J O I N T   I N S T I T U T E   F O R   N U C L E A R   R E S E A R C H

Laboratory for Theoretical Physics

P 109

L.I. Lapidus

ЖЭТФ, 1958, т34, в5, с1148-1153.

"POLARISATION IN ELASTIC SCATTERING AT HIGH ENERGIES"

1 9 5 7

JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory for Theoretical Physics

P 109

L.I. Lapidus

"POLARISATION IN ELASTIC SCATTERING AT HIGH ENERGIES"

1957

A b s t r a c t

Polarisation phenomena in elastic scattering at high energies are considered. It is shown that it is possible to obtain nucleon beams with considerable polarisation in the elastic scattering at small angles. The applicability of "black nucleus" and "gray absorbing nucleus" approximation to p-p scattering at high energies is discussed.

I. This paper deals with the problem of polarisation phenomena at high energies. An attempt has been made to clear up what peculiarities the polarization phenomena in the elastic scattering possess in the approximation when the "diffraction" expressions arise for the scattering cross section averaged over spins and what data may be obtained from the results of the polarization experiments at high energies when the elastic scattering to a great extent is due to inelastic processes.

Let us consider first the elastic scattering of particles with spin 1/2 on the spinless particles/event (0,1/2)/.

In the most of the published papers* different suppositions are made about the radial potential dependence after introducing the effective potential and the results of the comparison with experimental data are discussed from the standpoint of determining the effective potential parameters /see e.g. [3] /.

2. We shall do it in another way. Determine the absorbing coefficients K_1 and K_2 and the refraction indeces n_1 and n_2 by analogy with the spinless event (see, e.g. [4,5]), without introducing the interaction potentials. The quantities thus introduced are connected with the four functions V_{CI}, V_{SI}, V_{CR}, V_{SR} from [1] . The quantity K_I is, for instance, proportional to

---

* There is a lot of papers in which the polarization phenomena in the scattering on nuclei are considered using the optical model. We only mention Resenfeld's and Watson's[I] and Brown's[2] papers where the references to other papers may be found. The author takes the opportunity to thank Dr. Brown for sending some results unpublished yet.

the imaginary part whereas (n_2-1) is proportional to the real part of the mean nucleon-nucleon scattering amplitude at $\vartheta = 0^\circ$. In the notations of [1] K_I and $(n_1 - 1)\bar{K}$ are proportional to the imaginary and real parts of the quantity :

$$\bar{M}_0 = \frac{1}{8}\left\{\left[B+N+G\right]_{pp} + \left[B+N+G\right]_{np}\right\}_{\theta=0^\circ} \qquad (1)$$

and K_2 and $(n_2-1)k$,

are analogously proportional to the imaginary and real parts of

$$\bar{M}_1 = -\frac{1}{2i}\left[\frac{1}{\sin\theta}(C_{pp}+C_{np})\right]_{\theta=0^\circ} = -\frac{1}{2i}\left[\frac{d}{d\cos\theta}(C_{pp}+C_{np})\right]_{\theta=0^\circ} \qquad (2)$$

If one makes use of the usual transition from the Legendre polinomials $P_\ell(\cos\theta)$ to the Bessel functions $J_0(\ell\theta)$ then for the coefficients $A(\theta)$ and $B(\theta)$ of the amplitude

$$M = A(\theta) + B(\theta)(\vec{\sigma}\vec{n}) \quad , \quad \vec{n} = \frac{[\vec{k}_0\vec{k}]}{|[\vec{k}_0\vec{k}]|} \qquad (3)$$

one obtains

$$2A(\theta) = ik\int_0^R b\,db\,J_0(kb\theta)\left\{2 - e^{-[k_1-2i(n_1-1)]s}\left[e^{-[k_2-2i(n_2-1)]s} + e^{[k_2-2i(n_2-1)]s}\right]\right\}$$

$$2B(\theta) = -k\int_0^R b\,db\,J_1(kb\theta)\,e^{-[k_1-2i(n_1-1)]s}\left\{e^{-[k_2-2i(n_2-1)]s} - e^{[k_2-2i(n_2-1)]s}\right\} \qquad (4)$$

where

$$s^2 = R^2 - b^2 = R^2 - \ell^2\lambda^2$$

other notations are evident, and

$$\mathcal{K}_1 > \mathcal{K}_2 \geqslant 0 . \qquad (5)$$

The conditions (5) substitute the requirement that the sign of the imaginary part of the scattering phase shifts in the spinless event would be definite.

