JOINT INSTITUTE FOR NUCLEAR RESEARCH

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LABORATORY OF THEORETICAL PHYSICS

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"REMARKS ON THE MEASURABILITY OF ELECTROMAGNETIC FIELDS"

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A B S T R A C T :

Indeterminacy relations are derived, which express the facts, that only 4 -dimensional averages of electromagnetic fields are measurable and that also an average cannot be measured with unlimited accuracy simultaneously with the position of the averaging - region in space - time-continuum. Essentially in showing this is the use of test bodies, which obey Heisenbergs indeterminacy relations and possess a finite extension, finite mass and finite electric charge

1. I n t r o d u c t i o n

As is well-known there are growing in last time the doubts about the consistency of conventional, local quantum field theories and also the concrete difficulties inside these thories. It is quite clear, that the grave difficulties come all from very big moments or from the smallest space - time - regions. It is therefore accepted by most physicists, that the field theory should be alterated anyhow in the smallest space - time - regions. But up to date it seems not to be found a satisfactory way out of this situation, in spite of a lot of trials in much different directions[1]. Looking at this situation it might be right to discuss the question, how far the fundamental principles of local field are fit to describe our nature. It is hoped to get from such a consideration any hint from

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[1] Compare e.g. the review-article by Blokhintsev, |1|.

known properties of nature to the likely still unknown way, leading us to a better field theory. - In another formulation our question is: How deep are the fundamental principles of local field founded in reality, or even: Is it possible to measure in reality the fundamental quantities of conventional field theory exactly?1/

We shall restrict ourselves in this work to the best-known field theory, electrodynamics. Our concrete question is: How far are the field quantities of electrodynamics (in classical and in quantum theory) accessible to real measurements?

Looking hastily at this problem one could think it completely settled by the paper of Bohr and Rosenfeld |2|. These authors indeed have shown (by means of thought-experiments), that it is possible to measure the average of any component of electromagnetic field over a 4-dimensional region with unlimited accuracy. It seems that in |2| the extension of the region of averaging could get as small as you wish. But it is an essential supposition of this limiting process, as stressed by the authors themselves, that the atomic structure of measuring - device (especially of the test-bodies) can be neglected. From a logical point of view it is possible to make such supposition, because electrodynamics (without coupling to any other field) is free of any hint to atomic structure of matter. But of course for practical measurement the atomic structure of matter must be taken into account, if one wants to measure in space - time - dimensions of atomic size e.g.

In the following §§ it will be shown by very simple arguments that it is indeed impossible to determine simultaneously the average of one component of electromagnetic field over a microscopical

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1/ We return to the old principle, which in the hands of Heisenberg was very fertile in establishing quantum mechanics that it should be possible to formulate a physical theory by no other than measurable quantities.

space - time - region and the position of this region exactly by means of really existing test-bodies. Indeterminacy - relations for this phenomenon are derived.

2. Measurement of Electric Fields

At first we shall shortly summarize those points, which in the proof of Bohr and Rosenfeld | 2 | prevent its extension to very small space-time-regions, taking into account the atomic structure of matter. Once more we wish to stress, that of course Bohr and Rosenfeld were quite aware of these points, as can be seen from | 2 | and still more clear from a more recent article by Rosenfeld | 3 |. The most critical propositions of | 2 | are:

a) Mass and electric charge of the test-body can be brought to infinity.[1/]

b) One should be able to cut the test-body into many small parts each of them wearing a mirror.

c) Before and after measurement the test-body should be bound at the same point of a lattice, fixed in space.

d) During measurement the body should be connected with the just mentioned lattice by a linear spring.

It is clear, that such test-body, if it is existing in reality at all[2/], must have macroscopic dimensions. Of course you can think of a body with such qualities, having any size; therefore these propositions are logically correct. But for our problem we have to take as test-bodies for the smallest space-time-regions only such

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[1/] This point was critized already in the early work of Markov |4|.

[2/] i.e., if it is built up from matter, which is taken from the known part of our world.

bodies, which are found reglly in nature. Not useful for our purpose are also particles with a point-structure or bodies (particles) with a not well-defined[1] or with a not well-known structure. The first type of particles should be excluded because there are quantum fluctuations of infinite size acting on them (to be derived from conventional theory, look e.g. Corinaldesi | 5 |; about measurement with point-particles see however the paper by Landau and Peierls | 6 |).The second type of particles is not useful because the space-structure of electric charge of the tegt-body determines to an essential part the space-region of averaging the field; this structure therefore should be known as well as possible, it should be most simple and very stable. As we wish to measure in space-time-regions, being as small as possible, it seems to the author, that atomic nuclei are now the most usuful test-bodies.[2] The following discussion however is not only valid for atomic nuclei but also for any other test-bodies, fulfilling the above mentioned propositions.

