T H E J O I N T I N S T I T U T E F O R N U C L E A R R E S E A R C H

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O N A M E T H O D O F I N C R E A S I N G T H E D E N S I T Y O F A N E X T E R N A L P R O T O N B E A M F R O M T H E S I X-M E T E R S Y N C H R O C Y C L O T R O N

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ON A METHOD OF INCREASING THE DENSITY OF AN EXTERNAL PROTON BEAM FROM THE SIX-METER SYNCHROCYCLOTRON

Introduction

In this paper we describe a method of increasing the the density of the external proton beam from the six-meter synchrocyclotron.

This method comes to the creation on the way of an external beam, the magnetic field of definite configuration in the non-working region of the accelerator's electromagnet.

The calculation of the focusing effect of the magnetic field made on the basis of the analysis of particles motion along the beam trajectory.

The proposed method of focusing was realized at the six-meter synchrocyclotron of the Institute of Nuclear Problems in 1954*.

I. The motion of charged particles in azimuthal-symmetrical magnetic field

1. Horisontal plane

The equation for the motion of charged particles in azimuthal-symmetrical magnetic field in cylindrical coordinate system may be

* An analogous effect was observed at 184 "synchrocyclotron in Berkeley.

written in the following form

$$\frac{r^2+2r'^2-rr''}{(r^2+r'^2)^{3/2}}=\frac{eH_z(r)}{cp} \tag{1}$$

where $r'=\frac{dr}{d\varphi}$, p -momentum of the particle, $H_z(r)$-magnetic field strength.

The solution of the equation /1/ may be written in the form

$$\varphi=\int_{r_0}^{r}\frac{\lambda_0 r_0\rho_0 H_0+\int_{r_0}^{r}H_z(r)r\,dr}{r^2\sqrt{H_0^2\rho_0^2-(1/r^2)(\lambda_0 r_0\rho_0 H_0+\int_{r_0}^{r}H(r)r\,dr)^2}}\,dr, \tag{2}$$

where r_0 initial radius, $1/\rho_0$ trajectory curvature in the point r_0 , H_0-magnetic field strength in the point r_0 , r -variable radius. By inserting (2) into (1) we get $\lambda_0=1/\sqrt{1+[1/r_0(dr/d\varphi)_{r_0}]^2}$ sine of the angle between the initial radius and tangent to the trajectory.

On the bases of the expression (2) we shall determine the trajectory which further we shall call a mean one, initial conditions of λ_0, r_0 . The motion along the mean trajectory can be described by the functions

$$\varphi=\varphi(\lambda_0, r_0, r) \tag{3}$$

and

$$\mu=\mu(\lambda_0, r_0, r). \tag{4}$$

The functions (3) and (4) are connected by the ratio

$$tg\mu=r\,d\varphi/dr.$$

Let us consider the case of small deviations from the mean trajectory. By expanding the functions (3) and (4) in a series and limiting by the first terms of expanding if $\Delta\lambda_0 \ll 1$, $\Delta\tau_0/\tau_0 \ll 1$, $\Delta\tau/\tau \ll 1$ one can obtain the following equations, which connect initial and final deviations from the mean one ($\varphi = \text{const.}$).

These equations are the following

$$\frac{\partial\varphi}{\partial\tau}\Delta\tau = -\left(\frac{\partial\varphi}{\partial\tau_0}\Delta\tau_0 + \frac{\partial\varphi}{\partial\lambda_0}\Delta\lambda_0\right) \tag{5}$$

$$\Delta\mu = \frac{\partial\mu}{\partial\lambda_0}\Delta\lambda_0 + \frac{\partial\mu}{\partial\tau_0}\Delta\tau_0 + \frac{\partial\mu}{\partial\tau}\Delta\tau \;,$$

where $\dfrac{\partial\varphi}{\partial\tau} = -\dfrac{\partial\mu}{\partial\tau} = \dfrac{B}{\tau(\tau^2-B^2)^{1/2}}$

$$B = \lambda_0\tau_0 + \frac{1}{H_0\rho_0}\int_{\tau_0}^{\tau} H_z(\tau)\,\tau\,d\tau,$$

$$\partial\varphi/\partial\lambda_0 = \int_{\tau_0}^{\tau}\frac{\tau_0\,\tau\,d\tau}{(\tau^2-B^2)^{3/2}} \;,$$

$$\partial\varphi/\partial\tau_0 = \int_{\tau_0}^{\tau}\frac{(\lambda_0-\tau_0/\rho_0)\,d\tau}{(\tau^2-B^2)^{3/2}}, \tag{7}$$

$$\partial\mu/\partial\lambda_0 = \frac{\tau_0}{(\tau^2-B^2)^{1/2}}\,,\qquad \partial\mu/\partial\tau_0 = \left(\lambda_0 - \tau_0/\rho_0\right)\frac{1}{(\tau^2-B^2)^{1/2}}$$

