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JOINT INSTITUTE FOR NUCLEAR RESEARCH

2.3. Laboratory of High Energy

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ЖЭТФ, 1958, т34, в5, с1221-1229.

1958

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APPLICATION OF OPTICAL MODEL FOR HIGH ENERGY π-P AND P-P SCATTERING ANALYSIS

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Experimental data on π^--p and p-p scattering at the energies above 1 BeV in the lab. system have been analysed considering the nucleon as an optically uniform sphere with sharp edges.

It is shown that the data presently available may be described by means of the sphere of the radius $R = (1{,}08 \pm 0{,}07) \times 10^{-13}$ cm which is independent either of the type of the colliding particles or of their energy. The optical characteristics of this sphere are evaluated. The contributions from the imaginary and the real parts of the scattering amplitude are estimated.

One may assume that for the 1,37 BeV pions and for the proton energies above ~ 5 BeV the contribution from the real part of the scattering amplitude is small and the analysis of the scattering phenomena at high energies may be carried on using either the general scattering theory without taking tnto account the spin characteristics of the interaction as it was done in |5-8| or using the model of a purely absorbing sphere.

The possible behaviour of π^--p and p-p interaction cross sections with the increase of the energy of the colliding particles is discussed.

Introduction

In 1949 Fernbach, Serber and Taylor [1] used the optical analogy in the analysis of the high energy neutron scattering from the nuclei.

The nuclear matter was regarded as a refracting and absorbing medium. When the neutron passes through this medium its wave vector which is equal to k_0 outside the nucleus becomes equal to the complex value $k = k_0 + k_1 + iK$,
where

K - is the absorption coefficient,

k_1 - is the change of the real part of the neutron wave vector.

Then by analogy to the optics the nuclear matter may be characterized by the complex refractive index $n = \frac{k_0 + k_1 + iK}{k_0}$

Such an optical model of the nucleus was widely used in the analysis of the experimental data on the scattering of fast particles from the nuclei and gave a lot of valuable results.

The optical model formalism of the nucleus was further used in the analysis of pion and proton scattering from protons at 1 BeV and above.[2-4] Such a consideration may be called the optical nucleon model. So, the analysis of π-p scattering at 1.37 BeV[3] shows that the experimental data may be described by the nucleon model as a uniform sphere of the radius $R = (1{,}18 \pm 0{,}1) \cdot 10^{-13}$ cm. This model is characterized by $K = 0{,}67 \times 10^{13}$ cm^{-1} and $k_1 = 0$, i.e., it is a purely absorbing sphere. The data on p-p scattering for 0,8 BeV, 1,5 BeV and 2,75 BeV[2] have been analysed in the same way. The authors of this paper assumed the proton to be a uniform, purely absorbing sphere ($k_1 = 0$) with the radius R which is independent of energy, and considered the incoherent scattering to be insignificant. Under the above mentioned assumptions this sphere is characterized by $R = 0,93 \cdot 10^{-13}$ cm. and $K = 4,3 \times 10^{13} cm^{-1}$, $3,7 \times 10^{13} cm^{-1}$ and $2,7 \times 10^{13} cm^{-1}$ at the proton energies of 0,8 BeV, 15, BeV and 2,75 BeV respectively. It follows from these results of the analysis that the proton is more "transparent" for a pion than for a proton and the region of π-p interaction is larger than that of p-p interaction.

Recently the data on elastic p-p scattering at 2.24 BeV, 4,40 BeV and 6,15 BeV [4] have been published which were analysed from the standpoint of a rather Simple concept of the optical model which regards the nucleon as a disk with different parameters. It appeared that a certain choice of the parameters of such a disk has led to the agreement with the experiment.

In this paper an attempt to analyse all the available experimental data on p-p and π- p scattering has been taken in the BeV- energy region on the basis of the nucleon optical model. The analysis was performed under the assumption that:

a) the interaction region is determined by the optical uniform sphere with sharp edges.

b) the incoherent elastic scattering may be neglected.

In the light of the performed analysis some items of using the general scattering formalism for the solution of the problem under consideration have been examined by analogy to [5-8].

2. Experimental Data and Formalae used in the Analysis

The data including the magnitudes of the total cross sections (σ_t), of elastic scattering cross sections (σ_E) and inelastic scattering cross sections (σ_i) for p-p and π-p interactions at high energies are given in Table 1. The values of the wave length of the incident particle in the centre of mass-system are given in the second column of the Table. The values of σ_i for $E_p = 4.40$ BeV and $E_p = 6.15$ BeV are not measured experimentally. They are obtained on the basis of the speculations set forth in [8]. These magnitudes seem to be quite reasonable. Moreover, it is known that σ_i changes slowly with energy. One may judge about it by the results of the estimates of the mean value of the inelastic cross section in the nucleon-nucleon interaction at 50 BeV [12]. Moreover, as the calculation shows the results of the analysis are dependent upon the change of σ_i rather weakly in the wide value interval i.e., the choice of the magnitude of the inelastic cross section is not critical.

