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ON A POSSIBLE LIMIT OF THE APPLICABILITY OF QUANTUM ELECTRODYNAMICS

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In the present note the processes competing with the electromagnetic at high energies are considered. It is shown that such processes may be those involving the four-fermion interactions.

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I. Introduction

It was shown [1] that the application of the modern renormalization method in the quantum electrodynamics leads to the principle difficulty, i.e., to the vanishing of the renormalized charge. Although the absolute proof of this conclusion was argued, [2] none the less, the presence of the principle difficulties in the energy region E determined by the condition $\alpha \ln \frac{E}{mc^2} \sim 1 (\alpha = e^2/\hbar c)$ appears to be rather convincing. The space scale $l_0 \sim \frac{\hbar}{mc} e^{-3\pi/\alpha}$ corresponding to this energy is far beyond the limits of the gravitational radius of the electron as it was first pointed out in. [3] The extreme energy itself is enormously high $/E_0 \sim \sim mc^2 \cdot e^{3\pi/\alpha}/$

It can be expected, therefore, that the limits of the applicability of the modern electrodynamics will be revealed much earlier, e.g., due to a possible change of space-time structure in space-time regions which are small but still considerably greater than l_0.

There is, however, another possibility of limiting the significance of the quantum electrodynamics which is more accessible for the theoretical analysis.

Together with the purely electrodynamic interactions of photons, electrons and positrons there occur the processes involving mesons and nucleons. They may be induced in a purely electrodynamic way, e.g., by photon interaction with an electron.

If it turned out that the contribution of these non-electromagnetic processes exceeds that of electromagnetic then it would be not possible to consider the pure electrodynamics without essential involving other types of interactions. In particular, from a certain energy $E_{\kappa p}$ the expansion in a series $e^2/\hbar c$ would become not reasonable.

We will show that such a competing interaction may be the weak four-fermion Fermi interaction.

The validity of this interaction in high energy region is not experimentally checked and different theoretical doubts on the applicability of this interaction for the energies $E \gg mc^2$ may arise.

However, we shall start from the assumption about the applicability of this interaction up to very high energies and consider the conclusions resulting from this assumption.

The physical property of the purely fermion interactions is that the matrix elements of these interactions do not decrease with the increase of the energy of the fermions involved in the process while the matrix elements of the processes involving the bosons (photons, π- and K -mesons) decrease with the boson energy increase. It can be accounted for the fact that with the boson energy increase the boson field falls as follows $K^{-1/2}$

$$(\phi_K = \sqrt{\frac{\hbar}{2K}}\, e^{iKx} b_K^+ + conj.$$

Here K is the boson momentum, b_K^+ is the operator of boson production), and the fermion field with the fermion energy increase remains constant ($\Psi_K \sim u_K e^{iKx} a_K^+ + conj.$ where: u_K is the spinor amplitude, a_K^+ is the operator of the fermion production).

We shall show further that due to this property the fermion interactions become essential in the electromagnetic processes much earlier than the logarithmic limit $E \sim mc^2 e^{3\pi/\alpha}$ is reached.

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II. Fermion-Electromagnetic Interaction

Let us consider the photon interaction process |K| with the electron (e) leading to the production of μ meson (μ) and two neutrinos | ν , $\tilde{\nu}$ |:

$$K + e \longrightarrow \mu + \nu + \tilde{\nu}. \tag{1}$$

Such process will be described by the interaction Lagrangian

$$W = eW_e + eW_\mu + gW_{e\mu\nu} \tag{2}$$

where $eW_e = (\mathcal{J}_e A)$ is the electron interaction ($\mathcal{J}_e$ is the electron current) with the electromagnetic field (A is the vector potential), eW_μ has the same meaning for a meson. Finally, $gW_{e\mu\nu}$ is the four-fermion interaction of an electron, μ -meson and a neutrino; $g = \hbar c \Lambda_o^2 \cong 10^{-49}$ erg. cm^3 is the Fermi constant $\Lambda_o = 6.10^{-17}$ cm.^{4} whereas $W_{e\mu\nu} = (\tilde{\Psi}_e O_1 \tilde{\Psi}_e)(\tilde{\Psi}_\nu O_2 \Psi_\nu)$ + conj. Here $\Psi_e, \Psi_\mu, \Psi_\nu$ are the spinor fields of electrons, μ -meson and a neutrino respectively; O_1 and O_2 are certain spinor operators.

The total effective cross section for process (1) is:

$$\sigma_\mu = \frac{2\pi}{\hbar c} \int |W_{af}|^2 \frac{P_\nu^2 \, dP_\nu \, d\Omega_\nu \tilde{P} \, \tilde{P}_\nu'^2 \, d\tilde{P}_\nu' \, d\tilde{\Omega}_\nu'}{(2\pi\hbar)^6 \, dE_f} \tag{3}$$

where W_{af} is the matrix element from the interaction energy (2) for process (1), P_ν, $\tilde{P}_\nu$ are neutrino and antineutrino momenta, E_f is the finite state energy.

