JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Theoretical

Physics



P - 125

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# APPROXIMATE EQUATIONS FOR PHOTON SCATTERING AMPLITUDE ON NUCLEONS

ДАН, 1958, т119, №4, с. 690-693.



1958

The study of the photon scattering process on the nucleons may present important data about the meson structure of the nucleon. In the present paper the approximate equations for the physical amplitudes have been obtained using the dispersion relations for the sompton scattering[1].

1. The kinematic investigation of the amplitude. Let us denote the initial nucleon and photon momenta through P and K, respectively the finite ones through P' and K'. Taking into account the laws of energy - momentum conservation P + K = P' + K', one may constrnct two independent scalar products $V$ and $V_1$ of the vectors P and K, P' and K*.

$$V = (p + p')\cdot k \quad , \quad V_1 = k \cdot k' \tag{1}$$

From the relativistic invariance the amplitude of the process may be presented as follows:

$$\hat{R} = \sum_{i} \sum_{\nu,\mu=0}^{3} \Omega_i(V, V_1)\bar{u}(p')\hat{R}^{i}_{\mu\nu} u(p) e'_{\mu} e_{\nu} \tag{2}$$

where $e_{\nu}, e'_{\mu}$ - are the polarization vectors of the initial and finite photon, $\bar{u}(p'), u(p)$ are the spinors which characterize the nucleon in the finite and initial state, $\Omega_i(V, V_1)$ are invariant functions having only the isotopic structure, $\hat{R}^{i}_{\mu\nu}$ - are the operators containing the spin structure of the amplitude of the process. The summation is being performed over all the independent structures $\hat{R}^{i}_{\mu\nu}$.

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* In the system $\vec{p} + \vec{p}' = 0$ the photon energy $E = \frac{V}{2P_0}$ the recoil momentum $\vec{p}^2 = \frac{V_1}{2}$

From the relativistic and gradient invariance conditions one may find the number of independent structures $\hat{R}^{i}_{\mu\nu}$ and obtain their explicit expression [2].

Let us present the matrix element of the process in the following form:*

$$M = R \cdot X \tag{3}$$

$X$ - contains the production and annihilation operators of the particles and possible nonquantized external fields as well. R is composed of the four momenta of the participating particles and $\gamma$ -matrices and, in its turn, may be presented as follows:

$$R = \sum_{e,m} C_{em} (\Lambda_e T_m) \tag{4}$$

Here $\Lambda_1 = 1$, $\Lambda_{e\neq 1} = (\gamma \cdot P_1), (\gamma \cdot P_2) \ldots$ $(P_1, P_2, \ldots$ - four momenta of the participating particles/. $\Lambda_{e\neq 1} = 0$, if $\gamma$ - matrices do not einter into R. T are composed as all possible independent combinations of $\gamma$ - matrices and 4-momenta, open by the summation indices: $\gamma_\alpha \gamma_\beta \cdots \gamma_\nu, \gamma_\beta \gamma_\alpha \cdots \gamma_\nu, \gamma_\alpha \gamma_\beta \cdots P_{1\nu}, \ldots$ The summation in (4) is being performed over all possible combinations ( $\Lambda_e T_m$ ). When constructing $\Lambda_e$ and $T_m$ it is necessary to take into account the laws of conservation and equation of the motion of the participating particles. For the processes where the electromagnetic field takes part (3) may be rewritten in the form:

$$M = \sum_{\mu} R_\mu A_\mu X', \quad X = A_\mu X' \tag{5}$$

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* Here we follow M. Kawaguchi and N. Mugibayashi [2].

From the gradient invariance requirement

$$\sum_{\mu} iK_\mu R_\mu = 0 \tag{6}$$

Hence $M$ may be written as follows

$$M = \sum_{\mu} R_\mu A_\mu(K)\chi' = \sum_{\mu,\alpha} \left(\delta_{\mu\alpha} + \frac{if_\mu}{(f\cdot K)} iK_\alpha\right) R_\alpha A_\mu(K)\chi'. \tag{7}$$

Here $\frac{if_\mu}{(f\cdot K)}$ is an insingular function arbitrarily chosen. If $n$ -operators of the electromagnetic field enter into $M$

$$M = \sum_{\mu,\nu,\ldots\omega} R_{\mu\nu\ldots\omega}(K_1\ldots K_n) A_\mu(K_1) A_\nu(K_2)\ldots A_\omega(K_n)\chi^{(n)} \tag{8}$$

then the gradient invariance

$$iK_\mu R_{\mu\nu\ldots\omega} = iK_\nu R_{\mu\nu\ldots\omega} = \ldots = iK_\omega R_{\mu\nu\ldots\omega} = 0 \tag{9}$$

gives the following expression for $M$ if take into account (7)

$$M = \sum_{\mu,\nu\ldots\omega} \sum_{\alpha,\beta\ldots\gamma} G^{1}_{\mu\alpha}(K_1) G^{2}_{\nu\beta}(K_2)\ldots G^{n}_{\omega\gamma}(K_n) R_{\alpha\beta\ldots\gamma}(K_1\ldots K_n) \times$$

$$\times A_\mu(K_1) A_\nu(K_2)\ldots A_\omega(K_n)\chi^{(n)} \tag{10}$$

where

$$G^{j}_{\rho\xi}(K_j) = \left(\delta_{\rho\xi} + \frac{if_{j\rho}}{(f_j\cdot K_j)} iK_{j\xi}\right) \tag{11}$$

the particle. $j$ is the number of Thus, the gradient invariance considerations make it possible to present R in (4) in the form of R'

$$R' = G^1 G^2 \ldots G^n R = \sum_{e,m} C_{em} (\Lambda_e G^1 G^2 \ldots G^n T_m \tag{12}$$