3. In the infinite absorption ("black nucleus") $K_I \longrightarrow \infty$ and

$$A(\theta) = ik\int_0^R b\,db\,J_0(kb\theta) = \frac{iR}{\theta}J_1(kR\theta);\ B=0;\ \sigma_t = \frac{4\pi}{k}\,Jm\,A(0^\circ) = 2\pi R^2,\ \sigma_s = \pi R^2 \qquad (6)$$

and the polarization in the elastic scattering of the unpolarized beam P_{uu} is vanishing. In virtue of time reversal invariance the elastic scattering cross section of the polarized beam does not involve any azimuthal asymmetry and is consistent with the cross section $I_0(\theta)$ of the unpolarized beam scattering.

For the polarization of P_{pu} [6] after the scattering of the polarized (P^{in}) beam we obtain

$$I_0(\theta)\vec{P}_{pu} = P^{in}|A(\theta)|^2[\vec{n}\,\vec{K}_0];\ \vec{P}_{pu} = P^{in}[\vec{n}\,\vec{K}_0] \qquad (7)$$

if one chooses P^{in} to be perpendicular to $\vec{K}_0$ and to the normal $\vec{n}$. It follows from (7) that in the black nucleus approximation the polarization rotation does not occur. Thus, the "black nucleus" is characterized by the absence of any polarization effects. Therefore, the observation at high energies of the polarization P_{uu} different from zero may serve as a good "indicator" of the nonapplicability of the "black nucleus" concepts. Note right away that it is difficult to settle this question by studying only the unpolarized cross sections.

4. In the absence of refraction, i.e., if $n_1 = n_2 = 1$ ("grey absorbing nucleus")

$$A(\theta) = ik\int_0^R \rho d\rho\, J_0(k\rho\theta)(1-e^{-k_1 s}\,\mathrm{ch}\,k_2 s)$$

$$B(\theta) = k\int_0^R \rho d\rho\, J_1(k\rho\theta)\,\mathrm{sh}\,k_2 s \tag{8}$$

and

$$\sigma_t = \frac{4\pi}{k}\mathrm{Im}\,A(0^\circ) - 2\pi R^2\Big\{1 - \frac{1}{[(\varkappa_1-\varkappa_2)R]^2}\Big[1-e^{-(\varkappa_1-\varkappa_2)R}(1+\varkappa_1 R-\varkappa_2 R)\Big] - $$
$$ - \frac{1}{[(\varkappa_1+\varkappa_2)R]^2}\Big[1-e^{-(\varkappa_1+\varkappa_2)R}(1+\varkappa_1 R+\varkappa_2 R)\Big]\Big\}. \tag{9}$$

The expression for the differential elastic scattering cross section

$$I_0(\theta) = |A(\theta)|^2 + |B(\theta)|^2 \tag{10}$$

follows from (8). Integrating over (10) one may obtain for the total elastic scattering cross section

$$\sigma_s = \pi R^2\Big\{1 - \frac{1}{2(\varkappa_1 R)^2}\Big[1-e^{-2\varkappa_1 R}(1+2\varkappa_1 R)\Big] - \frac{2}{(\varkappa_1-\varkappa_2)^2R^2}\Big[1- \tag{11}$$
$$ - e^{-(\varkappa_1-\varkappa_2)R}(1+\varkappa_1 R-\varkappa_2 R)\Big] - \frac{2}{(\varkappa_1+\varkappa_2)^2R^2}\Big[1-e^{-(\varkappa_1+\varkappa_2)R}(1+\varkappa_1 R+\varkappa_2 R)\Big]\Big\}.$$

The expression for the total cross section of inelastic processes σ_c is obtained as the difference between (9) and (II).

Let us note when passing to the polarization phenomena that $A(\theta)$ is a purely imaginary quantity and $B(\theta)$ is a real one, i.e.,

$$A^+(\theta) = -A(\theta)\,,\quad B^+(\theta) = B(\theta) \equiv b \tag{12}$$

The expression for the polarization P_{uu}

$$I_0(\theta)P_{uu}(\theta)=A^{+}(\theta)B(\theta)+A(\theta)B^{+}(\theta) \tag{13}$$

vanishes even if A and B are different from zero. Since the azimuthal asymmetry in the cross section of the polarized beam scattering is proportional to the right-hand side of (13) it is also absent in this energy region where the scattering phase shifts are purely imaginary.

