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A B S T R A C T :

The polarization phenomena in the photoproduction and radiation capture of pions as well as in the Compton - effect on the nucleons are considered. The consequences of the invariance under time reversal are obtained. The Wolfenstein theorems are generalized for the case of the considered reactions with the participation of γ-rays.

I. Introduction

The progress in the experimental technique makes it possible to hope that the polarization experiments in such elementary reactions as the Compton - effect on the nucleons, the photo-pion production, the photo-deuteron disintegration will become possible in the nearest future. In connection with this the problem arises to consider the polarization phenomena in the reactions with γ-rays, to separate independent experiments, to establish a complete set of experiments necessary for the reconstruction of the reaction amplitude. When determining the number of independent experiments it is essential to take into account the symmetry under change of sign of time as well as the unitary conditions besides the invariance conditions under rotations and space reflections.

In the present paper the polarization phenomena in the pion-photoproduction and in the Compton-effect on the nucleons are considered. The interaction invariance under time reversal leads to the relations not only between the unpolarized (averaged over spins) cross sections but also between the polarization phenomena in the reverse reactions. Though it was shown |1-4| that the

Wolfenstein theorems are valid for the elastic scattering and for ... reactions in the phenomenological analysis the reactions with γ -rays require an especial consideration. In the present paper the Wolfenstein theorems are generalized for the case of such a reaction.

The unitarity of the S-matrix involving pion nucleon elastic scattering

$$-p \to -p; \quad on \to on; \quad +n \to +n; \quad op \to op$$

the charge exchange scattering $-p \rightleftharpoons on$, $+n \rightleftharpoons op$,

the photoproduction and the radiation capture of pions

$$+n \rightleftharpoons \gamma p, \quad -p \rightleftharpoons \gamma n, \quad on \rightleftharpoons \gamma n, \quad op \rightleftharpoons \gamma p$$

and the Compton - effect

$$\gamma p \to \gamma p; \quad \gamma n \to \gamma n$$

makes it possible to introduce three real phases and three mixing parameters into every state. This presents the possibility of determining the number of necessary experiments which becomes less if we take into account the isotopic invariance. The unitary condition is considered in the Appendix.

2. Photoproduction and Radiation Capture of Pions

To obtain the photoproduction amplitude we present it in the form

$$M = a + \vec{b}\vec{\sigma} \tag{1}$$

Since the amplitude M must be a pseudoscalar, the quantities

a and $\vec{b}$ must be a pseudoscalar and a vector constructed from the polarization vectors $\vec{e}, \vec{n}' = [\vec{q}'\vec{k}], \vec{\pi}' = \vec{q}' + \vec{k}'$ and $\vec{\Delta} = \vec{q}' - \vec{k}$ where $\vec{q}$ and $\vec{k}$ are the pion and photon momenta respectively. The construction of the photoproduction amplitude and the study of the time reversal invariance applied to the pion nucleon photoproduction has been performed in [5,6,7].

Using the gradient invariance the expression for M may be presented in the form:

$$M_{\pi\gamma} = A(\vec{\sigma}\vec{e}) + B(\vec{\sigma}\vec{q})(\vec{e}\vec{q}) + C(\vec{e}\vec{n}) + D(\vec{\sigma}\vec{k})(\vec{q}\vec{e}) \tag{2}$$

$$M_{\gamma\pi} = A'(\vec{\sigma}\vec{e}) + B'(\vec{\sigma}\vec{q})(\vec{e}\vec{q}) + C'(\vec{e}\vec{n}') + D'(\vec{\sigma}\vec{k})(\vec{q}\vec{e}). \tag{2'}$$

