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ON THE SPIN OF THE Λ°-PARTICLE

by

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On the spin of the Λ^0-particle

Some general formulae relating to the determination of the spin of the Λ^0-particle by angular distribution of its decay products, which have been obtained by the method [1,2], are given in this theoretical note.

The spin state of an assembly of Λ^0-particles is described by giving values of tensor moments T_ν^q (see for definition [1]), which enables us to describe any (the most general) spin state of the assembly.

The angular distribution of decay products of the Λ^0-particle ($\Lambda^0 \to p + \pi^-$) may be written as follows

$$\mathcal{F}(\vartheta,\varphi) = \frac{w}{2\sqrt{\pi}} \sum_{q=0,2}^{2S-1} (2q+1)^{-1/2} Q(S,q) \cdot \sum_{\nu=-q}^{q} (-1)^\nu \cdot Y_{q,-\nu}(\vartheta,\varphi) \cdot T_\nu^q \qquad (1)$$

where w is the complete decay probability of the Λ^0 by the scheme $\Lambda^0 \to p + \pi^-$; S is the Λ^0 spin.

$$Q(S,q) = (-1)^{S-1/2-q/2} \cdot (2S+1)^{-1/2} \cdot \Xi(\ell' S \ell' S;\, 1/2\, q)$$

The coefficients Ξ are tabulated in [3]. It may be shown that the $\Xi(\ell' S \ell' S;\, 1/2\, q) = (-1)^{q/2}(2\ell'+1)(2S+1)\, W(\ell' S \ell' S;\, 1/2\, q) \cdot C^{q0}_{\ell' 0 \ell' 0}$ does not depend upon ℓ' (equal to $S + 1/2$ or $S - 1/2$).

$\mathcal{F}(\vartheta,\varphi)$ does not depend therefore upon the parity of the Λ^0-particle. Formula (1) is written in the Λ^0 rest system; the axis Ξ of the system is assumed henceforth to be parallel to the unit vector $\vec{n}_\Lambda$ which is directed along the Λ^0-particle movement in the laboratory system (formula (1) is, of course, true at any choice of the axis $\overset{\circ}{z}$).

By integrating $\mathcal{F}(\vartheta,\varphi)$ with respect to cos ϑ we may obtain from (1) a distribution in an angle φ . The angle $\varphi\,(0\leq\varphi\leq 2\pi)$ may be defined as one between the normal $\vec{N}$ to the plane of the reaction in which the Λ^0-particle has been produced (more exactly, $\vec{N}=[\vec{n}_0\times\vec{n}_\Lambda]$, where $\vec{n}_0$ is the unit vector of the incident particles of this reaction) and the unit vector $\vec{n}=[\vec{n}_p\times\vec{n}_\Lambda]$, $\vec{n}_p$ is the direction of the movement of the decay proton.

$$\mathcal{F}(\varphi)=\frac{w}{4\pi}\cdot T_0^0\left\{1+\sqrt{2}\sum_{m=2}^{2S-1}\sum_{q=m}^{2S-1}\left[\cos m\varphi\cdot(\operatorname{Re}t_m^q)+\sin m\varphi\cdot(\operatorname{Im}t_m^q)\right]Q(S,q)\mathcal{I}_{qm}\right\}$$

$$\mathcal{I}_{qm}=\left[\frac{(q+m)!}{(q-m)!}\cdot\frac{2q+1}{2}\right]^{1/2}\cdot\frac{m}{2}\cdot 2^{(1-m/2)}\cdot\frac{(q/2-1)!\,(q-m-1)!!}{(q/2+m/2)!\,(q+1)!!} \qquad (2)$$

m and q take only even values, $t_m^q=(2q+1)^{-1/2}\cdot T_m^q/T_0^0$.

Unlike the analagous formula in [4,5] , formula (2) contains an expression of the coefficients A_M and B_M (see [4,5]) through characteristics of the initial spin state of the Λ^0-particle.

