere" may be determined as follows:

$$\eta_e = 0 \text{ if } D/F > R^2 \quad \text{and} \quad \eta_e = +i\infty, \text{ if } D/F < R^2 \tag{13}$$

for $F \gg P_0^2$ where P_0 is a certain great momentum which points out that the opacity occurs.

The quantity D/F is the operator, therefore, the inequalities (13) are determined for its eigenvalues.

It can be easily seen that in the c.m.s. ($\vec{\mathcal{P}} = 0$ $D/F = \frac{\vec{M}^2}{\vec{P}_0^2} = \frac{\hbar^2 \ell(\ell+1)}{\vec{P}_0^2}$, where $\vec{M}$ is the three-dimensional angular momentum, while $\vec{P}_0$ is the three-dimensional momentum of the relative motion. D/F determines the collision parameter in a relativistic - invariant way.

Note in conclusion that in the perturbation theory there are known "propagation functions" which lead to the divergences in the region of great frequencies. These functions are constructed with the help of the plane waves which were used as a zero approximation.

Meanwhile at great frequencies of the field in the presence of particles the plane wawe will be quite a bad approximation due to the diffraction scattering. Instead of the plane wave one ought to take the expansion in a series which takes into account the

the sharp change of the wave field at great relative momenta of the interacting particles. This gives rise to the relativistic invariant cut-off form-factors. However, these form-factors are due not be the weakining of the interaction at great frequencies as it is assumed in the conventional nonlocal theories but, on the contrary, due to its strengthening. As a whole, phenomenologically, this factor takes into account the intensive inelastic processes of any origin which occur at high energies.

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F I G U R E S

Fig. 1. The histogram of the values of the function $I(\ell)=2\,Im\,\eta_\ell$ calculated for the extreme experimental values of the differential diffraction scattering cross-section. Solid curves are drawn through the centres of the histogram rectangles.

Fig. 2. Relative contributions of the partial absorption cross-sections $\Delta_{in}(\ell)=\sigma_{in}(\ell)/\sigma_t$ and of the partial diffraction cross-sections $\Delta_d(\ell)=\sigma_d(\ell)/\sigma_t$ (in per cent). The indices " + " and " - " distinguish respectively the curves drawn for the cases of the differential diffraction scattering cross-section with the largest and the smallest curvatures. (See Fig. 3).

Fig. 3. Solid curves show the extreme experimental values of the differential diffraction scattering cross-section (with the largest or the smallest curvatures). Dashed curves show the angular distribution of the diffraction scattering which was calculated according to the optical model formulae.|11| Two curves correspond to the two curves for $I(\ell)$ (see, Fig. 1).

Fig. 4. The absorption coefficient $K = K(r)$ as the function of a distance from the nucleon centre. Two curves correspond to the two extreme experimental values of the angular distribution of the diffraction scattering (with the largest or the smallest curvatures).