If ω_0 and ω_1 are the functions describing a nucleon and a nucleon with a meson respectively, then for instance

$$A = (\omega_1, D\omega_0), \quad A' = (\omega_0, D\omega_1),$$

the effect of the operator of time reversal K on the function ω_1 and ω_0 is reduced to

$$K\omega_0 = \omega_0, \quad K\omega_1 = -\omega_1,$$

It follows from the invariance under time reversal that

$$KDK^{-1} = D^+$$

and following Watson [5] it can be shown that

$$A' = -A$$

By analogy

$$B' = -B, \quad C' = C, \quad D' = -D.$$

Thus

$$M_{\pi\gamma} = A(\vec{\sigma}\vec{e}) + B(\vec{\sigma}\vec{q})(\vec{e}\vec{q}) + C(\vec{e}\vec{n}') + D(\vec{\sigma}\vec{k})(\vec{q}\vec{e}) = M$$

$$-M_{\gamma\pi} = A(\vec{\sigma}\vec{e}) + B(\vec{\sigma}\vec{q})(\vec{e}\vec{q}) - C(\vec{e}\vec{n}') + D(\vec{\sigma}\vec{k})(\vec{q}\vec{e}) = M'. \qquad (3)$$

In virtue of γ-quantum being transverse $(\vec{e}\vec{k}) = 0$

and

$$B(\vec{\sigma}\vec{q})(\vec{e}\vec{q}) + D(\vec{\sigma}\vec{k})(\vec{q}\vec{e}) = \delta'(\vec{\sigma}\vec{\pi}')(\vec{e}\vec{\pi}') + \gamma'(\vec{\sigma}\vec{\Delta}')(\vec{e}\vec{\Delta}').$$

If we pass to the orthonormalized vectors $\vec{\pi}$, $\vec{\Delta}$, and $\vec{n}$ and take into account the relation

$$(\vec{\sigma}\vec{e}) = (\vec{\sigma}\vec{n})(\vec{n}\vec{e}) + (\vec{\sigma}\vec{\pi})(\vec{\pi}\vec{e}) + (\vec{\sigma}\vec{\Delta})(\vec{\Delta}\vec{e}),$$

then finally

$$M = \alpha(\vec{e}\vec{n}) + \beta(\vec{\sigma}\vec{n})(\vec{e}\vec{n}) + \gamma(\vec{\sigma}\vec{\Delta})(\vec{e}\vec{\Delta}) + \delta(\vec{\sigma}\vec{\pi})(\vec{e}\vec{\pi})$$
$$B(\sigma q)(e q) + D(\sigma k)(q e) \qquad (4)$$

and

$$M' = -\alpha(\vec{e}\vec{n}) + \beta(\vec{\sigma}\vec{n})(\vec{e}\vec{n}) + \gamma(\vec{\sigma}\vec{\Delta})(\vec{e}\vec{\Delta}) + \delta(\vec{\sigma}\vec{\pi})(\vec{e}\vec{\pi}). \qquad (4')$$

For the cross section of meson production by the unpolarized γ-quanta on the unpolarised proton target (the statistical factor is everywhere omitted) we have

$$2 I_0(\theta) = |\alpha|^2 + |\beta|^2 + |\gamma|^2 \cos^2\frac{\theta}{2} + |\delta|^2 \sin^2\frac{\theta}{2}. \qquad (5)$$

In virtue of the relation between the cross sections

$$I_0(\gamma n \to \pi^- p) = I_0(\pi^- p \to \gamma n)$$

the attempt to investigate the photoproduction experimentally by studying the inverse process- the radiation capture of a π^--meson by a proton may prove to be unsuccessful. Though such experiments are very difficult,, nevertheless, as a result of them the data may become available on the photoproduction on a free neutron by the monochromatic quanta.

The relationship between the nucleon polarization $\langle\vec{\sigma}\rangle_f$ in the photoproduction when the γ-quanta and the target are unpolarized and the addition I_p to the cross section of the photoproduction

$$I(\theta,\varphi) = I_0 + I_p = \frac{1}{4} Sp M^+ M + \frac{1}{4} Sp M^+ M \vec{\sigma}_1 \cdot \vec{N} \;,\; \vec{N} \equiv \langle\vec{\sigma}\rangle_{in}$$

when the target is polarized, i.e., the Wolfenstein theorem of the form

$$I_p = I_0 \langle\vec{\sigma}\rangle_f \vec{N}$$

follows from the representation of the amplitude in the form (1) and the arguments of time reflection [2]. Then expression for the nucleon polarization may be presented in the form

$$2 I_0(\theta)\langle\vec{\sigma}\rangle_f = \vec{n}\left\{\alpha^+\beta + \alpha\beta^+ + \frac{i}{2}(\gamma\delta^+ - \gamma^+\delta)\sin\theta\right\}. \qquad (6)$$