The structure of this matrix element W_{af} is such that in the first nonvanishing approximation it is equal to

$$W_{af} = eg \sum_i \left\{ \frac{(a|W_e|c)(c|W_{e\mu\nu}|f)}{E_0 - E_c} + \frac{(a|W_{e\mu\tilde{\nu}}|c)(c|W_\mu|f)}{E_0 - E_c'} \right. \qquad (4)$$

where E_0 is the initial state energy, and E_c is the intermediate state energy. In the system of center of gravity of a photon and electron $E_0 - E_c \sim \hbar ck$ (K is the photon wave vector), $(a|W_e|c) \sim K^{-1/2}$, $(c|W_\mu|f) \sim K^{1/2}$. Therefore, $|W_{af}|^2 \sim egK^{-3}$. The weight factor in (3) is proportional to K^5. Thus, the total cross section is as follows

$$\sigma_\mu \cong \alpha \Lambda_0^4 K^2 \cdot F \qquad (5)$$

where F is the factor of the order 1, which weakly depends on K*.

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* These qualitative conclusions are supported by more detailed calculations made by Dr. M. Mayer (Rumania). The author is very grateful to him.

Just in a similar manner one may consider the collision of two electrons with their simultaneous conversion into two mesons, in accordance with the scheme

$$e' + e'' \longrightarrow \mu' + \mu''. \qquad (1)$$

The differential cross section (in c.m.s.) for this process will be

$$d\sigma_{\mu\mu} \cong \Lambda_0^8 q^4 P^2 F \cdot d\Omega \qquad (6)$$

where q is the momentum transfer and P is the initial electron momentum both measured in reciprocal lengths.

On the other hand, the cross sections of purely electromagnetic processes are equal to

$$\sigma_c = \frac{1}{2}\pi\alpha^2\frac{1}{K^2}\left(\ln\frac{4K^2}{K_c^2} + 1/2\right) \qquad (7)$$

for the Compton effect,

$$d\sigma_{ee} = \alpha^2\frac{P^2}{q^4}\,d\Omega \qquad (8)$$

for electron elastic collision,

$$\sigma_p = \frac{28}{9}\alpha^3\frac{1}{K^2}\left(\ln\frac{4K^2}{K_c^2} - 3,5\right) \qquad (9)$$

for pair production/here $K = \frac{mc}{\hbar}$, $\alpha = \frac{e^2}{\hbar c}$ /

and

$$\sigma_\gamma = 4\alpha^3\frac{1}{K_c^2}\left|\ln\frac{4}{K_c^2} - 3,5\right| \qquad (10)$$

for the bremsstrahlung in the electron collision.

The comparison of these cross sections with those of mixed processes (1) and (1^1) shows that

$$\sigma_\mu > \sigma_c \qquad \text{with} \qquad K \gtrsim \alpha^{1/2}\frac{1}{\Lambda_0} \qquad (11)$$

$$\sigma_\mu > \sigma_p \qquad \text{with} \qquad K \gtrsim \alpha^{1/2}\left(\frac{\alpha}{\Lambda_0 K_c}\right)\frac{1}{\Lambda_0} \qquad (12)$$

$$d\sigma_{\mu\mu} > d\sigma_{ee} \qquad \text{with} \qquad q \gtrsim \alpha^{1/4}\frac{1}{\Lambda_0} \qquad (13)$$

$$\sigma_{\mu\mu} > \sigma_\gamma \qquad \text{with} \qquad q \sim p \gg \left(\frac{\alpha^3}{K^2\Lambda_0^2}\right)\frac{1}{\Lambda_0} \qquad (14)$$

{Here the factors ~1} are omitted *.

As is seen from these inequalities that if four-fermion interactions may be considered applicable in the energy range $K > 1/\Lambda_0$, the processes with neutrinos and μ - meson production are more intensive than the purely electromagnetic processes. The corresponding photon and electron energy in the system of center of gravity should be more than $\frac{\hbar c}{\Lambda_0} \sim 2$ BeV.

This is great energy but, nevertheless, it is much lower than the logarithmic one.

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* Note that pair production in this energy range is the main electromagnetic process. The Compton scattering in this case is minor in comparison with the diffractional one induced by pair production.

It should be noted that the production of nucleon and meson pairs will play a considerably smaller role since their production cross section will be $\left(\frac{m}{M}\right)^2$ times less than that of electron-positron pairs.

The processes involving the production of neutrinos and boson mesons will be essential later due to the above-mentioned difference in the behaviour of the boson and fermion matrix elements.

Thus, the fermion interaction may be the one which restricts the region of the applicability of electrodynamics by the scales $> \Lambda_o$ For smaller scales and, consequently, for the energies above $\frac{\hbar c}{\Lambda_o}$. it is not reasonable at all to study electrodynamics without considering the processes involving both μ -mesons and neutrinos and the Fermi constant g, together with $e^2/\hbar c$.

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R e f e r e n c e s

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2. N.N. Bogoliubov, D.V. Shirkov "Introduction to Quantum Field Theory" Moscow (1957), pp. 355-356.

3. M.A. Markov JETP, 17, 661 (1957).

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