For the polarization P_{pu} after the polarized beam scattering we get

$$I_0(\theta)\vec{P}_{pu}=P^{in}\left\{(a_0^2-b^2)[\vec{n}\cdot\vec{k}_0]-2a_0B\vec{k}_0\right\} \tag{14}$$

instead of (7). ($A = ia_0$). The quantities A and B are given in (8).

Note that the vanishing of the polarization P_{uu} when $A(\theta)$ and $B(\theta)$ are different from zero is known to take place also in the Bohrn approximation (for real potentials) as is seen from Expressions for $A(\theta)$ and $B(\theta)$ through the phase shifts. When real phase shifts are small

$$A_B^{+}(\theta)=A_B(\theta),\quad B_B^{+}(\theta)=-B_B(\theta).$$

The polarization (even when potentials are real) becomes different from zero if the expressions for A and B obtained in the Bohrn approximation would be made exactly unitary.

5. It can be seen from the consideration of the extreme cases that the presence of the polarization P_{uu} is interrelated with the difference of the refraction indeces and n_2 from unity. The maximum polarization will be when $n_1 = 1$, $K_2 = 0$. In this case

$$A(\theta)=ik\int_0^R b\,db\,J_0(kb\theta)\left\{1-e^{-\mathcal{K}_1 S}\cos[2(n_2-1)kS]\right\}=-A^+(\theta)=ia_0$$
$$B(\theta)=-ik\int_0^R b\,db\,J(kb\theta)e^{-\mathcal{K}_1 S}\sin[2(n_2-1)kS]=-B^+=ib_0\,. \qquad (15)$$

and the polarization reaches 100% at the small angle when $|a_0|=|b_0|$. The possible existence of such a point in the case under consideration follows from the fact that in the region of small angles $A(\theta)$ is a decreasing angle function and $B(\theta)$ is an increasing one. The intersection $a_0 = b_0$ is made near the first diffraction minimum.

As Brown's analysis has shown it is this case which occurs in the proton interaction with carbon at the energy of about 1 billion electron-volts. To describe the scattering two parameters K_1 and n_2 are necessary here (if the radius R is known/ and the study of the polarization P_{pu} may appear to be a verification of the accepted interpretation.

In this case we get instead of (14)

$$I(\theta)\vec{P}_{pu}(\theta)=P^{in}(a_0^2-b_0^2)[\vec{n}\vec{K}_0]+P_{uu}\vec{n} \qquad (16)$$

The polarization experiments necessary for the determination of the effective potential parameters if K_1, K_2, n_2-1, n_1-1/V_{CI} V_{SI} , V_{SK} , $V/_{SK}$ are different from zero were discussed by Riesenfeld and Watson [1].

In the present discussion the electromagnetic effects were neglected. As the consideration of the spinless event [4]. However, shows it is essential in some cases to take into account the electromagnetic interaction.

The change of the phase of the amplitude that will produce an effect upon the quite small angles is especially important for the polarization P_{uu}. The expression for the amplitude which takes the magnetic momentum into account and is obtained in the Bohrn approximation [7,8] has the following shortcoming. The expression for P_{uu} does not vanish if $\theta \longrightarrow 0^0$. The investigation of the electromagnetic effects in the polarization may appear to be extremely interesting for the study of the nucleon electromagnetic properties, (the relaxation of magnetic moments).

6. The elastic scattering of particles with spin 1/2 was considered above at the spinless target (event 0,1/2). The quantitative results concerning P_{uu} hold for the events (1/2,1/2) and (1,0). (nucleon-nucleon scattering and deutron scattering on spinless nuclei). Let us present the amplitude M for the nucleon-nucleon scattering in the form

$$M = BS + C(\vec{\sigma}_1 + \vec{\sigma}_2, \vec{n}) + \Big\{ \frac{1}{2} G\left[(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta}) + (\vec{\sigma}_1\vec{\pi})(\vec{\sigma}_2\vec{\pi})\right] \tag{17}$$

$$+ \frac{1}{2} H\left[(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta}) - (\vec{\sigma}_1\vec{\pi})(\vec{\sigma}_2\vec{\pi})\right] + N(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})\Big\} T .$$