Let us consider the cross section of meson photoproduction by the polarized γ-rays on the unpolarized target. The state of polarization of particles with the spin 1 is known to be set by the mean values of the operators $T_{1,\pm1}, T_{1,0}, T_{2,\pm1}$, $T_{2,0}$ and $T_{2,\pm2}$, which are constructed from the spin operator [8]. In virtue of the γ-quantum being transverse in the completely polarized γ-quantum beam $\langle T_{1,\pm1}\rangle = \langle T_{2,\pm1}\rangle \equiv 0$ [9] so that the expression for the cross section of the meson photoproduction by a partialy polarized beam of γ-rays may be presented in the form

$$I = I_0 + \langle T_{2,0}\rangle_{ih} \frac{1}{4} Sp M T_{20} M^+ - \langle T_{2,2}\rangle_{ih} \frac{1}{4} Sp M T_{2,2} M^+ +$$

$$+ \langle T_{2,-2}\rangle_{ih} \frac{1}{4} Sp M T_{2,-2} M^+ . \qquad (7)$$

Let us note first of all that after the averaging over the nucleon spin

$$\frac{1}{4} Sp M^+ M \vec{S} = 0 \qquad (8)$$

($\vec{S}$ is the spin of the photon), it can be immediately seen if we use the formula

$$(f \vec{S} g) = -i [\vec{f} \vec{g}]$$

for the functions of the type $f = \vec{f}\vec{e}$ and $g = \vec{g}\vec{e}$.
We may obtain ~~s~~ the same result if calculate the mean value of the spin vector of the photon arising in the radiation capture of a π-meson~~s~~ by the unpolarized protons. This result is rather general. For this reason ~~in~~ the term proportional to cos φ is absent in the expression for the cross section of the reaction induced by the polarized beam of γ-rays.

When calculating the rest components in (7) and the mean values of the tensors $T_{2,\pm 2}$, $T_{2,0}$ after the radiation capture let us use the formula |4|

$$(S_i S_k + S_k S_i) M = 2M\delta_{ik} - (M_i e_k + M_k e_i) . \qquad (9)$$

We have for the mean values of the tensors different from zero

$$\frac{1}{4}\,Sp M^{+} M T_{2,\pm 2} = \frac{\sqrt{3}}{4}\Big\{(|\alpha|^2+|\beta|^2)\left[n_y^2-n_x^2\mp 2in_x n_y\right]+$$

$$+|\gamma|^2(\Delta_y^2-\Delta_x^2\mp 2i\Delta_x\Delta_y)+|\delta|^2(\pi_y^2-\pi_x^2\mp 2i\pi_x\pi_y)\Big\} \tag{10}$$

and

$$\frac{1}{4}\,Sp M^{+} M T_{2,0} = \frac{1}{\sqrt{2}}\,I_0(\theta)$$

$$\left[2T_{2,\pm 2} = \sqrt{3}\left\{(S_x^2-S_y^2)\pm i(S_xS_y+S_yS_x)\right\};\ \sqrt{2}\,T_{2,0} = 3S_z^2-2\right].$$

If we choose the direction of the γ-quantum impulse for the z axis, then

$$\frac{1}{4}\,Sp M^{+} M T_{2,\pm 2} = \frac{\sqrt{3}}{4}\left\{(|\alpha|^2+|\beta|^2)-|\gamma|^2\cos^2\frac{\theta}{2}-|\delta|^2\sin^2\frac{\theta}{2}\right\}e^{\pm 2i\varphi} \tag{10'}$$

and finally the cross section of meson production by the polarized γ-quanta assumes the form

$$I(\theta,\varphi) = I_0(\theta)\left[1+\frac{\langle T_{2,0}\rangle_{in}}{\sqrt{2}}\right]+\langle T_{2,2}\rangle_{in}\frac{\sqrt{3}}{2}\left\{|\alpha|^2+|\beta|^2-|\gamma|^2\cos^2\frac{\theta}{2}-|\delta|^2\sin^2\frac{\theta}{2}\right\}\cos 2\varphi \tag{11}$$

with appropriate definition for $\langle T_{J,M}\rangle = (-1)^{J+M}\langle T_{J,-M}\rangle$.