The method of measuring the electric field by such test-bodies we take from Bohr and Rosenfeld | 2 |; basically is the formula:

$$\bar{\mathcal{E}} \rho V T = \mathcal{I}_2 - \mathcal{I}_1 . \tag{1}$$

The meanings of the symbols in (1) are:

ρ: Density of electric charge of the test-body (assumed to be constant);

1/ Here we mean also: Too much complicated!

2/ This also was the opinion of Markov, | 4 |.

V Volume of the test-body and of the averaging-region in space. $\overline{\mathcal{E}} = \frac{1}{VT}\int_{VT} \mathcal{E}(r,t)\, d^3r\, dt$: Average of electric field $\mathcal{E}$ over the 4-dimensional region V, T (in quantum theory: Eigenvalue of the average...);

T: Intervall of time, giving duration of measuring and averaging intervall in time; T could be taken arbitrary inside certain limits; $\mathcal{Y}_1, \mathcal{Y}_2$: Moments of the test-body at the beginning and end of T.

If ρ, V, are known, you can take by (1) from a measurement of $\mathcal{Y}_1 - \mathcal{Y}_2$ of course $\overline{\mathcal{E}}$. But by measuring exactly the moment of the test-body, you make, according to $\Delta p_\xi \Delta \xi \gtrsim \hbar, \ldots, \ldots,$ the position (here the position ξ of the centre) of the body uncertain. Also you need for an exact determination of the moment according to quantum mechanics an iffinite time. That means: By measuring exactly $\mathcal{Y}_1$ and $\mathcal{Y}_2$ the position of the averaging-region V, T in the space-time-continuum gets completely indefinite. Of course an exact determination of $\overline{\mathcal{E}}$ has no sense if it is not possible to know, where and when this field was existing. Therefore it is relevant to make both measurements of moment only with an inaccuracy Δp_ξ chosen such, that $\Delta \xi$ gets one order of magnitudy smaller than the diameter of the test-body[1]. A rough estimate shows, that the resulting indeterminacy $\Delta \tau$ of T is then extremely small (for an atomic nucleus $\Delta \tau \approx 10^{-25}$ sec.).

The need for a certain localization of the process of measuring

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[1] Of course we can confine our discussion to the determination of one component of $\overline{\mathcal{E}}$, say $\mathcal{E}_\xi$.

the field in space-time compells us therefore to let the field average itself uncertain in the order of:

$$\Delta\overline{\mathfrak{E}}_\xi \approx \frac{\Delta P_\xi}{\rho V T} \tag{2}$$

or also

$$\Delta\overline{\mathfrak{E}}_\xi \gtrapprox \frac{\hbar}{\rho V T \Delta\xi} \tag{3}$$

The inequality (3) of course is contained already in the paper by Bohr and Rosenfeld | 2 |. The essential difference being, that in | 2 | ρ is allowed to have any value. Therefore by letting $\rho \to \infty$, the right side of (3) can be brought to zero! Our proposition of taking only really existing particles as test-bodies clearly limits the variability of ρ. Indeed the sort of matter with the highest ρ we know now is nuclear matter. There seems to be very little probability of finding any other sort of matter with a well-defined ρ, being still more dense. In the authors opinion therefore (3) represents an essential limit of measurability of the electric field in reality.

Now we remark still two points, which were neglected in discussing (2):
Firstly the averaging-volume V, used to define $\overline{\mathfrak{E}}$, is not constant in time, according to the action of the field and because mostly we have $\vartheta_1 \neq \vartheta$. This is not convenient. Therefore one should manage, that in measuring the electric field one has $\vartheta_1 \approx \vartheta$. The problem gets most simple also if the field to be measured is very weak and if the test-body is as heavy as possible. It may be, that for the measurement of stronger fields any sort of compensation-device will proof convenient.

Secondly we would like to correct the measured field $\overline{\mathfrak{E}}$ for the own-field of the test-body. Of course this is possible as soon as we know the world-line of the test-body in the past and as soon as classical electrodynamics are competent. In our problem however neither classical electrodynamics are exactly competent nor is it possible to make exact dates on the past of our test-body. It seems therefore to the author, that it is impossible to perform this correction; we should consider the own-field of a microscopic test-body as inseparable part of the measured field.