If T_m^q with even q are real, i.e. if $\operatorname{Im}t_m^q=0$ (as is shown in [2] , this will happen, for instance, when Λ^0 is created in a reaction whose incident particles and target particles are entirely unpolarized) $\mathcal{F}(\varphi)$ will contain only terms with $\cos m\varphi$. The same expression (no matter whether T_m^q are complex of real numbers) will hold for the distribution in an angle η between the plane of Λ^0 -creation and its decay plane. (cf. [5] formula (2)). By appropriate integration we may also obtain from (1) the distribution $\mathcal{F}(\theta)$ which is a distribution over the number of particles emitted in a unit solid angle at an angle θ to the axis z ($\parallel \vec{n}_\Lambda$), averaged over all azimuths φ (see [6]):

$$\mathcal{F}(\theta) = \frac{w}{4\pi} \cdot T_0^0 \left\{ 1 + \sum_{q=2,4\ldots}^{2S-1} Q(S,q) P_q(\cos\theta) \cdot T_0^q / T_0^0 \right\} \qquad (3)$$

When comparing (2) and (3), we see that $\mathcal{F}(\varphi)$ (or $\mathcal{F}(\eta)$) is determined by the tensor moments T_m^q $q = 2, 4 \ldots 2S-1$, $m = 2, 4 \ldots q$ while $\mathcal{F}(\theta)$ depends upon quite other tensor moments: T_0^q. $T_0^{\prime q}$ ~~and~~ $\cancel{T_0^q}$ and T_m^q are not completely independent characteristics of the Λ^0 spin state. But if T_0^q or T_m^q do not take their maximum values, there may be rather an arbitrary (within certain limits) distribution in an angle η when $\mathcal{F}(\theta)$ is fixed (cf. [7] formulae (17) and (18)). Therefore, the distributions in η and $\cos\theta$ which were given in [6], do not contradict each other, which has already been emphasized in [7] where the cases $S = 3/2$ and $S = 5/2$ have been discussed. These distributions indicate, perhaps (poor statistics !), that Λ^0-particles observed are not completely polarized perpendicularly to the plane of the reaction $\pi^- + p \to \Lambda^0 + \theta^0$ (cf. footnote **).

The observed cases of reactions $\pi^- + p \to \Lambda^0 + \theta^0$, $\Lambda^0 \to p + \pi^-$ belong to energies 1 and 1,5 BeV. Increasing statistics at these energies, we may chiefly hope to define more accurately only the lower limit of the Λ^0 spin value. Let us show that the measurement of the angular distribution of the decay products of the Λ^0 which is created at the threshold of the reaction $\pi^- + p \to \Lambda + \Theta^0$ (~ 755 MeV) will just give the Λ^0 spin value itself if two natural assumptions are made, namely:

1) Θ^0-particle spin is zero (which does not contradict the available data, at least) and 2) forces between Λ^0 and Θ^0 are short-ranged.

The latter implies that we may neglect all elements of R-matrix $(i_\Lambda i_\Theta S' \ell' \alpha' | R^{\mathcal{JE}} | \tfrac{1}{2} 0 \tfrac{1}{2} \ell \alpha)$ with $\ell' > 0$ when compared to the element with $\ell' = 0$ near the threshold* (see for notation [1], the

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* Angular distribution of Λ^0, Θ at 1 BeV indicates that at least $\ell' = 2$ is present. [6] Hence the range of forces $\Lambda^0 - \Theta^0$ is equal to $\sim 2 \cdot 10^{-13}$ and we are justified in neglecting the above elements in the energy interval of an incident π-mesons from 755 MeV to 780-800 MeV in the laboratory system. It is possible, of course, that the element $(i_\Lambda 0 i_\Lambda 0 \alpha' | R^{\mathcal{JE}} | \tfrac{1}{2} 0 \tfrac{1}{2} \ell \alpha)$ is small owing to some peculiarities of the mentioned reaction. In this case the angular distribution of Λ^0, Θ^0 will be non-isotropic.

matrix R is connected with the known S-matrix in the following way $R = S - 1$).