Let us draw out attention, to the polarization of γ-rays in the radiation capure of π-mesons by the unpolarized protons. For the mean values of the tensors not equal to zero

$$I_0\langle T_{2,m}\rangle = \frac{1}{4}\,Sp M^{+} T_{2,m} M.$$

We obtain the expressions coinciding with (10) if in the radiation capture we choose the direction of the outgoing γ -quanta for the z axis.

If represent (11) in the form

$$I(\theta,\varphi)=I_0(\theta)+\langle T_{2,0}\rangle_{in} I_{2,0}+\langle T_{2,2}\rangle_{in} I_{2,2}$$

the obtained result may be written in the form of the relations

$$I_{2,m}=I_0(\theta)\langle T_{2,m}\rangle_f \tag{12}$$

expressing the Wolfenstein theorem for the considered reaction. It means that the study of the γ -quantum polarization in the reaction $\pi^- p \to \gamma n$ gives the same information as the study of the cross section for the meson photoproduction by the polarized γ -quanta. The above mentioned result concerning the nucleon polarization must be added to this. The study of the nucleon polarization in the radiation capture gives the same data as well as the investigation of the cross section for the photoproduction on the polarized proton target whereas the proton polarization in the photoproduction is connected with the cross section of the rediation capture on the polarized protons.

Completing the consideration of the polarization phenomena in the reactions $\gamma N \rightleftarrows \pi N$ we compare the expressions for the correlation of the polarization in the radiation capture

$$I_0(\theta)\langle(\vec{\sigma}\vec{a})(T_{ik}b_i c_k)\rangle=\frac{1}{4}Sp M'^{+}(\vec{\sigma}\vec{a})(T_{ik}b_i c_k)M'$$

with the addition to the- cross section of the photoproduction I_{pp} when both the γ -quantum beam and the target are polarized

$$I_{pp} = \frac{1}{4} S_p M^+ M (\vec{\sigma}\vec{a})(T_{ik} b_i c_k)$$

For the correlation $I_0 \langle (\vec{\sigma}\vec{a})(\vec{S}\vec{b}) \rangle$ we have

$$-2 I_0(\theta) \langle (\vec{\sigma}\vec{a})(\vec{S}\vec{b}) \rangle = -i \left\{ (\alpha^+\gamma - \alpha\gamma^+)(\vec{a}\vec{\Delta})(\vec{\pi}\vec{b}) - (\alpha^+\delta - \alpha\delta^+)(\vec{a}\vec{\pi})(\vec{\Delta}\vec{b}) \right\} +$$

$$+ \left\{ (\beta\gamma^+ + \beta^+\gamma)(\vec{a}\vec{\pi})(\vec{b}\vec{\pi}) + (\beta\delta^+ + \beta^+\delta)(\vec{a}\vec{\Delta})(\vec{b}\vec{\Delta}) + (\gamma\delta^+ + \gamma^+\delta)(\vec{a}\vec{n})(\vec{b}\vec{n}) \right\}, \qquad (13)$$

it can be particularly seen from here that

$$\langle (\vec{\sigma}\vec{n})(\vec{S}\vec{\pi}) \rangle = \langle (\vec{\sigma}\vec{n})(\vec{S}\vec{\Delta}) \rangle = \langle (\vec{\sigma}\vec{\pi})(\vec{S}\vec{n}) \rangle =$$

$$= \langle (\vec{\sigma}\vec{\Delta})(\vec{S}\vec{n}) \rangle = 0 \qquad (14)$$

$$2 I_0(\theta) \langle (\vec{\sigma}\vec{n})(\vec{S}\vec{n}) \rangle = -(\gamma\delta^+ + \gamma^+\delta)$$

etc.