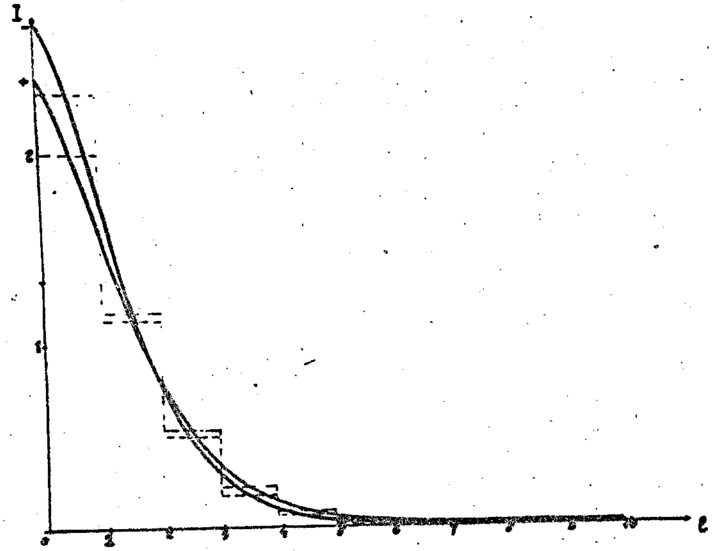

Fig 1

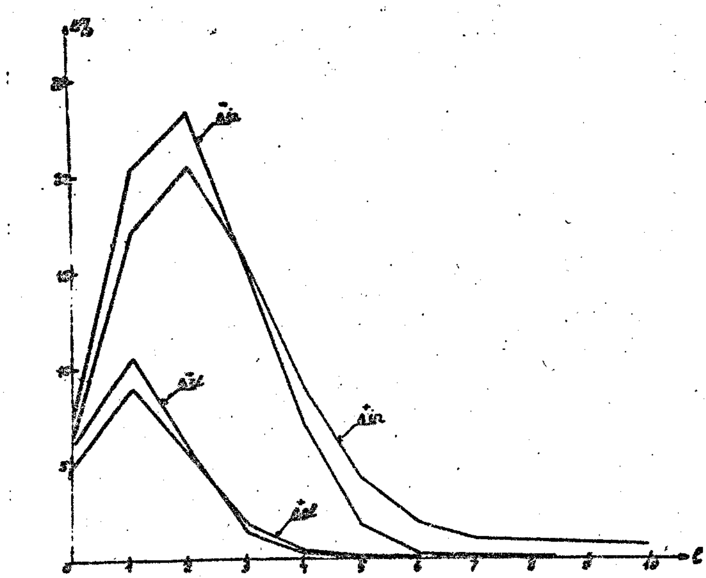

Fig 2

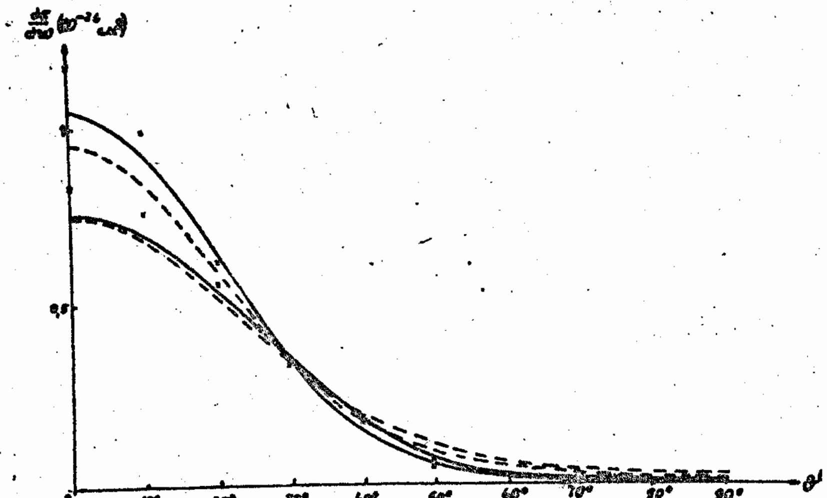

Fig 3

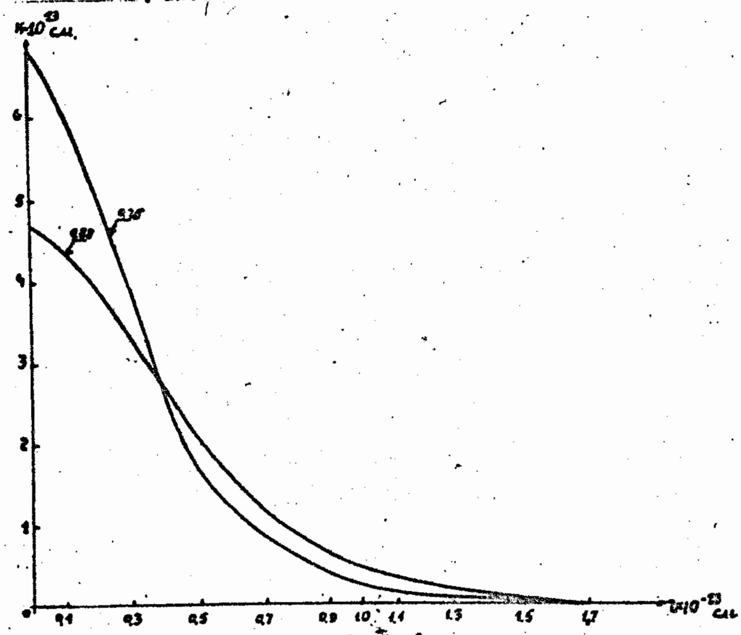

Fig 4