Besides the values of the characteristics of p-p and π - p scattering given in Table 1 the differential cross sections of pion and proton elastic scattering on protons for the energies mentioned in the Table are available. This information serves as one of the criteria that the choice of the parameters of the system describing the scattering is correct.

As is known if we neglect the incoherent elastic scattering and assume the elastic scattering to be purely diffractional ($\sigma_e = \sigma_d$) then the scattering cross sections are determined by the parameters of the optical model in the following way: the nucleon is given as a sphere of the radius R^1; the total cross section of the inelastic scattering

$$\sigma_i = \pi R^2 \left\{ 1 - \frac{1-(1+2kR)\exp(-2kR)}{2k^2R^2} \right\} , \qquad (I)$$

the total cross section of the elastic scattering

$$\sigma_d = \pi R^2 \Big\{ 1 + \frac{1-(1+2kR)e^{-2kR}}{2k^2R^2} \tag{2}$$

$$- \frac{(\frac{1}{4}k^2-k_1^2)+e^{-kR}[2k_1R(\frac{1}{4}k^2+k_1^2)+k_1k]\sin 2k_1R-e^{-kR}[(\frac{1}{4}k^2-k_1^2)+kR(\frac{1}{4}k^2+k_1^2)]\cos 2k_1R}{(\frac{1}{4}k^2+k_1^2)^2R^2}$$

the differential cross section of the elastic scattering

$$\frac{d\sigma}{d\omega} = |f(\vartheta)|^2, \tag{3}$$

where the scattering amplitude is determined by the expression

$$f(\vartheta) = i k_0 \int_0^R [1-e^{(-k+2ik_1)s}] J_0(k_0\rho \sin\vartheta)\rho\, d\rho, \tag{4}$$

in which ϑ is the scattering angle, $s = \sqrt{R^2-\rho^2}$ and $J_0(k_0\rho\sin\vartheta)$ are the zero-order of Bessel's function. Rather a clumsy formula (2) is obtained for the cross section of the elastic diffraction scattering. It is known, however, that by opaque $\frac{\sigma_i}{\pi R^2} \leq 0{,}9$ corresponding to $kR \leq 2{,}3$ and $\frac{k_1}{k} < 1$ the expression for the diffraction scattering cross section may be reduced to the simpler form [9]:

$$\sigma_d = \sigma_d(k, k_1=0)\Big\{1+4\Big(\frac{k_1}{k}\Big)^2\Big[1-\frac{1}{18}(kR)^2+\ldots\Big]\Big\}, \tag{5}$$

where

$$\sigma_d(k, k_1=0) = \frac{\pi R^2}{B^2}\Big\{B^2-14-2(1+B)e^{-B}+8e^{-B/2}(2+B)\Big\} \tag{6}$$

and $B = 2kR$. This gives the result which is different from that obtained in expression (2) not more than by 1%. Since the experimental data satisfy the abovementioned requirements the use of expression (5) is completely justified. The given expressions are correct at the energies when in the centre-of-mass-system of the colliding particles the condition $\lambda \ll R$ is fulfilled.

3. The Parameters of the NucleonOptical Model for the Description of Scattering at High Energies

Making use of the data given in Table I and using the relations (1) and (5), the sets of the parameters of the nucleon optical model have been obtained with the help of which one could describe the known results of the measurements of the cross sections for high energy interactions of pions and protons with protons. Each set of the parameters was determined both for the mean values of the cross sections and for the extreme ones in accordance with the accuracy of their determining by the errors of experimental data. The radius of the interaction sphere was taken as an original magnitude for the set of the parameters. The relation between the cross sections of elastic and inelastic scattering $(\sigma_d \leq \sigma_i)$ made it possible to determine immediately the lower limit of the interval of the considered radii by the values somewhat greater than the radius for the "black" sphere. The minimum values of R were determined from the dependence of the opacity $\frac{\sigma_i}{\pi R^2}$ upon KR and upon $\frac{\sigma_d(K, k_1 = 0)}{\pi R^2}$, They are given in Table II.

However, the data about the elastic cross sections and inelastic cross section are not sufficient for choosing one or another set of the parameters of the optical model. Additional experimental data, for instance the results of the measurements of differential cross sections of elastic scattering are necessary for this. Calculating the differential cross sections of elastic scattering for each set of the values R, K and k_1 according to the formulae (3) and (4) one may prefer either one or another set by means of comparing the calculated angular distributions of elastic scattering with those measured experimentally. The numerical integration was performed using Simpson's formula with the accuracy ~ 1% for small angles and some percents for large scattering angles. While making these calculations it became clear that the changes of σ_i' by 20%; R and σ_d being constant, lead to the changes in the magnitudes of $\frac{d\sigma}{d\omega}$ within the accuracy of numerical integration. Thus, in these calculations the choice of the values of σ_i is not critical.