3. Measurement of magnetic fields

In macro-physics it is possible to measure a magnetic field by the power, with which it acts on an electric current or on a magnet. In micro-physics one has to remember, that a current consists of single charged particles and it seems to be most simple to use for measurement the power, by which a magnetic field $\mathfrak{H}$ acts on a single, moving, charged body (velocity $\mathfrak{v}$, charge density ρ). The density of this power is, as is well-known,

$$\mathfrak{k} = \rho \, \frac{\mathfrak{v} \times \mathfrak{H}}{c} = \frac{\rho}{c} \begin{cases} \vartheta_y \mathfrak{H}_z - \vartheta_z \mathfrak{H}_y \\ \vartheta_z \mathfrak{H}_x - \vartheta_x \mathfrak{H}_z \\ \vartheta_x \mathfrak{H}_y - \vartheta_y \mathfrak{H}_x \end{cases} \qquad (4)$$

For the measurement of a magnetic field of the special form $\mathfrak{H} = (0, 0, \mathfrak{H}_z)$, one has therefore two simple possibilities: Taking either a particle of velocity $\mathfrak{v} = (0, \vartheta_y, 0)$ or $\mathfrak{v} = (\vartheta_x, 0, 0)$. We should take $|\mathfrak{v}|$ not too small, in order to make $\mathfrak{k}$ strong and well measurable. Of course also here we can measure only an average-value of the field over a finite space-time-region V,T ;

1/ Taking the power on a magnet we need a new test-body; we can take however the same test-body as in § 2 by using (1).

the fundamental equation for the second possibility reads:

$$\bar{\mathfrak{L}}_\xi \, \rho V T \frac{\vartheta_\xi}{c} = -(P_{\eta_1} - P_{\eta_2}). \tag{5}$$

The symbols in (5) are defined quite parallel to (1). Also the further considerations are quite analogous to those of § 2. Only ϑ_ξ in (5) must be considered separately. In order to know something about the position ξ or the test-body in the ξ -directions, ϑ_ξ cannot be known exactly. It is however convenient and possible to choose $P_\xi >> \Delta P_\xi$; $\Delta\vartheta_\xi$ has then practically not any influence on $\Delta\bar{\mathfrak{L}}_\xi$. Under this proposition we have:

$$\Delta\bar{\mathfrak{L}}_\xi \approx \frac{\Delta P_\eta \cdot c}{\rho V T \vartheta_\xi},$$

or also

$$\Delta\bar{\mathfrak{L}}_\xi \gtrapprox \frac{\hbar c}{\rho V T \vartheta_\xi \Delta\eta} \tag{6}$$

Of course there are still (5) other relations for the uncertainties in measuring magnetic fields, each being quite analogous to (6). From (6) it follows quite similar to the discussion of (3), that it is impossible to determine simultaneously the average of the magnetic field with the position of the averaging-region. Also it is impossible to take the limit of V, T $\longrightarrow 0$, without getting an infinite indeterminacy in the field-measurement.

4. 4-dimensional formulation

The inequalities (3), (6) and the 7 other analogous relations can be comprehended very clearly, using the notation of the theory of relativity. Introducing:

$j_\nu = \rho(c, \vartheta_x, \vartheta_y, \vartheta_z)$; $\xi_\nu = (c\tau, \xi, \eta, \zeta)$ *) ; $\Delta\xi_\nu$ analogous;

*) By ξ_ν we symbolize the coordinates of the centre of V, T.

$$f_{\mu\nu} = \begin{pmatrix} 0 & \mathfrak{E}_x & \mathfrak{E}_y & \mathfrak{E}_z \\ -\mathfrak{E}_x & \vartheta & \mathfrak{B}_z & -\mathfrak{B}_y \\ -\mathfrak{E}_y & -\mathfrak{B}_z & \vartheta & \mathfrak{B}_x \\ -\mathfrak{E}_z & \mathfrak{B}_y & -\mathfrak{B}_x & \vartheta \end{pmatrix} ; \quad \bar{f}_{\mu\nu}(\xi) = \frac{1}{VcT} \int\limits_{V,cT(\xi)} f_{\mu\nu}(x)\, d^4x ;$$

$\Delta \bar{f}_{\mu\nu}$ analogous, it is possible to comprehend the mentioned 9 inequalities to:

$$\Delta \bar{f}_{\mu\nu}(\xi)\, \Delta \xi_\lambda \, j_\kappa \gtrsim \frac{\hbar c}{VT}\, \delta_{\mu\kappa,\nu\lambda} ; \tag{7}$$

the tensor δ having the following meaning:

$\delta_{00,ii} = 1 = -\delta_{ii,00}$ for $i = 1, 2, 3$;

$\delta_{ii\,kk} = 1 = -\delta_{kk,ii}$ for $i \neq K$; $i, K = 1, 2, 3$;

$\delta_{\mu\kappa,\nu\lambda} = \vartheta$ else.