$\Delta\tau_0, \Delta\lambda_0, \Delta\tau, \Delta\mu$ corresponding deviations ~~of~~ the parameters ~~from~~ of the mean trajectory. Thus, the change of the sizes of the beam in the horizontal plane can be described by the system of linear equations

$$\Delta\tau = a_1\Delta\tau_0 + a_2\Delta\lambda_0$$
$$\Delta\mu = a_3\Delta\tau_0 + a_4\Delta\lambda_0 \;, \tag{8}$$

where the coefficients a_1, a_2, a_3, a_4 are determined by the numerical integration from the expression (7).

2. Vertical plane

The motion of the charged particles in the vertical plane can be described by the differential equation

$$m\frac{d^2 z}{dt^2} = -\frac{e}{c}\lambda\vartheta\frac{\partial H_z}{\partial r}z, \qquad (9)$$

where ϑ -velocity of the particle, λ-sine of the angle between the radius and tangent to the trajectory. After substituting $\frac{d}{dt}$ for $\vartheta\frac{d}{dS}$ the equation (9) may be written as follows

$$\frac{d^2 z}{dS^2} + \alpha z = 0 \qquad (10)$$

where $\alpha = \frac{\partial H_z}{\partial r}\lambda\frac{1}{H_0\rho_0}$

Dividing the trajectory into parts, for which $\alpha \approx Const$, we can easily obtain the solution of the equation (10) in the form

$$\begin{pmatrix} z_K \\ z'_K \end{pmatrix} = \begin{pmatrix} \cos\sqrt{\alpha_1}S_1 & \sin\sqrt{\alpha_1}S_1/\sqrt{\alpha_1} \\ -\sqrt{\alpha_1}\sin\sqrt{\alpha_1}S_1 & \cos\sqrt{\alpha_1}S_1 \end{pmatrix} \times \dots$$

$$\times \begin{pmatrix} \cos\sqrt{\alpha_K}S_K & \sin\sqrt{\alpha_K}S_K/\sqrt{\alpha_K} \\ -\sqrt{\alpha_K}\sin\sqrt{\alpha_K}S_K & \cos\sqrt{\alpha_K}S_K \end{pmatrix} \begin{pmatrix} z_0 \\ z'_0 \end{pmatrix}, \qquad (11)$$

where $z_0, z'_0 = (dz/dS)_0$-initial conditions at the input; z_K, z'_K-initial conditions for (K + 1) section of the trajectory.

The equation (11) allow to determine the geometrical sizes of the beam in the vertical plane along the trajectory.

II. Motion of the charged particles in azimuthal-symmetrical magnetic field in the presence of perturbation

We can write the differential equation, describing the trajectory of the charged particle in the horisontal plane as follows

$$\frac{\tau^2+2\tau'^2-\tau\tau''}{(\tau^2+\tau'^2)^{3/2}}=\frac{eH(\tau,\varphi)}{cP} \tag{12}$$

where $\tau'=d\tau/d\varphi$, $H(\tau,\varphi)$ -magnetic field strenth in the presence of perturbation. To make the analysis of motion more convenient let us determine the mean trajectory as one the perturbating magnetic field along which is equal to zero.

Applying the coordinate system connected with the mean trajectory we can write the equation (12) in the form

$$\frac{1-\left(\frac{d\tau}{dS_0}\frac{dS_0}{dS}\right)^2-\tau\left[\frac{d^2\tau}{dS_0^2}\left(\frac{dS_0}{dS}\right)^2+\frac{dS_0}{dS}\frac{d\tau}{dS}\frac{d}{dS_0}\left(\frac{dS_0}{dS}\right)\right]}{\tau\sqrt{1-\left(\frac{d\tau}{dS_0}\frac{dS_0}{dS}\right)^2}}=\frac{eH_z(\tau,S)}{cP} \tag{13}$$

where the parameters S_0, τ_0 determine the mean trajectory and S, τ the considered one. By expanding the magnetic field in a range along the mean trajectory and limiting by the first terms of expanding we obtain

$$H_z(\tau,S)=H_z(\tau_0,S_0)+\left(\frac{\partial H_z}{\partial\tau}\right)_{S_0}\rho+\left(\frac{\partial H_z}{\partial S}\right)_{S_0}\Delta S, \tag{14}$$

where $\rho=\tau-\tau_0$, $\Delta S=S-S_0$.