The value of the radius of the interaction sphere R being fixed the angular distributions of elastic scattering have been calculated for the extreme values of σ_d . Thus, the region of the possible angular distributions for the intermediate values of σ_d has been determined in accordance with the accuracy of experimental data concerning this magnitude. The results of the calculations and their comparison with experimental data are given below.

a) p-p scattering at 1,5; 2,24 and 2,75 BeV

For 1,5 and 2,75 BeV the experimental results have been obtained using the diffusion hydrogen chamber [3] . These data have a poor statistical accuracy and because of this the experimental angular distributions are described equally satisfactorily by the nucleon optical model with the parameters changing in wide limits (see Fig. 1a and b).

However, the available counter data [4] for $E_p = 2,24$ BeV permit to determine the limits of the passible values of the sphere radius for this energy region. It appears that the uniform sphere may have the maximum radius $1,15 \times 10^{-13}$ cm since $R = 1,20 \times 10^{-13}$ cm leads to the explicit discrepancy with the results of the experiments (see Fig. 2a). It should be noted that for $R \leq 1,15 \times 10^{-13}$ cm the nucleon optical model agrees with the experiment only for the angles not more than 30. The minimum radius for $E_r = 2,24$ BeV does not contradict to the value $0,93 \times 10^{-13}$ cm which the authors of the paper give for the energies of 1,5 and 2,75 BeV. The purely absorbing sphere ($k_1 = 0$) may have only the radius R less than $1,0 \times 10^{-13}$ cm.

It is necessary to note that some suggestions by Rarita [13] on the application of the nucleon optical model for p-p collisions at $E_p \approx 1$ BeV are correct to some extent also at the energies under consideration.

b) p-p scattering at 4,4 and 6,15 BeV

Experimental data for $E_p = 4,4$ BeV [4] if $\vartheta < 30^\circ$ are described satisfactorily by the uniform sphere with the radius from $0,95 \times 10^{-13}$ cm up to $1,15 \times 10^{-13}$ cm since the values $R = 0,92 \times 10^{-13}$ cm and $R = 1,2 \times 10^{-13}$ cm lead to explicit discrepancy with experimental results (see Fig. 2b and 3a).

The purely absorbing sphere may have the radius R not more than $1,10 \times 10^{-13}$ cm.

The measured angular distribution of the elastic scattering if $E_p = 6,15$ BeV is described satisfactorily by the uniform sphere with the radius R from $1,0 \times 10^{-13}$ cm up to $1,15 \times 10^{-13}$ cm.

The discrepancy with the experiments for $R = 0{,}95 \times 10^{-13}$ cm and for $R = 1{,}20 \times 10^{-13}$ cm is seen in Fig. 3b and 2c. At all possible values of the radius the sphere may be purely absorbing at this energy.

c) π -p scattering at $E_{\pi} = 1{,}37$ BeV

The available experimental data on pion-proton scattering [3] are in good agreement with the representation of a nucleon as a uniform sphere the radius of which changes within a wide interval. We would remind that these data are obtained with the use of the diffusion chamber and have an insufficient statistical accuracy. Due to this the restriction of the region of R values were made using the dispersion relations which point out that the contribution from the real part of the scattering amplitude to the elastic scattering cross section at 0^{0} at E_{π} = 1,37 BeV is of the order of 7% [10]. Applied to the optical model of the uniform sphere, it means that the radius of the sphere R may have the values from $1{,}01 \times 10^{-13}$ cm up to $1{,}25 \times 10^{-13}$ cm.

4. On the Applicability of the Nucleon Optical Model with $\zeta k_1 = 0$

Attempts have been recently made to analyse the scattering in the BeV - energy-region from the standpoint of the general scattering theory [5-8]. These works were based upon the assumptions that the imaginary part of the scattering amplitude (see, for instance [14]),

$$f(\vartheta) = \frac{i\lambda}{2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-\beta_\ell) P_\ell(\cos\vartheta) \qquad (7)$$

is much greater than the real one; that the considered scattering characteristics are independent of the spins of the interacting particles and the "charge-exchange" effect in π-p scattering is small. In formula (7) $\hbar \ell$ is the orbital momentum, $P_\ell(\cos\vartheta)$ are the Legendre polynomials, $\beta_\ell = \exp\{2i\eta_\ell\}$, where η_ℓ is the phase shifts. The relations obtained under these assumptions are very simple and convenient for analysing the scattering at high energies. This fact stimulated to draw our attention to the problem about the limits of the application of these assertions. This was considered in this section. The parameters of the optical model which does not also take into account the dependence of the considered scattering characteristics upon the spins are interrelated with the scattering phase in the following way [1]

$$\eta_\ell = \left(k_1 + \frac{1}{2} i k\right) s_\ell . \qquad (8)$$

Here $2 s_\ell$ is the range length of an incident particle with the orbital momentum $\hbar \ell$ in the nucleon matter.