(It is clear that $G^{j} = G_{\rho\xi}(K_j)$ affects by the indices $\rho, \xi$ ). Therefore, if (4) was expanded into $N(\Lambda)\, N(T)$ independent forms $(\Lambda_e T_m)$, $N(\Lambda)$ is the number of the possible $\Lambda_e$, $N(T)$ is the number of the possible $T_m$ , then (12) will be expanded in $N(\Lambda)N(GT)$ independent forms. It is clear that $N(T) > N(GT)$ since the arbitrary functions enter into $^{if_j\mu}/(f_j\, k_j)$ Se (11).

Thus, R may be expanded in $N(\Lambda)N(GT)$ independent invariant and gradient-invariant forms, the number of which is determined by the initial and finite states of the process and is independent either of the intermediate states or of the form of the interaction.

Let us apply these considerations to the Compton effect. If one takes the matrix element of the Compton effect in the form

$$M = \sum \bar{U}(p') A_\mu(K') R_{\mu\nu}(K, K', p, p') A_\nu(K) U(p) \tag{13}$$

then

$$R_{\mu\nu} = \sum_{\alpha,\beta} G_{\mu\alpha}(K') G_{\nu\beta}(K) R_{\alpha\beta} = \sum C_{em} (\Lambda_e G_{\mu\alpha} G_{\nu\beta} T_{\alpha\beta m}) \tag{14}$$

a) $\Lambda_1 = 1$, $\Lambda_2 = i\gamma\cdot K$, $\gamma^+ = \gamma$, $N(\Lambda) = 2$

$(\gamma\cdot p)$, $(\gamma\cdot p')$ - are excluded by the equations of the motion, $(\gamma\cdot K')$ is excluded by the law of conservation;

b) one may construct the following independent forms of

$$\gamma_\alpha\gamma_\beta,\ \gamma_\beta\gamma_\alpha,\ \gamma_\alpha K_\beta :\ K'_\alpha\gamma_\beta,\ K_\alpha K'_\beta,\ K_\alpha P_\beta,\ K'_\beta P_\alpha,\ \gamma_\beta P_\alpha$$
$$N(T) = 8 \tag{15}$$

( $p'$ comes out due to the law of conservation)

It can be seen from here that

$$N(\Lambda) \times N(T) = 2 \times 8 = 16 .$$

Let us take into account the gradient invariance $M$ : choosing $f'_\mu = P_\mu$, $f_\nu = P_\nu$ and making use of (11), (13), (14), (15) one may just obtain that $\gamma_\beta P_\alpha$, $K_\alpha P_\beta$, $K'_\beta P_\alpha$ drop out, i.e. $N/GT/=5$

Thus, the gradient invariance considerations reduce the number of independent forms to ten 10: $N(\Lambda)\times N(GT) = 2\times 5 = 10$.

If one takes into account the amplitude invariance with respect to the time reflection this number is reduced to ~~xis~~ six. As six independent structures we choose the following expressions:

$$\hat{R}_1 = \frac{1}{p\cdot K\, p\cdot K'}\left\{e\cdot e'\, p\cdot K\, p\cdot K' + (e\cdot p'\, e'\cdot p\, p\cdot K - e'\cdot p'\, e\cdot p\, p'\cdot K)\right\}$$

$$\hat{R}_e = \frac{(K\cdot K')^{1/2}}{p\cdot K\, p\cdot K'}\left\{\hat{e}'(e\cdot p'\, p\cdot K - e\cdot p\, p'\cdot K) + \hat{e}(e'\cdot p\, p'\cdot K' - e'\cdot p'\, p\cdot K')\right\}$$

$$\hat{R}_3 = \frac{1}{(K\cdot K')^{1/2}}\left\{\hat{e}'(\hat{K}+\hat{K}')\hat{e} - \hat{e}(\hat{K}+\hat{K}')\hat{e}'\right\}$$

$$\hat{R}_4 = \frac{1}{p\cdot K\, p\cdot K'}\left\{\hat{e}'\hat{K}'(e\cdot p'\, p\, K - e\, p\, p'\cdot K) + \hat{e}\hat{K}(e'\cdot p'\, p\, K' - e'\, p\, p'\, K')\right\}$$

$$\hat{R}_5 = \frac{(K\cdot K')^{1/2}}{p\cdot K\, p\cdot K'}\left\{\hat{e}'(\hat{K}+\hat{K}')\hat{e}\, p\cdot K' + \hat{e}(\hat{K}+\hat{K}')\hat{e}'\, p\cdot K\right\} - 2\hat{R}_2\,.$$

$$\hat{R}_6 = \frac{(k\,k')^{-1/2}}{p\,k\;p\,k'}\Big\{2(\hat{k}+\hat{k}')(e\,p'\,e'\,p\,p\,k + e'\,p'\,e\,p\,p\,k') -$$

$$-2p\,k\,p\,k'\left[\hat{e}'(e\,p'+e\,p)+\hat{e}(e'\,p+e'\,p')\right]\Big\}+\hat{R}_3$$

$$a\,b = a_0 b_0 - \vec{a}\,\vec{b}\,, \qquad \hat{a} = a_0\gamma_0 - \vec{a}\,\vec{\gamma} \tag{16}$$