With these assumptions made, let us re-write formulae (7), (8), (9) in [1] (the target is considered to be unpolarized). The Λ^0-particle tensor moments will be then expressed as the sum over $J_1, J_2, \ell_1, \ell_2, J$ (see for notation [1]). Using properties of the coefficients G_{τ_Λ} and G_0 in this expression, we obtain that $J_1 = J_2 = i_\Lambda$ and $J = q_\Lambda$. Further, $\ell_1, \ell_2 = i_\Lambda \pm 1/2$ and since space parity is conserved and $\ell_1 + \ell_2$ must be even, $\ell_1 = \ell_2 = \ell = i_\Lambda + 1/2$ or $\ell_1 = \ell_2 = \ell = i_\Lambda - 1/2$ depending on the parity of the Λ^0 with respect to the proton (note that θ^0-parity is equal to (parity π $)^2 = +1$). So there is no summation over $J_1, J_2, \ell_1, \ell_2, J$ at all. Denoting the Λ^0 spin by S instead of i_Λ and taking off the indices Λ from q and τ, we have for the Λ^0 tensor moments with even q:

$$T^{q0}_{\tau 0}(\vec{n}_\Lambda, P_\Lambda) = A \cdot (2S+1)(2q+1)^{-1/2} \cdot Q(S, q) \cdot Y_{q\tau}(\vartheta_\Lambda, \pi) \tag{4}$$

A is a constant proportional to the total cross section of the reaction $\pi^- + p \to \Lambda^0 + \theta^0$ at $E_\pi \cong 755 \div 780$ MeV. It must be kept in mind that the indices τ are related to $\vec{n}_\Lambda$ as quantization axis and ϑ_Λ is the angle between the direction of the emergence of Λ^0-particles and the π-meson beam.

Substituting concrete expressions (4) for the Λ^0 tensor moments into (2) and (3), we shall obtain distributions in an angles η and θ which will depend only upon the Λ^0-particle spin (and upon ϑ_Λ).

We shall not give here these general formulae**. It is interesting to note, however, that, if these formulae are integrated with respect to ϑ_Λ, the distribution in an angle θ will be isotropic whereas the distribution in an angle η (or φ) will be the following:

$$I_S(\eta) = C\left\{1 + \sum_{m=2}^{2S-1} \cos m\eta \cdot \sum_{q=m}^{2S-1} \left[Q(S,q)\cdot \mathcal{Y}_{qm}\right]^2 (2q+1)^{-1}\right\} \tag{5}$$

For example, $I_{3/2}(\eta) \sim 1 + 1/3 \cdot \cos 2\eta$.

If the tensor moments (4) are inserted in (1) -

$$\mathcal{F}(\vartheta,\varphi) = \frac{A w}{2\sqrt{\pi}} \sum_q (2q+1)^{-1} [Q(S,q)]^2 \sum_\nu Y^*_{q\nu}(\vartheta,\varphi)\cdot Y_{q,\nu}(\vartheta_\Lambda,\pi) = \tag{6}$$
$$= A w [8\pi\sqrt{\pi}]^{-1} \sum_{q=0}^{2S-1} [Q(S,q)]^2 \cdot P_q(\cos\gamma) \equiv \mathcal{F}_S(\gamma)$$

we shall obtain a general formula for the distribution in an angle γ. γ is the angle between the directions ϑ_Λ, π and ϑ, φ or, as may be shown, the directions of the flight of the decay proton and the incident π-meson beam. The formula $\mathcal{F}_{3/2}(\gamma) \sim 1 + 3\cos^2\gamma$ was first given in [8], $\mathcal{F}_{5/2}(\gamma) \sim 1 - 2\cos^2\gamma + 5\cos^4\gamma \sim 1 + 4/5 \cos 2\gamma + 1/3 \cos^4\gamma$

** When $S = 3/2$ we have $\mathcal{F}_{3/2}(\eta) \sim 1 + 0{,}5 \sin^2\vartheta_\Lambda \cdot \cos 2\eta$ and $\mathcal{F}_{3/2}(\theta) \sim 1 - (1 - 3\cos^2\vartheta_\Lambda)(5 - 3\cos^2\vartheta_\Lambda)^{-1} \cos^2\theta$
At an angle $\vartheta_\Lambda = 90°$ we have $\mathcal{F}_{3/2}(\eta) \sim 1 + 0{,}5 \cos 2\eta$ while $\mathcal{F}_{3/2}(\theta) \sim 1 - 0{,}2 \cos^2\theta$ i.e. probability does not increase at all at $\cos\theta = \pm 1$ though the distribution in an angle η is the same as that from completely polarized Λ^0 particles.

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