Further we will consider small deviations from the mean trajectory in the azimuthal dimension of the perturbaion region $\varphi < 1$ and $_0(\partial H_z/\partial r)_{S_0} \gg (dH_z/dS)_{S_0}$.

Distingushing the curvature of the mean trajectory from the equation (12) and neglecting the terms of the order $\left(\frac{\rho}{r_0}\right)^2$ and $\left(\frac{d\rho}{dS_0}\right)^2$ we can obtain differential equation for motion of charged particles in the horisontal plane for the cases of small deviation from the maan trajectory.

$$\frac{d^2\rho}{dS_0^2} + \beta\frac{d\rho}{dS_0} + \rho(\alpha+\gamma) = 0,$$

where $\beta = -2\frac{dr_0}{dS_0}\frac{d^2 r_0}{dS_0^2} \Big/ \left[1-\left(\frac{dr_0}{dS_0}\right)^2\right]$; (15)

$$\alpha = \frac{e}{cP}\left(\frac{\partial H_z}{\partial r}\right)_{S_0} \Big/ \sqrt{1-\left(\frac{dr_0}{dS_0}\right)^2}$$

$$\gamma = \frac{\frac{1}{r_0}\left[1-\left(\frac{dr_0}{dS_0}\right)^2\right] - 2\frac{d^2 r_0}{dS_0^2} + 3\left(\frac{dr_0}{dS_0}\right)^2\frac{d^2 r_0}{dS_0^2}}{r_0\left[1-(dr_0/dS_0)^2\right]}$$

P -momentum of the particle. If the coefficients satisfy the inequality $\beta_{S max} \ll 1$ and $\gamma \ll \alpha$, the equation (15) can be written in the form

$$\frac{d^2\rho}{dS_0^2} + \alpha\rho = 0. \qquad (16)$$

The motion of particles int the vertical plane may be given by the analogous differential equation

$$\frac{d^2 Z}{dS_0^2} - \alpha Z = 0. \qquad (17)$$

The obtained equations (16) and (17) allow to calculate the geometrical sizes of the beam in the magnetic field perturbation zone.

III. Calculation of focusing effect

The estimation of increasing the average density was performed by comparing the cross-sections of the external beam in the absence of presence of the focusing magnetic field. The determination of the cross-sections of the beam and the choice of the parameters of the magnetic field perturbation for the six-meter synchrocyclotron was performed by transforming the areas of initial conditions inte horizontal and vertical planes along the trajectory of the beam. The areas of initial conditions at the input to the perturbation zone were obtained from the conditions of the maximum geometrical opacity of the magnetic channel.

Fig. 1 and 2 show area change of initial conditions along the trajectory of the external beam.

The calculations made showed that the optimum of focusing effect on the external proton beam with the energy 680 MeV from the six-meter synchrocyclotron chamber can be obtained when changing the magnetic field strength 750-1000 Oe/cm in the perturbation zone with the dimension of 45 cm.

As a result of confronting the calculated cross-sections of the beam it has been found that the external beam density at the distance of 12 cm from the outlet and of the focusing arrangement can be increased three times.

IV. Magnetic field in the zone of focusing arrangement

The focusing field was created in the non-working region of the accelerator's magnetic field with the help of iron masses of certain configuration. Fig.3 shows the iron-masses cross sections of the focusing arrangement. The plates (1) and (2) procide approximately constant field gradient in the non-working region of the focusing arrangement.

In order to obtain the necessary range of derivative magnetic field density in the perturbation area the possibility was provided for changing the distance between the plates of the focusing arrangement without the violation of the vacuum in the accelerator's chamber.

Fig. 3 shows also the calculating and experimental values of the magnetic field strength in the perturbation zone with different positions of the focusing arrangement plates.

The calculation of the magnetic field strength was based on the assumption of iron masses axial magnitizing up to saturation[3/]. The measurements of the magnetic field strength inside the focusing arrangement were made with the help of a fluxmeter.

The correction of the magnetic field perturbation in the zone of the last orbits caused by the plates of the focusing arrangement was realized with the help of shims (the plates 3, Fig.3). The measurements of the magnetic field strength in this region was made with the accuracy of 01% by the divice, which employed the phenomenon of nuclear resonance.

The general scheme of accelerating external proton beam from the six-meter synchrocyclotron chamber is shown in Fig.4.

C o n c l u s i o n

The experiments made at the six-meter synchrocyclotron showed that the mentioned focusing arrangement allows to increase the proton beam density 2,8-3 times. The measurements of the proton beam density were made at the distance of 12 meters from the outlet end of the focusing arrangement according to the reaction. $C_{12}^{6}(p, p, n)C_{11}^{6}$.

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