The case under consideration when k_1 is equal to zero indicates that the phase shift η_ℓ must be purely imaginary and the magnitude β_ℓ equal to $e^{-k s_\ell}$ must be real and positive for any form of the nucleon matter distribution. That is, the case $k_1 = 0$ in the nucleon optical model is equivalent to the assumptions made in the abovementioned consideration of the scattering from the standpoint of the general scattering theory. Let us make a simple assumption when considering the limits of the applicability of the case $K_1 = 0$. Since it can be seen from the experimental data that the differential cross section of the elastic interaction $\frac{d\sigma}{d\omega}$ is a monotonuus function, tending to zero one may assume that if $\frac{d\sigma}{d\omega}$ is equal to zero for a certain angle ϑ_0 then for all $\vartheta > \vartheta_0$ it is equal to zero.

Then $f(\vartheta)$ will be a positive function for all angles and β_e is simply expressed through the measured differential cross section of elastic scattering:

$$\beta_e = 1 - \frac{\int_0^{\pi} P_e(\vartheta)\sqrt{\frac{d\sigma}{d\omega}}\sin\vartheta d\vartheta}{\bar{\lambda}} \quad (9)$$

We have calculated β_0 using formula (8) for all the discussed energies of the interaction and making use of the analytic form $\frac{d\sigma}{d\omega}$ from [8]. It was assumed that the angular distribution is known with the accuracy of $\pm$ 15% for all angles. The results of the calculations are given in Table III.

Thus the performed consideration of p-p scattering which does not concretize the form of the interaction region leads to the negative values of β_0 for the energies below $\sim$ 5 BeV.

It means that the original suggestions of this consideration are not correct and the phase shift is not purely imaginary at the energies below 5 BeV i.e., the nucleon optical model with $K_1 = 0$ is not applicable in the frame of the made assumptions.

For the energy E_p = 4,4 BeV the application of these assumptions seems to be possible for the minimum values of σ_d.

In the terms of the uniform sphere model the purely absorbing sphere must have $R \gg 1.0 \times 10^{-13}$ cm.

It follows from the abovementioned consideration that for E_p = 6,15 BeV the assumptions both about K_1 = 0 and about the spin independence of the scattering characteristics for the case of the maximum possible value of σ_d are not correct. Thus, the radii below $1{,}0 \times 10^{-13}$ cm in the purely absorbing sphere are forbidden.

An analogous consideration of π^{-}-p scattering at $E_{\pi} = 1{,}37$ BeV gives $\beta_0 = 0{,}15 \pm 0{,}08$. It means that the assumption about the absence of the potential scattering ($k_1 = 0$) for pions at this energies does not contradict to the experimental data.

5. The Discussion of the Results of the Analysis

At the energies $E_p = 1{,}5 \div 2{,}75$ BeV the comparison of the theoretical calculations for the model of the purely absorbing sphere with the experimental angular distributions does not give any evidence that it is not applicable due to insufficient accuracy of the experimental data. However, this comparison also shows that there are no grounds to suggest that the region of pion-proton interaction is greater than that of proton-proton interaction.

The authors of the paper [4] have shown on the basis of the experimental data that the "form-factor" is rather probably independent of the energy at 2,24, 4,4 and 6,15 BeV for the optical model taken in the most general form. When passing to the concrete form of the optical model of the nucleon the independence of the "form-factor" upon the energy makes it possible to speak about the constancy or about the weak change of the radius of the uniform sphere as the most probable case. In view of this the choice $R = 0{,}93 \times 10^{-13}$ cm for the energies $E_p = 1{,}5$. 2,75 BeV becomes hardly probable since at $E_p = 6{,}15$ BeV the minimum radius describing the experimental data becomes equal to $1{,}0 \times 10^{-13}$ cm.

More general consideration based on a smaller number of assumptions which follow from the experimental data (see section IV) is a convincing argument against the analysis of p-p scattering in the frame of the purely absorbing sphere model at the proton energy being below ~ 5 BeV in the lab. system.

All the experimental data on elastic π-p and p-p scattering available at high energies may be satisfactorily described by the nucleon optical model, if the interaction region will be presented in the form of the uniform sphere with sharp edges. The radius of the sphere may be the same for all the considered energies and for both modes of interaction. The magnitude of this radius is within the limits from $1{,}01 \times 10^{-13}$ cm up to $1{,}15 \times 10^{-13}$ cm. The corresponding values of the absorbing coefficient K as well as of the contribution from the real part of the scattering amplitude to the elastic cross section are given in Table IV for

$$R = (1{,}08 \pm 0{,}07) \times 10^{-13} \text{ cm.}$$

It can be seen from Table IV that the uniform sphere becomes "lighter" with the energy increase and approximates the purely absorbing one. The satisfactory agreement with the experimental results which the uniform sphere model gives if $k_1 = 0$ for $E_p = 6{,}15$ BeV and $E_\pi = 1{,}37$ BeV reaffirms the suggestions about the possibility of making the analysis, assuming that at the mentioned energies the imaginary part of the scattering amplitude $f(\vartheta)$ is considerably greater than the real one and the considered characteristics of the scattering are independent of the spins of the interacting particles. Since the above mentioned assumptions are correct then for the analysis of p-p scattering above 5 BeV one may make use of the consideration like it was done in [8].