(15)

For the addition to the cross section of the photoproduction we obtain

$$\frac{1}{4} S_p M^+ M (\vec{\sigma}\vec{a})(\vec{S}\vec{b}) = \frac{i}{2} \left\{ (\alpha\gamma^+ - \alpha^+\gamma)(\vec{a}\vec{\Delta})(\vec{\pi}\vec{b}) - \right.$$

$$\left. - (\alpha\delta^+ - \alpha^+\delta)(\vec{a}\vec{\pi})(\vec{\Delta}\vec{b}) \right\} + \frac{1}{2} \left\{ (\beta\gamma^+ + \beta^+\gamma)(\vec{a}\vec{\pi})(\vec{b}\vec{\pi}) + \right.$$

$$\left. + (\beta\delta^+ + \beta^+\delta)(\vec{a}\vec{\Delta})(\vec{b}\vec{\Delta}) + (\gamma\delta^+ + \gamma^+\delta)(\vec{a}\vec{n})(\vec{b}\vec{n}) \right\}. \qquad (16)$$

(16a)

If the results of the expreiments on the investigation of the polarization correlation in the radiation capture are available. i.e., being aware of the combinations $(\alpha\gamma^{+}-\alpha^{+}\gamma),(\alpha\delta^{+}-\alpha^{+}\delta)$ etc one may predict the results of the experiments with the polarized beam and target. This can be seen from the comparison of (16) and (13). Therefore, these expreiments are not independent.

It can be also seen from (13) and (16) that the relations of the type (12) occur when even if one of the vectors $\vec{a}$ or $\vec{b}$ are directed along the normal $\vec{n}$. The analogous conclusions arise when in (13) and (16) $\vec{S}$ is changed for $T_{2,m}$.

3. Compton Effect.

The amplitude of the Compton effect may be presented in the form (1) but for the elastic scattering M must be a scalar. Using the usual arguments including the symmetry under time reversal we have the most general expression for M

$$M=A(\vec{e}\vec{e}')+B([\vec{e}\vec{e}']\vec{n})+C(\vec{\sigma}\vec{n}')(\vec{e}\vec{e}')+D(\vec{\sigma}\vec{n}')(\vec{n}'[\vec{e}\vec{e}'])+$$

$$+F(\vec{\sigma}[\vec{e}\vec{e}'])+E\left\{(\vec{\pi}'\vec{e}')(\vec{\sigma}[\vec{\Delta}'\vec{e}])+(\vec{\pi}'\vec{e})(\vec{\sigma}[\vec{\Delta}'\vec{e}'])\right\} \qquad (17)$$

where $\vec{e}$ and $\vec{e}'$ are the polarization vectors before and after the collision $\vec{n}',\vec{\Delta}'$ and $\vec{\pi}'$ are constructed as earlier from the unit vectors $\vec{k}$ and $\vec{k}'$ at the directions of the quantum momenta before and after scattering. Expression (17) has 6 terms as it must be. The expression for M may be presented in another form

$$M = R_1(\vec{e}\,\vec{e}\,') + R_2(\vec{\mathfrak{z}}\,\vec{\mathfrak{z}}\,') + R_3(\vec{\sigma}[\vec{e}\,'\vec{e}\,]) + R_4(\vec{\sigma}[\vec{\mathfrak{z}}\,'\vec{\mathfrak{z}}\,]) +$$

$$+ R_5\left\{(\vec{\sigma}\vec{k})(\vec{\mathfrak{z}}\,'\vec{e}) - (\vec{\sigma}\vec{k}\,')(\vec{\mathfrak{z}}\,\vec{e}\,')\right\} + R_6\left\{(\vec{\sigma}\vec{k}\,')(\vec{\mathfrak{z}}\,'\vec{e}) - (\vec{\sigma}\vec{k})(\vec{e}\,'\vec{\mathfrak{z}})\right\} \tag{17'}$$

if we introduce the vectors

$$\vec{\mathfrak{z}} = [\vec{k}\vec{e}\,], \quad \vec{\mathfrak{z}}\,' = [\vec{k}\,'\vec{e}\,'] .$$

Under time reversal

$$\vec{e} \rightleftarrows \vec{e}\,', \quad \vec{\mathfrak{z}}\,' \rightleftarrows \vec{\mathfrak{z}}$$