Here
$$S = \frac{1}{4}(1 - \vec{\sigma}_1\vec{\sigma}_2) , \quad T = \frac{1}{4}(3 + \vec{\sigma}_1\vec{\sigma}_2)$$

singlet and triplet projection operators, and

$$\vec{\pi} = \frac{\vec{k}_0 + \vec{k}}{|\vec{k}_0 + \vec{k}|} , \quad \vec{\Delta} = \frac{\vec{k}_0 - \vec{k}}{|\vec{k}_0 - \vec{k}|} .$$

It can be seen from the expressions for the coefficients B,C....., obtained by Wright [9] that when phase shifts are imaginary B,G,H and N are imaginary quantities whereas C is a ~~n~~ real one. Let us expecially note that it is this case for $\overline{M}_0$ and for $\overline{M}_1$ in (1) and (2) that leads to the maximum polarization in nucleon scattering on nuclei.

When the phase shifts are imaginary in N-N scattering the polarization P_{uu} vanishes whereas the scattering cross section of the polarized beam with the unpolarized target does not involve azimuthal asymmetry. The study of the polarization P_{uu} may, therefore, appear to be a good means to verify whether the diffractional approach with pure imaginary phase shifts to the analysis of the experimental data is correct. Some other polarization effects (including the correlation of polarization) are different from zero whereas the correlation of polarization when the beam is polarized is not different from the case of the unpolarized beam on the unpolarized target.

The addition to the correlation

$$I_0(\theta)P_{1pq} = \frac{1}{4} S_p M \sigma_{1l} M^+ \sigma_{1p} \sigma_{2q} .$$

vanishes.

The number of independent experiments when phase shifts are imaginary coincides with that of terms in the amplitude, thus the coefficient phases become equal to 0 and $\pi/2$.

In order to get an idea how it is for elastic scattering of particles with higher spin let us consider in brief the case

(1,0) restricting ourselves again by imaginary phase shifts. The amplitude M for this case coincides, in fact, with M for the triplet neutron-proton scattering, i.e., the expression for M in this case in obtained if we put B = 0, T = 1 in (17) and in the remaining expressions we substitute $(\vec{\sigma}_1 \vec{a})(\vec{\sigma}_2 \vec{a})$ for $2(\vec{S}\vec{a})^2 - 1$ where S is the operator of spin 1. Then, as can be seen from 9/ and 10/, the polarization P_ω also vanishes whereas the mean values of the tensors

$$\hat{D}_{ik} = \frac{1}{2}(S_i S_k + S_k S_i) - \frac{2}{3}\delta_{ik}$$

which together with S_i characterize the state of polarization are different from zero. The cross section of the polarized beam scattering will, generally speaking, involve the terms, proportional to $\cos\varphi$ and $\cos 2\varphi$ but the term involving $\cos\varphi$ in proportional only to the mean value

$$T_{2,1} = -\frac{\sqrt{3}}{2}\left\{(S_x S_z + S_z S_x) + i(S_z S_y + S_y S_z)\right\} = -\sqrt{3}\left\{\hat{D}_{xz} + i D_{zy}\right\}.$$

7. When discussing the supposed experiments with nucleon beams at the energies of some BeV the nucleon elastic scattering by nucleons and nuclei is sometimes considered to be consistent with the simple diffraction picture on the "black nucleus" while the polarization phenomena will be absent. This is supported by a good agreement of the available experimental data on p-p spattering and nucleon scattering by nuclei with simple formulae for the cross sections in the approximation of the "black nucleus" or "grey absorbing nucleus" with n = 1, although the values

of the so obtained optical parameters in the energy region about 1 BeV make the authors be surprised at this good agreement.

Some objections against the applicability of similar consideration for p-p scattering in the energy region about 1 BeV are given by Rarita II/.

The results of the discussion given in this paper indicate that at high energies when the elastic scattering is due to a great extent to inelastic processes it may be possible to obtain the nucleon beams with the considerable polarization. The presence of such beams makes it possible to arrange additional experiments. The polarization experiments present a effective means of studying the spin effects in the elastic scattering, the presence of which may not appear when studying the differential cross sections.

To realize the predictions of the "black nucleus" or "grey absorbing nucleus" approximation to the polarization phenomena is rather doubltful, since $(n_1-1)k$, $(n_2-1)k$ are proportional to the real part of the forward amplitude and its derivative by the angle when $\theta \to 0^{\circ}$, divided into the momentum k. Howver, even at extremely high energies these quantities, as if follows from the dispersion relations, are not equal to zero but tend to the constant values different from zero. In additions, the existence of the nucleon magnetic moment leads to the presence in amplitude (3) of the coefficient $B(\theta)$ with the imaginary part different from zero.