An analogous statement may be made on π-p scattering starting from a certain pion energy below 1,37 BeV. This boundary must be more specific by measuring π-p scattering at smaller energies. Evidently (the knowledge of only the magnitudes σ_i, σ_e and $\frac{d\sigma}{d\omega}$ does not give the possibility of getting the complete data about the properties of the scattering amplitude. It is necessary to have a more complete set of the experimental data. In particular the investigation of the polarization effects might essentially clear up the problem under discussion.

The obtained value for the radius of the interaction region $R = (1,08 \pm 0,07) \times 10^{-13}$ cm which can be assumed to be independent neither of the energies of the interacting particles nor of their type corresponds to the model of the sphere with sharp edges. It is known, however, that under the assumption of more "smearing" distributions (for instance a Gaussian one) of the nucleon matter the roof-mean-square radius of the interaction region will have the magnitude which is less than the mentioned one. Its concrete value will be certainly dependent upon the form of the distribution accepted for the calculations. However, the various speculations point out that the value of the root-mean-square radius of the interaction region for p-p and π-p scatterings will be close to the magnitude of the electromagnetic dimension of the nucleon, obtained from the measurments of the electron scattering from the protons [15].

It is known that at high energies the cross section of pion and proton interactions with nucleons are tending to the constant limit because the nucleon dimensions are finite (we neglect the Coulomb interaction). Making use of the dispersion relations, P.V. Vavilov [16] has calculated the limiting value for the total cross section of pion interaction with the nucleons. It was found to be $\approx$ 30 mb. The total cross section is already equal to the limit one for the considered pion energy of 1,37 BeV. There are some grounds that the values of elastic and inelastic cross sections of π-p interaction will not change with the energy increase. As it became clear above the elastic cross section at these energies may be considered as the consequence of the inelastic one.

The increase or decrease of the inelastic interaction will be resulted in the increase or in the decrease of the elastic scattering, respectively, that will lead to the change of σ_t (see, e.g., formulae (1) and (6)).

If the above mentioned considerations are correct then at

$E_\pi \to \infty,\ \sigma_t \approx 30\,mb,\ \sigma_i \approx 24\,mb$

and $\sigma_e \approx (6 \div 7)\,mb$.

It is known for the nucleon-nucleon interaction that $\sigma_i = (21 \pm 4)\,mb$ if $E_N = 50$ Bev. It means that the inelastic cross section changes slowly with energy. As the above performed consideration has shown the elastic cross section of p-p interaction at $E_p = 6{,}15$ Bev may be interpreted as the consequence of the inelastic scattering. If the inelastic cross section of the interaction changes slowly or remains constant with the energy the increase of the elastic cross section will be also approximately constant. Then, starting from the considerations set forth above one may expect that if $E_p \to \infty$ $\sigma_i \approx 24\,mb$ and $\sigma_e \approx 7\,mb$ at $\sigma_t \approx (30 \div 31)\,mb$.

Thus, we arrive at the conclusion that at high energies of the colliding particles the total elastic and inelastic cross sections of pion and nucleon interactions with nucleons have identical values.

We are grateful to Isaeva L.A. and Shustrova L.A. for their assistance in the numerical calculations.

E_p BeV	$\lambda\!\!\!^{-} \times 10^{-14}$ cm	σ_t mb	σ_e mb	σ_i mb
1,5	2,35	$47,2 \pm 0,9$ [11]	20 ± 2 [2]	27 ± 2 [2]
2,24	1,92	$44,1 \pm 4,$ [8]	$17, \pm 3$ [4]	26 ± 2 [8]
2,75	1,73	$41 \pm I$ [11]	15 ± 2 [2]	26 ± 2 [2]
4,40	1,37	34 ± 2 [8]	$9,7 \pm 1,5$ [4]	$24,2 \pm 2$ [8]
6,15	1,16	$31,3 \pm 1,5$ [8]	$7,5 \pm 1,5$ [4]	$23,8 \pm 2$ [8]
E_π BeV				
1,37	2,7	$30,6 \pm 2,8$ [10]	$6,6 \pm 1$ [3]	$24 \pm 1,5$ [3]

T a b l e II

E_p BeV	1,5	2,24	2,75	4,4	6,15	E_π(BeV) 1,37
$R_{min} \times 10^{-13}$ cm	0,90	0,89	0,90	0,92	0,95	1,01