$$\hat{R}_i = \sum_{\mu,\nu} e'_\mu\, R^{\mu i}_{\mu\nu}\, e_\nu\,. \tag{17}$$

Let us establish some properties of the function symmetry. With the help of the S matrix formalism, the matirx element of the Compton scattering may be written as follows: |1|

a) $$\langle\gamma'|S|\gamma\rangle = \frac{i}{\sqrt{4k_0k_0'}}\int e^{i(k'x-ky)}\langle\Phi_{p'}|\frac{\delta j_\nu(y)}{\delta A_\mu(x)}|\Phi_p\rangle\,dx\,dy \tag{18}$$

b) $$\langle\gamma'|S|\gamma\rangle = i\,\frac{(2\pi)^4}{\sqrt{4k_0k_0'}}\,\delta(p+k-p'-k')R$$

where $A_\mu(x)$ -is the operator of the electromagnetic field, and $a/\Phi_p\rangle$ is the vector of the nucleon state,

$$j_\nu(y) = i\,\frac{\delta S}{\delta A_\nu(y)}\,S^+ \tag{19}$$

One may establish from (18) that

$$R(k,k',p,p') = R^*(-k,-k',p',p)\,. \tag{20}$$

In fact:

$$\left[\frac{i}{\sqrt{4K_0K_0'}}\int e^{i(k'x-ky)}\langle\Phi_{p'}\left|\frac{\delta j_\nu(y)}{\delta A_\mu(x)}\right|\Phi_p\rangle dx\,dy\right]^* =$$

$$= -\frac{i}{\sqrt{4K_0K_0'}}\int e^{-i(k'x-ky)}\langle\Phi_p\left|\frac{\delta j_\nu^+(y)}{\delta A_\mu(x)}\right|\Phi_{p'}\rangle dx\,dy$$

$$j_\nu^+(y) = -iS\frac{\delta S^+}{\delta A_\nu(y)}$$

But since $SS^+ = 1$, then

$$\frac{\delta S}{\delta A_\mu}S^+ + S\frac{\delta S^+}{\delta A_\mu} = 0 \qquad \text{and} \qquad j_\nu^+(y) = j_\nu(y)$$

~~(Therefore)~~

It can be seen from here that

$$\left[\langle\gamma'|S|\gamma\rangle\right]^*_{p\rightleftarrows p',\,k\to -k,\,k'\to -k'} = -\langle\gamma'|S|\gamma\rangle$$

Comparing with (18b) one obtains (20).

Substituting (19) into (2) and taking into account (20) we obtain the following important property $\Omega_i(\nu,\nu_1)$

~~or~~

$$\Omega_i(\nu,\nu_1) = \Omega_i^*(-\nu,\nu_1)$$

or

$$\operatorname{Re}\Omega_i(\nu,\nu_1) = \operatorname{Re}\Omega_i(-\nu,\nu_1)$$

$$\operatorname{Im}\Omega_i(\nu,\nu_1) = -\operatorname{Im}\Omega_i(-\nu,\nu_1) \qquad (21)$$

The isotopic dependence $\Omega_i$ is evident:

$$\Omega_i = \Omega_i' + \Omega_i^e\tau_3 = \Omega_i^p\frac{1+\tau_3}{2} + \Omega_i^n\frac{1-\tau_3}{2} \qquad (22)$$

where $\Omega_i^p$ describes the scattering on the proton, $\Omega_i^n$ that on the neutron.

## 2. Dispersion Relations for Relativistic Amplitudes

Making use of the analytic property of the functions $\Omega$ in the upper half-plane of the variable $\nu$ [3] we have

$$\mathrm{Re}\,\Omega_i(\nu,\nu_1)=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\mathrm{Im}\,\Omega_i(\nu,\nu_1)}{\nu'-\nu}\,d\nu' \tag{23}$$

The region of the negative $\nu$ in (23) may be excluded using (21). The method of the exclusion of the region $0<\nu<2M\mu+\mu^2-\nu_1$ for the amplitude R is given in [1]. (See, also [4],[5]). In this region the Hermitian part of the amplitude R is written as follows

$$D=\sum_i \bar{u}(p')\hat{R}_i\,u(p)\overset{o}{\Omega}_i=-\bar{u}(p')\Big\{4(\hat{\mu}^2M+\hat{\mu}e\tau_p)\hat{R}_1+$$

$$+(2\hat{\mu}^2M+\hat{\mu}e\tau_p)R_4+\frac{1}{4\nu_1^{1/2}}(2\mu M+e\tau_p)^2\hat{R}_5+\frac{\nu_1^{1/2}}{2}\hat{\mu}^2\hat{R}_6\Big\}u(p)$$

$$\hat{\mu}=\mu_p'\tau_p+\mu_n\tau_n\,,\quad \tau_p=\frac{1+\tau_3}{2}\,,\quad \tau_n=\frac{1-\tau_3}{2} \tag{24"}$$

where $\mu_p'$ and $\mu_n$ are anomal magnetic momenta of the proton and the neutron. Taking into account (21) and (24), (23) will be written as follows

$$\mathrm{Re}\,\Omega_i(\nu,\nu_1)=\overset{o}{\Omega}_i(\nu,\nu_1)+\frac{1}{\pi}P\int_{2M\mu+\mu^2-\nu_1}^{\infty}\Big(\frac{1}{\nu'-\nu}+\frac{1}{\nu'+\nu}\Big)\mathrm{Im}\,\Omega_i(\nu,\nu_1)\,d\nu' \tag{25}$$