In some papers [10-12] the expression for Compton effect amplitude was found which contains the terms not greater than the linear in frequency

$$M = \frac{e^2}{m}(\vec{e}\,\vec{e}\,') - \frac{ie}{m}\left(2\mu - \frac{e}{m}\right)k(\vec{\sigma}[\vec{e}\,'\vec{e}\,]) - 2\mu^2 i\,\frac{(\vec{\sigma}[\vec{\mathfrak{z}}\,\vec{\mathfrak{z}}\,'])}{k} -$$

$$- i\frac{e}{m}\,\frac{\mu}{k}\left[(\vec{e}\,\vec{k}\,')(\vec{\sigma}\,\vec{\mathfrak{z}}\,') - (\vec{e}\,'\vec{k})(\vec{\sigma}\,\vec{\mathfrak{z}})\right] \tag{18}$$

where μ is the magnetic moment of the nucleon ($\vec{k}$ in (18) is not a unit vector).

We get from (17) for the cross section of the unpolarized beam of γ-rays on the unpolarized target

$$2I_0(\theta)=\{|R_1|^2+|R_2|^2+4\mathrm{Re}[R_3^*(R_5+R_6)+R_4^*R_5]\}(1+\cos^2\theta)+(|R_3|^2+$$
$$+|R_4|^2)(3-\cos^2\theta)+2(|R_5|^2+|R_6|^2)(3+\cos^2\theta)+4\mathrm{Re}\{(R_1^*R_2)+R_4^*(R_3-R_6)+$$
$$+(R_5^*R_6)(2+\cos^2\theta)\}\cos\theta. \qquad (19)$$

The unitary condition of the S-matrix leads to the optical theorem [13]

$$k\sigma_t = 4\pi\, \mathrm{Im}[R_1(0^\circ)+R_2(0^\circ)] \qquad (20)$$

where σ_t is the total cross section of the interaction involving both elastic scattering and inelastic processes. The inequality *

$$I_0(0^\circ) \geqslant \left(\frac{k\sigma_t}{4\pi}\right)^2 \qquad (21)$$

follows from (20) for the cross section of forward elastic scattering $I_0(0^\circ)$. In virtue of this inequality the elastic scattering at high energies is found to be peaked forward and concentrated in the small solid angle.

$$\Delta\omega=\pi\theta^2\leq\left(\frac{4\pi}{k\sigma_t}\right)^2\sigma_s\ ;\qquad \sigma_s=\int I_0(\theta)\,do. \qquad (22)$$

---

* The existence of inequality (21) for the elastic scattering of particles was shown in [14] and in a less general case by Rarita [15] as well as by Karplus and Ruderman (see [15]). This inequality was first published in a brief note by Wick [16] which remains unnoticed.

(22)

Inequalities (21) and (22) reflect the main features of elastic scattering at high energies without using the optical model. The expression for the polarization of the recoil nucleon when the target and the beam are unpolarized

$$2I_0(\theta)\langle\vec{\sigma}\rangle_f=\vec{n}\sin\theta\, 2\,\mathrm{Re}\left\{(R_1^*R_4)+\left[(R_2^*R_4)-(R_1^*R_3)\right]\cos\theta-(R_2^*R_3)\cos 2\theta\right. \tag{23}$$

coincides with the expression determining the azimuthal asymmetry in the cross section of the Compton effect on the polarized nucleon. Let us note that ~~the magnitudes~~ R_5 and R_6 did not enter into (23).

The ~~mea~~n value of the photon spin (and the corresponding addition to the cross section) when the target and the beam are unpolarized turn out into zero as well as in the photoproduction. This result can be seen from the dimensional considerations. The mean value of the spin under these conditions may be directed as the only pseudovector - the normal $\vec{n}$. Since for the photons spreading at the z axis $\langle S_x\rangle=\langle S_y\rangle=0$ there remains to consider only $\langle S_z\rangle$, but $\langle S_z\rangle\sim n_z=0$

Using (9) we have for the mean values $T_{2,m}$ different from zero

$$2I_0(\theta)\langle T_{2,\pm 2}\rangle=\sqrt{3}\left\{|R_4|^2+\mathrm{Re}\left[R_4^*(R_5+R_6\cos\theta)\right]\right\}e^{\pm 2i\varphi}$$

$$\sqrt{2}\,I_0(\theta)\langle T_{2,0}\rangle=I_0(\theta). \tag{24}$$

For the addition to the cross section of the Compton-effect when the incident γ-quanta are polarized the expressions coinciding with (24) are obtained.