T a b l e III

E_p (BeV)	1,5	2,24	2,75	4,4	6,15
β_0(S-Wave)	$-0,46 \pm 0,13$	$-0,52 \pm 0,13$	$-0,39 \pm 0,12$	$-0,11 \pm 0,10$	$+0,06 \pm 0,08$

T a b l e IV

E_p (BeV)	K (10^{13} cm^{-1})	$\frac{\lvert Re\, f(\vartheta)\rvert^2}{\lvert f(\vartheta)\rvert^2}$ % R = 1,05 x 10^{-13} cm	R=1,1x10^{-13} cm	R=1,15x10^{-13} cm
1,5	0,64 + 2,6	6 + 21	12 + 27	20 + 35
2,24	0,60 + 2,1	8 + 23	15 + 30	22 + 35
2,75	0,60 + 2,0	5 + 22	9 + 28	15 + 35
4,4	0,53 + 1,3	0 + 21	0 + 29	5 + 35
6,15	0,51 + 1,0	0 + 16	0 + 23	0 + 30
E_π 1,37	0,53 + 1,0	0 ~ 7	~ 7	~ 7

Fig. 1 Solid curves show the region of possible angular distributions of elastic p-p scattering in the centre-of mass-system for the values of the radius a) $R = 1{,}3 \times 10^{-13}$ cm and b) $R = 1{,}2 \times 10^{-13}$ cm. The dashed curves show the angular distributions calculated for the purely absorbing sphere of the radius $R = 0{,}93 \times 10^{-13}$ cm for both energies. The experimental angular distributions are presented in the histogram[2].

Fig. 2 The solid curves show the region of the possible angular distributions of elastic p-p scattering in the centre-of-mass-system for $R = 1{,}10 \times 10^{-13}$ cm, the dashed curves determine the region of angular distributions for $R = 1{,}20 \times 10^{-13}$ cm. Experimental points are taken from [4].

Fig. 3 Solid curves show the region of possible angular distributions of elastic p-p scattering in the centre-of-mass-system calculated with the help of the purely absorbing sphere model starting from the following data:

a) $\sigma_i = 24{,}2$ mb $\sigma_d = (9{,}7 \pm 1{,}5)$ mb $R = (0{,}97 + 1{,}05) \times 10^{-13}$ c

b) $\sigma_i = 23{,}8$ mb $\sigma_d = (7{,}5 \pm 1{,}50)$ mb $R = (1{,}0 + 1{,}13) \times 10^{-13}$ cm

The dashed curves show the angular distributions calculated for the case of a purely absorbing sphere with the minimum radius:

a) $R = 0{,}92 \times 10^{-13}$ cm.

b) $R = 0{,}95 \times 10^{-13}$ cm.