As for the agreement of the diffraction formulae for the cross sections with the experimental data in the energy region of about 1 BeV it seems to be due to a simple circumstance, which consists in the following. The main features of the elastic scattering at high energies (the scattering is stretched forward and concentrated in the small angle region are given by two simple inequalities 11, 12, 13: a)

$$I_0(0^\circ) \geqslant \left(\frac{k\sigma_t}{4\pi}\right)^2 \tag{18}$$

with the help of which one may obtain [12], that b)

$$\pi\theta^2 \leqslant \left(\frac{4\pi}{k\sigma_t}\right)^2 \sigma_s \tag{19}$$

for n-p scattering

$$I_0(\pi) \geqslant \left(\frac{k}{4\pi}\right)^2 \left(\sigma^t_{pp} - \sigma^t_{np}\right)^2 \tag{20}$$

is added to them.

Eqs. (18)-(20) are based upon the optical theorem, i.e., follow from the general property of the unitarity of the S-matrix, which is taken into account in the optical model as well. In the frame of the optical model (if n = 1) (18) turns out into the equality whereas the main region of the scattering angles

$$\theta^2 \leqslant \left(\frac{2}{kR}\right)^2$$

follows from (19). The latter circumtances seem to account for good consistence of the simple "diffraction" formulae with the experimental data on the elastic scattering at high energies.

The arguments analogous to those given by Okun' and Pomeranchuk [14] point out that the quantity $A(\theta)$ in amplitude (3) considerable exceeds $B(\theta)$ on the whole and, therefore, the latter may be neglected when considering the unpolarized cross sections. In spite of this, as the discussion in this paper shows the considerable polarization is possible in the angle region where $A(\theta)$ and $B(\theta)$ are found to be comparable.

Although the discussion was mainly concerned with the nucleon interaction with nucleons and nuclei it may, of course, be applied to the beams of other nuclei as well as to those of antinucleons, hyperons, electrons etc.

Note that the beams of antinucleons, hyperons and antihyperons turn out, generally speaking, to be partially polarized in the production processes. This creates the possibility of obtaining the data about the magnitude of the spin of the hyperons and antihyperons [15] when studying the dependece of the interaction cross section of these beams with the nucleons and nuclei upon the azimuthal angle φ.

To study the polarization $P_{\rho u}$ proves to be of interest. As is seen from (16) $P_{\rho u} P(\theta)$ coincides practically with P^{in} in the angle region where $a_0^2 \gg b_0^2$. The polarization rotation is maximum in the angle region where $|a_0| = |b_0|$. There P_{uu} is maximum and $P_{\rho u}$ given by the second term in (16) is rotated relative to P by $\pi/2$.

The author is grateful to S.M. Bilenky, I.I. Levitnov, R.M. Ryndin, J.A. Smorodinsky and L.M. Soroko for useful discussions and valuable advice.

L. Lapidus.

References

1. W.B. Riesenfeld and K.M. Watson Phys., 102, 1157, 1956.
2. G.E. Brown Proc. Phys. Soc., A 70, 361, 1957.
3. I.I. Levintov Doklady Akad. Nauk SSSR 107, 240, 1956.
4. A.I. Akhiezer, I.I. Pomeranchuk "Some Problems of Theory of a Nucleus".
5. R.M. Sternheimer Phys. Rev., 101, 384, 1956.
6. R. Oehme Phys. Rev., 98, 147, 1955.
7. J. Schwinger Phys. Rev., 73, 407, 1948.
8. J.T. Sample Canad. Journ. Phys., 34, 36, 1956.
9. S.C. Wright Phys. Rev., ~~98~~ 99, 996, 1955.
10. O.D. Cheishvili JETP, 30, 1147, 1956.
11. W. Rarita Phys. Rev., 104, 221, 1956.
12. L.I. Lapidus JETP, 31, 1099, 1956.
13. G. Wick Phys. Rev., 75, 1459 (A), 1949.
14. L.B. Okun', I.I. Pomeranchuk JETP, 30, 424, 1956.
15. L.I. Lapidus JETP, 31, 342, 1956.