4. Conclusion

The extension of the consequences of the time-reversal invariance to the reactions with the participation of γ-rays is the main result of the present paper. The examples of the Compton-effect, photoproduction and radiation capture of pions were considered. The generalization for the case of any binary reaction (with two particles in the initial and final states) may be made in an analogous way.

Apart from theoretical this result is of experimental interest. For instance, the study of the nucleon polarization in the deuteron photoproduction gives the same information as well as the radition capture of the polarized nucleon, whereas the study of the γ-quantum polarization in the neutron radiation capture by a proton is equivalent to the investigation of the deuteron photodisintegration cross section by the polarized γ-quanta.

The relations between the polarization phenomena in the reverse reactions as well as the relation between the averaged cross sections are based on the invariance of the interaction under time reversal. However, in contrast to the "semi detailed" balance for the Wolfenstein relations it is essential whether the space parity is conserved or not. For the illustration let us consider the elastic scattering of neutrino on the spinless particles. If one will not require the invariance under space and time inversion the most general form of the scattering amplitude will be

$$M = a + b(\vec{\sigma}\vec{n}) + c(\vec{\sigma}\vec{\pi}) + d(\vec{\sigma}\vec{\Delta}) \tag{25}$$

The last two terms being pseudoscalars under space reflection are transformed in a different way under time reversal. If the time reversal invariance is conserved in the absence of the space parity conservation (the combined parity) the last term in (25) is absent. Though in the considered case the detailed equivalence remains trivially the Wolfenstein relation is absent. If d = 0 the polarization as a result of the unpolarized beam scattering is given by

$$I_o(\theta)\langle\vec{\sigma}\rangle_f = (a^+b + ab^+)\vec{n} + (ac^+ + a^+c)\vec{\pi} + i(b^+c - bc^+)\vec{\Delta} \tag{26}$$

whereas the addition to the cross section I_p is found to be proportional to

$$(a^+b + ab^+)\vec{n} + (a^+c + ac^+)\vec{\pi} - i(b^+c - c^+b)\vec{\Delta}. \tag{27}$$

Although both the polarization and azimuthal assymetry present the same information (the identical combinations of quantities enter into (26) and (27)) but the Wolfenstein relation is absent.

Appendix

Unitary Conditions

The unitarity conditions together with the invairance requirements determine the number of independent parameters necessary for the phenomenological analysis of the exp~~/~~erimental data. Let us consider the energy region of γ -quanta close to 300 MeV when in the photon interaction with the nucleons besides the~~ø~~ elastic scattering only the photoproduction of single pions occurs. We introduce the 3 x 3 S-matrix todescribe the Compton effect, the photoproduction and the radiation capture, the scattering and the charge exchange of mesons. To be more definite let us consider the Compton effect on the proton. In this case we may introduce the unitary S-matrix to describe the processes

$$
\begin{aligned}
&+n \longrightarrow +n\ (S_{11}); \quad 0p \longrightarrow +n\ (S_{12}); \quad \gamma p \longrightarrow +n \\
&+n \longrightarrow 0p\ (S_{21}); \quad 0p \longrightarrow 0p\ (S_{22}); \quad \gamma p \longrightarrow 0n \\
&+n \longrightarrow \gamma p\ (S_{31}); \quad 0p \longrightarrow \gamma p\ (S_{32}); \quad \gamma p \longrightarrow \gamma p
\end{aligned}
\qquad (A.1)
$$

(the elements of the S-matrix are given in the brackets and

$S_{ik} = S_{ki}$)

If we do~~ø~~ not take into account the isotopic invariance it is necessary to introduce another unitary matrix, involving

$$-p \to -p\ (S'_{11});\quad on \to p\ (S'_{12});\quad \gamma n \to p\ (S'_{13})$$

$$-p \to on\ (S'_{21});\quad on \to on\ (S'_{22});\quad \gamma n \to on\ (S'_{23})$$

$$-p \to \gamma n\ (S'_{31});\quad on \to \gamma p\ (S'_{32});\quad \gamma n \to \gamma n\ (S'_{33}) \qquad \text{(A.2)}$$

for the processes including the Compton-effect on the neutron. The unitary conditions make it possible to express separately for (A.1) and (A.2) 6 independent complex quantities S_{ik} through 6 real ones by introducing for instance, three phase shifts $\delta_1, \delta_2, \delta_3$ and three mixing parameters φ, ψ, θ.