## 3. Dispersion Relations for Physical Amplitudes.

The obtaining of the dispersion relations for the relativistic amplitudes $\Omega_i$ is an intermediate stage. To obtain them for the physical amplitudes $M_K(\nu, \nu_1)$ let us present the Compton scattering amplitude expanded in three-dimensional structures $\hat{r}_i$ (all the calculations are being carried on in c.m.s.):

$$\hat{R} = \sum_{i=1}^{6} M_i(\nu, \nu_1)\hat{r}_i \qquad (26)$$

where

$$\hat{r}_1 = i\left[\bar{\sigma}(\bar{e}\times\bar{n})\,\bar{e}'\bar{n} - \sigma(\bar{e}'\times\bar{n}')\bar{e}\cdot\bar{n}'\right], \quad \hat{r}_2 = \bar{e}\cdot\bar{n}'\bar{e}'\cdot\bar{n}$$

$$\hat{r}_3 = i\bar{\sigma}(\bar{e}'\times\bar{n}')\times(\bar{e}\times\bar{n}), \quad \hat{r}_4 = i\bar{\sigma}(\bar{n}'\times\bar{n})\bar{e}\;\bar{e}'$$

$$\hat{r}_5 = i\bar{\sigma}(\bar{e}\times\bar{e}'), \quad \hat{r}_6 = \bar{e}\cdot\bar{e}' \quad \vec{n} = \frac{\vec{K}}{K_0}, \quad \bar{n}' = \frac{\vec{K}'}{K_0}$$

Note that $\bar{R}_i = \bar{u}(p')\hat{R}_i u(p)$ expand in $\hat{r}_i$ in the following way:

$$\bar{R}_1 = \frac{N^2}{(p'n)}\left\{\hat{r}_2 + \delta p'n\hat{r}_4 - p'n(1-\bar{n}n'\delta^2)\bar{r}_6 - \delta^2\hat{r}_7\right\}$$

$$\bar{R}_2 = \frac{N^2(n\cdot n')^{1/2}}{p'n}\delta\left\{\hat{r}_1 + 2\hat{r}_2 - \hat{r}_8\right\}$$

$$\bar{R}_3 = \frac{N^2}{(n\cdot n')^{1/2}}\left\{2\delta\hat{r}_1 - 2\delta\hat{r}_8 + 4\delta\beta\hat{r}_3 - 4\delta\beta\hat{r}_4 - 4\beta\hat{r}_5 - 4\delta^2\hat{r}_2\right\}.$$

$$\bar{R}_4 = \frac{N^2\beta}{p'n}\left\{\hat{r}_1 - 2\delta\hat{r}_2 + \delta\hat{r}_8\right\}$$

$$\bar{R}_5 = \frac{N^2(n\cdot n')^{1/2}}{p\cdot n\, p'\cdot n}\Big\{-4\delta(p\cdot n)\hat{r}_1 + 2\delta(\beta n n' - 4pn)\hat{r}_2 + 4\delta(pn)\hat{r}_8 - $$
$$- 2\delta\beta n\cdot n'\hat{r}_3 + 4\delta\beta(p\cdot n)\hat{r}_4 + 2\beta n\cdot n'\hat{r}_5 + 2\beta(1+\delta n n')(p+r_1' n)\hat{r}_6\Big\}$$

$$\bar{R}_6 = \frac{N^2 4\beta}{(n\cdot n')^{1/2} p'n}\Big\{(1-\delta p'n)\hat{r}_2 + \delta p'n\hat{r}_3 - \delta p'n\hat{r}_4 - p'n\hat{r}_5 + \delta\hat{r}_7\Big\}$$

$$\hat{r}_7 = \hat{r}_1 + \bar{n}\cdot\bar{n}'(\hat{r}_2 - \hat{r}_3 + \hat{r}_4) - \hat{r}_5, \quad \hat{r}_8 = \hat{r}_3 - \hat{r}_4 + \bar{n}\bar{n}'\hat{r}_5$$

$$\delta = \frac{K_0}{E+M}, \quad \beta = 1+\delta, \quad N^2 = \frac{E+M}{2E}$$

$$pn = \frac{E}{K_0} + 1, \quad nn' = 1 - \bar{n}\bar{n}', \quad p'n = \frac{E}{K_0} + \bar{n}\bar{n}'. \tag{27}$$

**Substituting (27) into (2) and comparing with (26) we are establishing the relation between** $M$ **and** $\Omega_i$

$$M_i(r, v_1) = \sum_{j=1}^{6} C_{ij}(v, v_1)\Omega_j(v, v_1) \tag{28}$$

$$M_1 = \frac{N^2\delta}{p'n}\Big\{-\delta\Omega_1 + (n\cdot n')^{1/2}\Omega_2 + 2\frac{p'n}{(n\cdot n')^{1/2}}\Omega_3 + \beta/\delta\,\Omega_4 - 4(n\cdot n')^{1/2}\Omega_5 + \frac{4\beta}{(n\cdot n')^{1/2}}\Omega_6\Big\}$$

$$M_2 = \frac{N^2\delta}{p'n}\Big\{\frac{1-\delta^2\bar{n}\bar{n}'}{\delta}\Omega_1 + 2(n\cdot n')^{1/2}\Omega_2 - 4\delta\frac{p'n}{(n\cdot n')^{1/2}}\Omega_3 - 2\beta\Omega_4 +$$
$$+ \frac{2(n\cdot n')^{1/2}(\beta n\cdot n' - 4p\cdot n)}{pn}\Omega_5 + \frac{4\beta}{\delta}\;\frac{\beta - \delta pn}{(n-n')^{1/2}}\Omega_6\Big\}$$