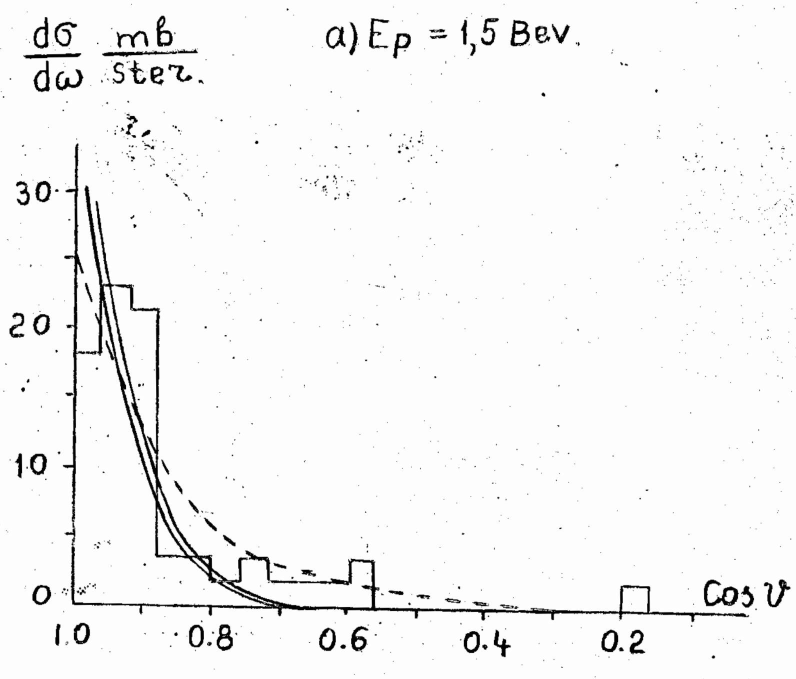

dσ/dω mb/ster.
a) Ep = 1,5 Bev.
30
20
10
0
1.0
0.8
0.6
0.4
0.2
Cos ϑ

Fig. 1a

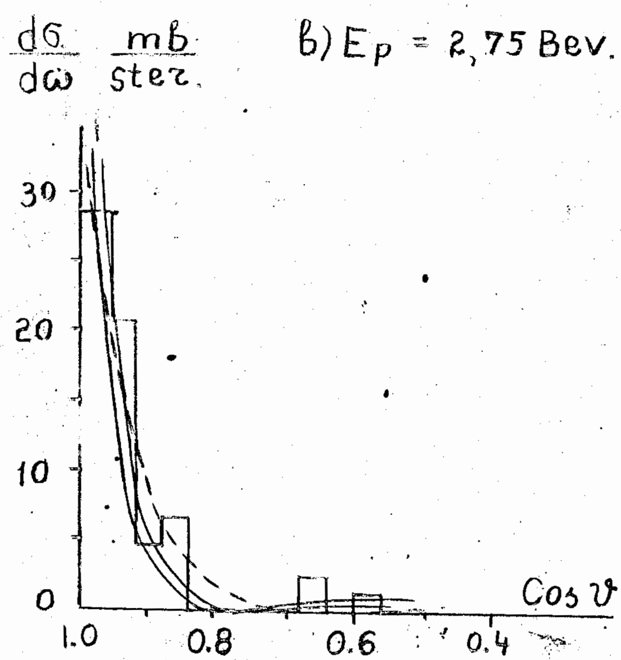

dσ/dω mb/ster.
b) Ep = 2,75 Bev.
30
20
10
0
1.0
0.8
0.6
0.4
Cos ϑ

Fig. 1b

$\frac{d\sigma}{d\omega}$ $\frac{mb}{ster}$ a) $E_p = 2,24$ Bev

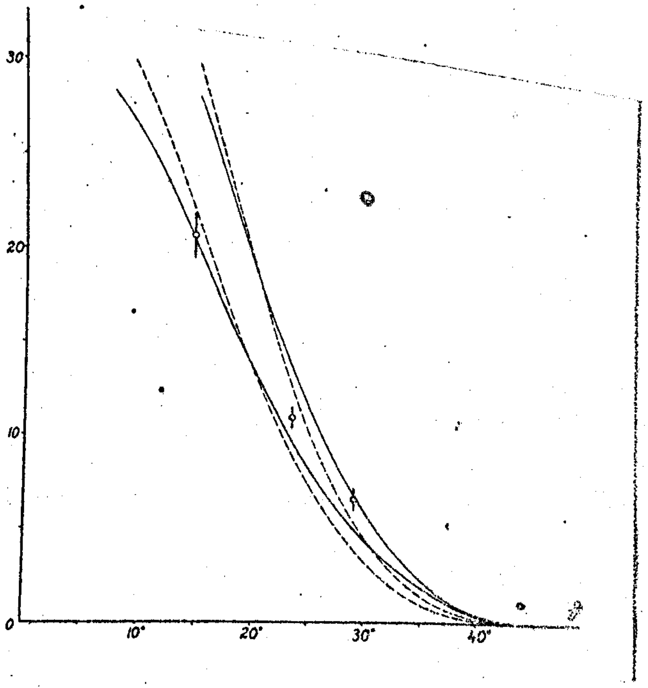


Fig 2a

$\frac{d\sigma}{d\omega}$ $\frac{mb}{ster}$ b) $E_p = 4,40$ Bev

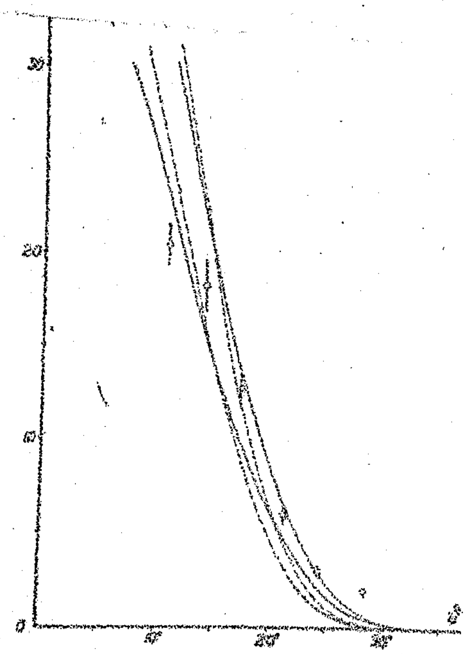