Then

$$S_{11} = e^{2i\delta_1}(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi)^2 + e^{2i\delta_2}(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi) + e^{2i\delta_3}\sin^2\theta\sin^2\psi$$

$$S_{12} = e^{2i\delta_1}(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi)(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi) + e^{2i\delta_2}(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi)(\sin\varphi\sin\psi - \cos\theta\cos\varphi\cos\psi) - e^{2i\delta_3}\sin^2\theta\sin\psi\cos\psi$$

$$S_{13} = e^{2i\delta_1}(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi)\sin\theta\sin\varphi - e^{2i\delta_2}(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi)\sin\theta\cos\varphi + e^{2i\delta_3}\sin\theta\cos\theta\sin\psi$$

$$S_{22} = e^{2i\delta_1}(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi)^2 + e^{2i\delta_2}(\sin\varphi\sin\psi - \cos\theta\cos\varphi\cos\psi)^2 + e^{2i\delta_3}\sin^2\theta\cos^2\psi$$

$$S_{23} = e^{2i\delta_1}\sin\theta\sin\varphi(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi) +$$

$$+ e^{2i\delta_2}\sin\theta\cos\varphi(\cos\theta\cos\varphi\cos\psi - \sin\varphi\sin\psi) - e^{2i\delta_3}\sin\theta\cos\theta\cos\psi$$

$$S_{33} = e^{2i\delta_1}\sin^2\theta\sin^2\varphi + e^{2i\delta_2}\sin^2\theta\cos^2\varphi + e^{2i\delta_3}\cos^2\theta.$$

Neglecting the reactions with γ-rays i.e., if $\delta_3 = \theta = 0$ only S_{11}, S_{12} and S_{22} are different from zero and the familiar expressions may be obtained from (A.3) for the elements of the 2 x22 S - matrix, $\psi + \varphi$ being the mixing parameter. The quantity θ characterized the effect of the radiation processes.

Instead of introducing (the real) phase shifts and the mixing parameters one may do it another way. The unitarity conditions may be used in the form of the optical theorem generalization [17,18] to determine the complete set of the experiments is by anology to [19] in the presence of only the elastic scattering. The unitary conditions make it possible to reduce the number of necessary experiments (from the standpoint of the completely phenomenological analysis) up to the total number of ~~indepe~~ independent terms in the amplitudes of all the reactions which the unitary S-matrix involves. When taking into account the radiation processes in the amplitudes of the meson nucleon scattering there appear the isotopically non-invariant additions and in the general case the number of the terms reaches 20. Assuming the matrix elements for the photoproduction and the Comptron effect to be proportional to e and e^2 respectively and expanding (A.3) we obtain that in the processes of meson scattering the taking into account of the radiation processes leads to small corrections (of the order of e^2). We shall consider, therefore, the phases δ_1 and δ_2 coinciding with phases $\pi - N$ scattering in the states with the isotopic spins $T = 3/2$ and $T = 1/2$ respectively (see Appendix in [20]). On the other hand the effect of πN scattering on elastic scaftering of γ rays on a nucleon is found to be comparable with the magnitude of the whole effect.

Some considerations concerning the decrease of the number of the parameters in the S-matrix are given in [21,22]. Therefore they are not discussed here.

The expression for the amplitude of the Compton effect (18) is unitary only approximately since, for instance, the right hand side of (20) is equal to zero. However, the limitation

$$Jm\left[R_1(0) + R_2(0)\right] \ll Re\left[R_1(0^\circ) + R_2(0^\circ)\right]$$

necessary for (18) being correct, is realised with good approximation.

It should be noted in conclusion that when taking into account the considerations concerning the isotopic invariance the same phases of $\pi - N$ scattering enter into the S-metrix for the Compton effect on the neutron. This gives use to some similarity in the Compton effect on the neutron and elastic scattering of γ-rays on the proton.

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