$$M_3 = \frac{N^2\delta}{p'n}\Big\{\delta(1-nn')\Omega_1 - (n\cdot n')^{1/2}\Omega_2 + \frac{2(1+2\delta)p'n}{(n\cdot n')^{1/2}}\Omega_3 + \beta\Omega_4 +$$
$$+ \frac{2(n\cdot n')^{1/2}(-\beta n\cdot n' + 2pn)}{p\cdot n}\Omega_5 + \frac{4\beta(pn-1)}{(n\cdot n')^{1/2}}\Omega_6\Big\}$$

$$M_4=\frac{N^2\delta}{p'n}\Big\{\delta(pn-1)\Omega_1+(n\cdot n')^{1/2}\Omega_2-2(1+2\delta)\frac{p'n}{(n\cdot n')^{1/2}}\Omega_3-\beta\Omega_4+$$
$$+4\delta(n\cdot n')^{1/2}\Omega_5+\frac{4\beta(-pn+1)}{(n\cdot n')^{1/2}}\Omega_6\Big\}$$

$$M_5=\frac{N^2\delta}{p'n}\Big\{\delta\Omega_1-\bar{n}\bar{n}'(n\cdot n')^{1/2}\Omega_2-\frac{2(2+3\delta-\delta nn'}{\delta}\frac{p'n}{(n\cdot n')^{1/2}}\Omega_3+\beta\bar{n}\bar{n}'\Omega_4$$
$$+\frac{2(n\cdot n')^{1/2}(\beta nn'+2\delta pn\bar{n}\bar{n}')}{pn\cdot\delta}\Omega_5-\frac{4\beta}{\delta}\frac{(\delta+p'n)}{(n\cdot n')^{1/2}}\Omega_6\Big\}$$

$$M_6=N^2\Big\{-(1-\delta^2\bar{n}\bar{n}')\Omega_1+\frac{2\beta(n\cdot n')^{1/2}(\beta-\delta nn')(p+p'n)}{pn\;p'n}\Omega_5\Big\}.$$

One may obtain the reverse relations

$$\Omega_i(\nu,\nu_1)=\sum_j b_{ij}(\nu,\nu_1)M_j(\nu,\nu_1) \tag{29}$$

$$\Omega_1=\frac{1}{N^2\delta\beta}\Big\{(\beta-\delta nn')(M_3+M_4)-\delta M_6\Big\}$$

$$\Omega_2=\frac{p'n}{2\beta N^2(n\cdot n')^{1/2}}\Big\{2M_1+\Big[\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}n\cdot n'+\frac{pn-2\beta}{1-\delta}\Big]M_2+$$
$$+\Big[\frac{2pn}{p'n+pn}\frac{1-\delta^2+2\delta^2pn}{\delta\beta}+\frac{pn}{p'n}\frac{pn\delta-\beta}{\delta\beta}\Big(1-\frac{(pn-2\beta)(pn\delta-\beta)}{pn(1-\delta)}\Big)+\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}-$$
$$-\frac{pn(\beta+2\delta-pn\delta)}{1-\delta^2}\Big]M_3+\Big[\frac{2p}{p'n+pn}\frac{1-\delta^2+2\delta^2pn}{\delta\beta}+\frac{pn}{p'n}\frac{pn\delta-\beta}{\delta\beta}\Big(1-\frac{(pn-2\beta)(pn\delta-\beta)}{pn(1-\delta)}\Big)$$
$$+\frac{2\beta}{\delta}\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}-\frac{pn(\beta+2\delta-pn\delta)}{1-\delta^2}-\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}nn'\Big]M_4-$$

$$-\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}M_5+\left[\frac{2pn}{p'n+pn}\,\frac{\delta}{\beta}+\frac{pn}{p'n}\,\frac{1}{\beta}\left(1-\frac{(pn-2\beta)(pn\delta-\beta)}{pn(1-\delta)}\right.\right.-$$

$$\left.-\frac{\beta+2\delta^2-pn\delta}{1-\delta^2}\right]M_6\Bigg\}$$

$$\Omega_3=\frac{(n\cdot n')^{1/2}}{4N^2\beta}\left\{\left[\frac{1-\delta+pn\delta}{1-\delta^2}\,n\cdot n'-\frac{pn}{1-\delta}\right]M_2+\left[\frac{2pn}{p+p_1'n}\,\frac{1-\delta^2+2\delta^2pn}{\delta\beta}+\frac{pn}{p'n}\,\frac{(pn-2)(pn\delta-\beta)}{1-\delta^2}\right.\right.+$$

$$\left.+\frac{1-\delta+pn\delta}{1-\delta^2}+\frac{pn(-\beta+2\delta^2-pn\delta)}{1-\delta^2}\right]M_3+\left[\frac{2pn}{p+p_1'n}\cdot\frac{1-\delta^2+2\delta^2pn}{\delta\beta}+\frac{pn}{p'n}\,\frac{(pn-2)(pn\delta-\beta)}{1-\delta^2}\right.-$$

$$\left.-\frac{1-\delta+pn\delta}{1-\delta^2}\,n\cdot n'+\frac{pn(\beta+2\delta^2-pn\delta)}{1-\delta^2}\right]M_4-\frac{1-\delta+pn\delta}{1-\delta^2}M_5+\left[\frac{2pn}{p+p_1'n}\,\frac{\delta}{\beta}\right.+$$