As it should be, also the other 3 components of (7), which do not belong to the above mentioned 9 inequalities, have physical meaning. One of these 3 relations is:

$$\Delta \bar{f}_{\vartheta 1}(\xi)\, \Delta \xi_0 \, j_1 \gtrsim -\frac{\hbar c}{VT} , \quad \text{or} \quad \Delta \mathfrak{E}_\xi(\xi)\, \Delta\tau\, \vartheta_\xi \gtrsim \frac{\hbar}{\rho VT} . \tag{8}$$

According to (2) we can give (8) also the form: $\Delta p_\xi \Delta\tau\, \vartheta_\xi \gtrsim \hbar$ using $\Delta p_\xi \vartheta_\xi \approx \Delta E$, we get instead of (8): $\Delta E \Delta\tau \gtrsim \hbar$. Here ΔE means the uncertainty of the energy of our test-body, $\Delta\tau$ is the indeterminacy of time of measurement. (8) represents therefore the well-known relation between energy and time.

5. Concluding remarks

At first still some remarks on the validity of the above estimates:

Everywhere we did assume, that the test-body during all the measuring-process does not suffer any deformation or other alteration (of course, translations are allowed). This is only valid for weak fields and for not too small $\Delta\xi$ 1/.

It might further be remarkable, that the relations (7) are valid in classical <u>and</u> in quantized electrodynamics, as soon as the test-bodies are particles, which obey quantum-mechanics.

Of course it must be stressed also, that our discussion does not exclude the possibility of the existence of any other methods of measuring electromagnetic fields (also indirect methods), being more exact than the method analysed by us. The author however does not give a big probability to this possibility.

Concerning the meaning of the inequalities (7), one can see already the following: Clearly they are not in accord with the conventional, local electrodynamics. They withdraw the direct empirical foundation of that theory by saying: The field-quantities of Maxwells theory at space-time-points are not measurable. Only the averages of the fields over finite space-time-regions are measurable. But even the averages cannot be determined quite exact, if one wishes to measure simultaneously the position of the averaging-region in space-time-continuum. Therefore we can have no doubt, that one has to try to reformulate electrodynamics essentially, if one wishes to eliminate all non-observable quantities. It shall be most important to limit anyhow the concept of the local field. It might be typical for this limitation, that certain qualities of the elementary test-bodies shall be contained organically inside the fundamental equations of the field. 2/

1/ For very small $\Delta\xi$ namely the process of measuring the position ξ shall become a strong perturbation, by which the test-body and the field might be alterated essentially and statistically.

2/ A similar opinion was put forward by Markov | 4 |.

The up-to-date usual sharp separation between field and test-body likely must be given up. May be, this can be interpreted as a new example for the general trend of modern physics to take into account the essential statistical interaction between measuring apparatus and measuring-object.

It is quite clear that this new theory should contain also the conventional theory as a limiting case for big averaging-regions. It should contain especially all the successes of the conventional theory.

Of course there is also the possibility that the just sketched program cannot be fulfilled. In that case the author would vote for searching a theory without any field-quantities.

Concluding we wish to mention a deep-lying difficulty, appearing in the process of measuring more than one field-averages simultaneously be atomic (or nuclear) test-bodies 1/. Surely it would be the natural way to take identical test-bodies. But this seems to be impossible, because identical atomic objects cannot be distinguished. At the end of the measurement one could not know, how to associate the bodies to those of the beginning of measurement. Author does not yet see any way out of this difficulty.

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1/ Discussion of such processes lead in | 2 | to the conventional uncertainty-relations between two field-components.

R e f e r e n c e s :

|1| D. Blokhintsev, Usp. Fiz. Nauk LXI., 137 (57).

|2| N. Bohr and L. Rosenfeld, Kgl. Danske Vid. Selsk., Math., - fyz. Medd., Vol. 12, No. 8 (33).

|3| L. Rosenfeld, in "Niels Bohr and the Development of Physics", London 1955; esp. pp. 83, 84.
Look also: Jauch and Rohrlich, Theory of Photons and Electrons, Cambridge 1955, p. 81.

|4| M. Markov, Phyz. Zs. Sowj. 12, 105 (37);
look also: JETF 8, 124 (38).

|5| E. Corinaldesi, Suppl. al Vol. X del Nuovo Cim., p. 83 (53).

|6| L. Landau and R. Peierls, Zs. f. Physik 69, 56 (31).