Fig 2b

$\frac{d\sigma}{d\omega}$ $\frac{mb}{ster}$ c) $E_p = 6,15$ Bev

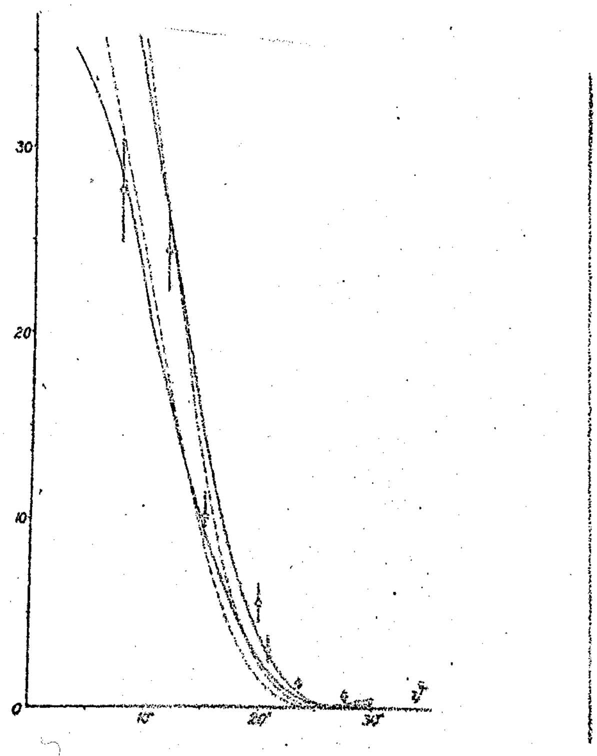
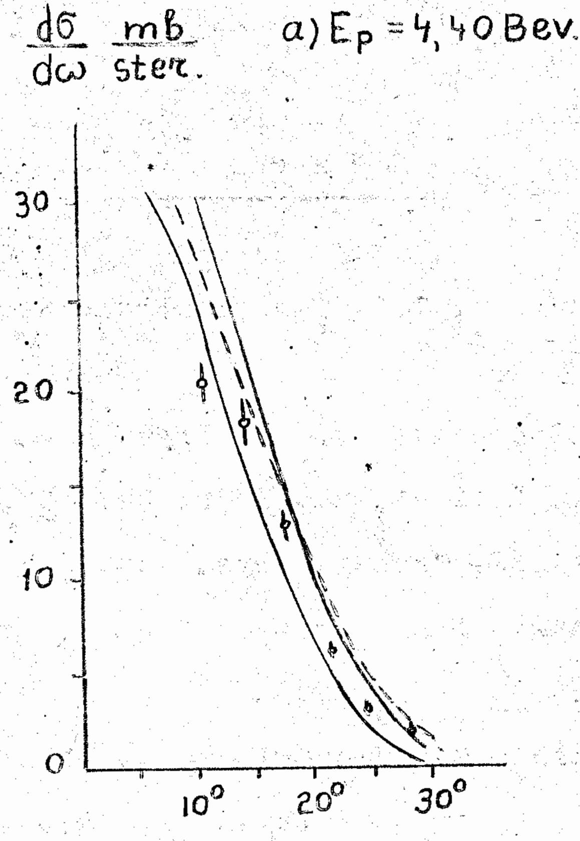
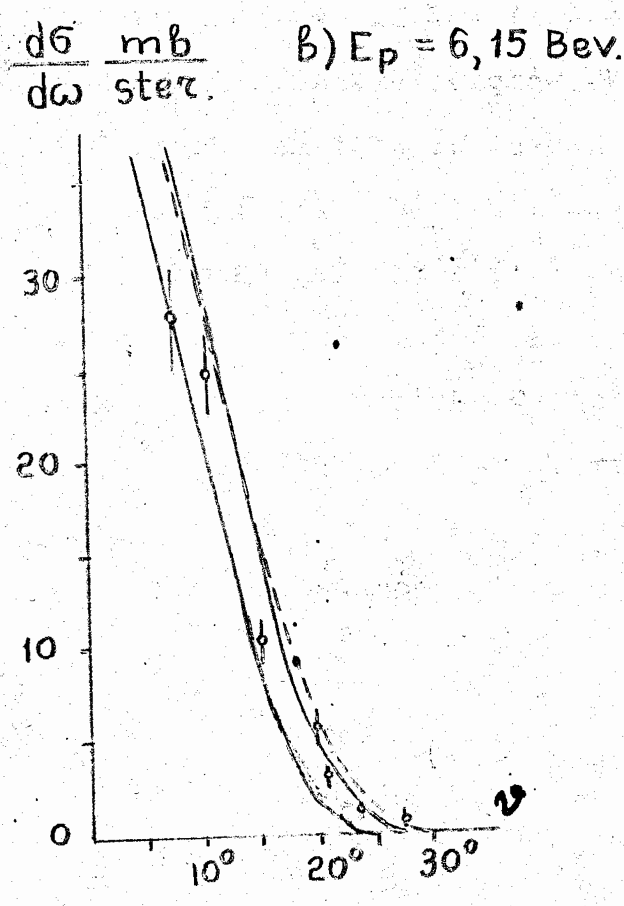


Fig 2c



$\frac{d\sigma}{d\omega}$ $\frac{mb}{ster.}$
a) $E_p = 4,40$ Bev.
30
20
10
0
10°
20°
30°

Fig. 3a



$\frac{d\sigma}{d\omega}$ $\frac{mb}{ster.}$
b) $E_p = 6,15$ Bev.
30
20
10
0
10°
20°
30°
ϑ

Fig. 3b

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