$$\left.\frac{pn\delta(pn-2)}{p'n(1-\delta^2)}-\frac{1-\delta+pn\delta}{1-\delta^2}\right]M_6\Bigg\}$$

$$\Omega_4=\frac{1}{N^2\beta^3}\Big\{\beta p'nM_1-\delta p'nnn'M_2+(pn-\delta nnn'+\delta nn')M_3+$$

$$+nn'(\beta-\delta nn')M_4+\delta p'nM_5-\delta nn'M_6$$

$$\Omega_5=\frac{(n\cdot n')^{1/2}pn\,p'n}{2N^2\delta\beta^2(p+p_1'n)}\Big\{(1-\delta^2\bar{n}\bar{n}')(M_3+M_4)+\delta^2M_6\Big\}$$

$$\Omega_6=\frac{(n\cdot n')^{1/2}}{4N^2\beta^2(1-\delta)}\Big\{p'n(\beta-\delta nn')M_2+\left[\delta(\delta-pn)n\cdot n'+pn(2+\delta)-{}^{\beta}/_{\delta}(1+\delta^2)\right]M_3$$

$$+(\delta nn'-1-\delta^2)\,\frac{\beta-\delta nn'}{\delta}M_4+\delta p'nM_5-(\delta nn'-1-\delta^2)M_6\Big\}.$$

Taking into account (25), (28) and (29) one may obtain the dispersion relations for the physical amplitudes $M_K$

$$Re\, M_i(\nu,\nu_1) = \sum_j C_{ij}(\nu,\nu_1)\Omega_j^0 +$$

$$+ \sum_{j,k} \frac{1}{\pi} P \int_{2M\mu+\mu^2-\nu_1}^{\infty} d\nu' \left(\frac{1}{\nu'-\nu} + \frac{1}{\nu'+\nu}\right) C_{ij}(\nu,\nu_1) B_{jk}(\nu',\nu_1)\, Im\, M_k(\nu',\nu_1). \tag{30}$$

As the matrices $C_{ik}, B_{ik}$ are rather clumsy the formulae (30) are not explicitly written out. In the energy region, however, where the terms of the $\left(\frac{K_0}{M}\right)^2$ order the matrices $C_{ik}(\nu,\nu_1), B_{ik}(\nu,\nu_1)$ are simplified and we obtain the following approximate equations for the physical amplitudes $M_i = D_i + iA_i$

$$D_1 = \frac{\omega}{\pi} P \int_{\mu}^{\infty} \rho(\omega',\omega) \frac{d\omega'}{\omega'+\omega} \left\{ \frac{2A_1}{\omega'-\omega} + \frac{A_2 - 2A_4}{2\omega'} + \left[ \left(1 + 2\frac{\nu_1}{\omega\omega'}\right)\frac{A_1}{M} \right.\right.$$

$$- \left(\frac{\omega}{\omega'} + \frac{3}{4}\frac{\nu_1}{\omega'^2} - \frac{\nu_1}{2\omega\omega'}\right)\frac{A_2}{M} - \frac{3}{4}\left(1 + \frac{\nu_1}{3\omega'^2}\right)\frac{A_3}{M} + \left(1 + 2\frac{\omega}{\omega'} - \frac{\nu_1}{\omega\omega'} +\right.$$

$$\left.\left.\left. + \frac{\nu_1}{2\omega'^2}\right)\frac{A_4}{M} + \frac{3A_5 + 2A_6}{4M}\right]\right\} + 2\omega\hat{\mu}_Б(\hat{\mu} + \hat{\mu}_Б)$$

$$D_2 = \frac{\omega}{\pi} P \int_{\mu}^{\infty} \rho(\omega',\omega) \frac{d\omega'}{\omega'} \left\{ \frac{A_2}{\omega'-\omega} + \frac{2A_4}{\omega'+\omega} + \frac{\omega'}{\omega'+\omega}\left[\frac{2A_1}{M} + \frac{\nu_1}{\omega\omega'}\left(1 + \frac{3}{2}\frac{\omega}{\omega'}\right)\frac{A_2}{M} \right.\right.$$

$$\left.\left. + \left(\frac{1}{2} + \frac{\nu_1}{\omega\omega'} + \frac{\nu_1}{2\omega'^2}\right)\frac{A_3}{M} + 2\left(1 - \frac{\nu_1}{\omega'^2} + \frac{3}{2}\frac{\nu_1}{\omega\omega'}\right)\frac{A_4}{M} - \frac{A_5 + 2A_6}{2M}\right]\right\} + 4\omega\hat{\mu}_Б^2$$

$$D_3 = \frac{\omega}{\pi} P \int_{\mu}^{\infty} \rho(\omega',\omega) \frac{d\omega'}{\omega'+\omega} \left\{ \frac{2A_3}{\omega'-\omega} - \frac{A_2 - 2A_4}{2\omega'} + \left[ -\frac{A_1}{M} + \left(\frac{\omega}{\omega'} + \frac{3}{4}\frac{\nu_1}{\omega'^2} - \right.\right.\right.$$

$$\left. - \frac{\nu_1}{2\omega\omega'}\right)\frac{A_2}{M} - \left(1 - \frac{\nu_1}{\omega'^2} + 4\frac{\nu_1}{\omega\omega'}\right)\frac{A_3}{4M} - \left(1 + 2\frac{\omega}{\omega'} + \frac{3}{4}\frac{\nu_1}{\omega'^2}\right)\frac{A_4}{M} +$$

$$\ast A_5 + \frac{A_5 - 2A_6}{4M}\Big]\Big\} - 2\omega(\hat{\mu} + \hat{\mu}_{\text{б}})^2$$

$$D_4 = \frac{\omega}{\pi} P \int_{\mu}^{\infty} \rho(\omega', \omega) \frac{d\omega'}{\omega'} \Big\{ \frac{A_4}{\omega' - \omega} + \frac{A_2}{2(\omega' + \omega)} + \frac{\omega'}{\omega' + \omega} \Big[ \frac{A_1}{M} - \Big( \frac{\omega}{\omega'} + \frac{3}{4}\frac{V_1}{\omega'^2} - \frac{V_1}{2\omega\omega'} \Big) \frac{A_2}{M} + \Big(1 - \frac{V_1}{\omega'^2}\Big) \frac{A_3}{4M} + \Big(1 + 2\frac{\omega}{\omega'} + \frac{V_1}{\omega\omega'} + \frac{3}{2}\frac{V_1}{\omega'^2}\Big) \frac{A_4}{M} - \frac{A_5 - 2A_6}{4M} \Big] \Big\}$$

$$D_5 = \frac{\omega}{\pi} P \int_{\mu}^{\infty} \rho(\omega', \omega) \frac{d\omega'}{\omega' + \omega} \Big\{ \frac{2A_5}{\omega' - \omega} - \Big(1 - \frac{V_1}{\omega^2}\Big) \frac{A_2}{2\omega'} - \Big(1 + \frac{V_1}{\omega^2} + \frac{V_1}{\omega\omega'}\Big) \frac{A_4}{\omega'}$$

$$+ \Big[ -\frac{A_1}{M} + \Big( \frac{\omega}{\omega'} - \frac{3}{2}\frac{V_1}{\omega\omega'} + \frac{3}{4}\frac{V_1}{\omega'\omega'} + \frac{V_1^2}{2\omega'\omega^3} + \frac{V_1^2}{4\omega^2\omega'^2} + \frac{V^2}{\omega\omega'^3} \Big) \frac{A_2}{M} +$$

$$+ \Big( \frac{7}{4} + \frac{V_1}{4\omega^2} + \frac{V_1}{4\omega'^2} - \frac{V_1^2}{4\omega^2\omega'^2} \Big) \frac{A_3}{M} - \Big( 1 + 2\frac{\omega}{\omega'} - \frac{V_1}{\omega^2} - \frac{V_1}{2\omega'^2} - \frac{V_1^2}{\omega'\omega^3} +$$

$$+ \frac{V_1^2}{2\omega^2\omega'^2} + \frac{V_1^2}{\omega\omega'^3} \Big) \frac{A_4}{M} - \Big( \frac{7}{4} + \frac{V_1}{4\omega^2} + \frac{V_1}{\omega\omega'} \Big) \frac{A_5}{M} - \Big(1 - \frac{V_1}{\omega^2}\Big) \frac{A_6}{2M} \Big] \Big\} -$$

$$- 2\omega \hat{\mu}_{\text{б}} (2\hat{\mu} + \hat{\mu}_{\text{б}})$$

$$D_6 = \frac{1}{\pi} P \int_{\mu}^{\infty} \rho(\omega', \omega) d\omega' \Big\{ \frac{2\omega'}{\omega'^2 - \omega^2} A_6 + \frac{V_1^2}{\omega'^2\omega^2} \frac{A_3 + A_4}{M} \Big\} - \frac{e^2}{M}$$

$$\hat{\mu}_{\text{б}} = \frac{e}{2M} \tau_p$$

$$\frac{2V' dV'}{V'^2 - V^2} \longrightarrow \frac{2\omega' d\omega'}{\omega'^2 - \omega^2} \rho(\omega', \omega)$$

$$\rho(\omega', \omega) = 1 + \frac{\omega'}{M} - \frac{2\omega^2}{M(\omega' + \omega)} + \frac{\omega' - \omega}{\omega' + \omega} \frac{V_1}{2M\omega'} \qquad (31)$$

4. The Unitarity Condition. The dispersion relations (30), (31) bind the Hermitian and antiHermitian part of the reaction amplitude. The unitarity condition $SS^{+} = 1$ written in the one-meson approximation makes it possible to express the antiHermitian part of the Compton scattering in terms of the photoproduction amplitude. Note, that the onemeson consideration is wholly justified in the interval $\mu < \omega < 2\mu$ [1]. For $\omega > 2\mu$ it may be considered only as an approximation. As a result (30) and (31) acquire the meaning of equations. From $SS^{+} = 1$ and $S = 1 + R$ in the onemeson approximation we have

$$\langle \gamma' | R^{+} + R | \gamma \rangle = -\frac{1}{(2\pi)^6} \int d\vec{q}\, d\vec{p}\,'' \langle \gamma' | R^{+} | \pi \rangle \langle \pi | R | \gamma \rangle \tag{32}$$

$|\pi\rangle$ characterizes nucleon and meson in the intermediate state with momenta $\vec{p}\,''$ and $\vec{q}$ and other quantum numbers.

Taking into account (18) and the determination of the photoproduction amplitude

$$\langle \pi | R | \gamma \rangle = i \frac{(2\pi)^4}{\sqrt{4 K_0 q_0}} \delta(p + k - p'' - q) T \tag{33}$$

where

$$T = i\vec{\sigma}\vec{e}F_1 + \vec{\sigma}\vec{m}\vec{\sigma}(\vec{n} \times \vec{e})F_2 + i\vec{\sigma}\vec{n}\vec{e}\,\vec{m}F_3 + i\vec{\sigma}\vec{m}\vec{e}\,\vec{m}F_4 \qquad \vec{m} = \vec{q}/|\vec{q}|$$

after the simple calculations we get the relationship between $A_i$ and $F_i$

$$A_2 = (4\pi)^{-2} \lambda \int d\Omega \Big\{ \frac{m_x}{n'_x} F_{12} + \beta F_{21} + \frac{m_x}{n'_x} F_{13} + \beta F_{31} - F_{22} +$$

$$+ \frac{n_z'^2 m_y^2 - m_x^2}{n_x'^2} F_{23} + (\alpha n'_z - \beta m_z) F_{32} + \alpha ( F_{14} + F_{41} + \vec{n}\vec{m} F_{34} + \vec{n}\vec{m} F_{43} + F_{44} ) \Big\}$$

$$A_6 = (4\pi)^{-2}\lambda\int d\Omega\Big\{F_{11} - \bar{n}\bar{m}F_{12} - \bar{n}'\bar{m}F_{21} + n'_z F_{22} + m_y^2(F_{14} + F_{41} + n'_z F_{23} + n'_z F_{32} + \bar{n}'\cdot\bar{m}F_{34} + \bar{n}\cdot\bar{m}F_{43} + F_{44})\Big\}$$

$$A_1 n_x'^2 - A_3 n'_z + A_5 = (4\pi)^{-2}\lambda\int d\Omega\Big\{-F_{11} + \bar{n}\cdot\bar{m}F_{12} + \bar{n}'\cdot\bar{m}F_{21} - m_x^2 F_{14} - m_y^2 F_{41} - n'_z F_{22} + n'_x m_x\beta F_{23} - n'_z m_y^2 F_{32} + n'_x m_x F_{24} + n'_x m_x m_y^2 F_{34}\Big\}$$

$$A_3 - n'_z A_5 = (4\pi)^{-2}\lambda\int d\Omega\Big\{n'_z F_{11} - \bar{n}'\bar{m}F_{12} - \bar{n}\cdot\bar{m}F_{21} + n'_z m_y^2 F_{14} - n'_x m_x\beta F_{41} + F_{22} + m_y^2 F_{23} + n_x'^2\beta^2 F_{32} - n_x'^2\beta F_{31} - \beta n_x'^2 m_y^2 F_{34}\Big\}$$

$$2A_1 + n'_z A_3 + A_5 = (4\pi)^{-2}\lambda\int d\Omega\Big\{-F_{11} + (\beta - m_x\frac{n'_z}{n'_x})F_{12} - (\beta n'_z - \frac{m_x}{n'_z})F_{21} - m_x\frac{n'_z}{n'_x}F_{13} - \beta n'_z F_{31} - \bar{n}'\bar{m}\frac{m_x}{n'_x}F_{14} - \beta\bar{n}\cdot mF_{41} + n'_z F_{22} + (n'_z m_y^2 + \bar{n}\cdot\bar{m}\frac{m_x}{n'_x})F_{23} + (n'_z m_y^2 + \beta\bar{n}'\cdot\bar{m})F_{32} + \frac{m_x}{n'_x}F_{24} + \beta F_{42} + \alpha n_x'^2 F_{33} + \alpha n_x'^2(\beta F_{34} + \frac{m_x}{n'_x}F_{43})\Big\}$$

$$A_3 + A_4 = (4\pi)^{-2}\lambda\int d\Omega\Big\{-\frac{m_x}{n'_x}F_{12} - \beta F_{21} + F_{22} + m_y^2(F_{23} + F_{32}) + m_y^2(F_{33} + \beta F_{34} + \frac{m_x}{n'_x}F_{43})\Big\}. \tag{34}$$

Here
$$n'_x = \sin\theta, \quad n'_z = \bar{n}\bar{n}' = \cos\theta, \quad m_x = \sin\theta' \cos\varphi', \quad m_y = \sin\theta' \sin\varphi'$$

$$m_z = \bar{n}\,\bar{m} \cos\theta', \quad \bar{n}'\bar{m} = \cos\theta'' = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos\varphi'$$

$$d\Omega = \sin\theta' d\theta' d\varphi', \quad \beta = \frac{(\bar{m}\times\bar{n}')_y}{n'_x}, \quad \alpha = \frac{1}{n'_x}\left[m_x m_z + \frac{n'_z}{n'_x}(m_y^2 - m_x^2)\right]$$

$$F_{ik} = F_i^+(\bar{n}'\bar{m})\,F_k(\bar{n}\,m) \qquad \lambda = \frac{\omega E_k}{W} \tag{34}$$

Note that (34) may be easily integrated over in $S$ and $P$ approximation. In this approximation $F_1 = a + b\cos\theta$, $F_2 = c$, $F_3 = d$, $F_4 = 0$ [45] ($a, b, c, d$ - do not depend upon the angles any longer. Substitating (34) into (31) one may determine $D_i$. A similar consideration as well as the comparison of (31) with the experiment is supposed to be made in future.

In conclusion we express our deep gratitude to Academician Bogoliubov N.N. and Logunov A.A. for valuable discussions and constant interest in the course of work.

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