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SPACE CHARGE EFFECTS IN AXIAL INJECTION LINE FOR U-400 CYCLOTRON

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1 Introduction

The problem of the space charge influence on the beam dynamics in transport lines can be studied by different methods, depending on the goals. The only solution for a uniform density continuous beam is the Kapchinsky-Vladimirsky distrtibution [1]. Many computer codes have been developed to take into account non-linear space charge fields, various magnetic and electrostatic elements, beam bunching etc. [2, 3, 4, 5, 6, 7, 8]. The space charge influence on the transverse ion beam dynamics in the axial injection line for the U400 cyclotron has been investigated by use of the method of the distribution function moments [9], [10]. No particles are traced in this approach. The information computed concerns averaged values (transverse beam sizes and velocities, cross terms). The method previously had been applied for the calculation of a high intensity beam injection line for the INR Meson Factory (Troitsk) [11]. The program used then has been substantially revised now to extend magnetic elements available for a user, to make it comfortable in the interactive mode and to provide it with the proper graphics.

2 Moment Method Use for Beam Transport Calculation

Let us define second order distribution function moments matrix M

$$M^{II} = \begin{pmatrix} M_{xx} & M_{xv} \\ M^{\star}_{xv} & M_{vv} \end{pmatrix}, \tag{1}$$

$$M^{i,j}_{xx} = \overline{x_i x_j} = \int f x_i x_j d\vec{x} d\vec{v}, \quad M^{i,j}_{xv} = \overline{x_i v_j} = \int f x_i v_j d\vec{x} d\vec{v},$$
$$M^{i,j}_{vv} = \overline{v_i v_j} = \int f v_i v_j d\vec{x} d\vec{v}, \quad i,j = 1,2. \tag{2}$$

Here f is the beam distribution function, and x_i, v_j are particle coordinates and velocities. The evolution of the matrix M under beam transportation through a beamline with linear electromagnetic fields is defined by the system of equations [10]

$$\frac{dM_{xx}}{ds} = M_{xv} + M^{\star}_{xv},$$
$$\frac{dM_{xv}}{ds} = M_{vv} + M_{xx}B^{\star} + M^{\star}_{xv}A^{\star},$$
$$\frac{dM_{vv}}{ds} = BM_{xv} + M^{\star}_{xv}B^{\star} + AM_{vv} + M_{vv}A^{\star}, \tag{3}$$

where $s = v_z t$ and $v_z = \beta c$ are longitudinal coordinate and velocity, the matrix B includes external and space charge forces, matrix A includes external forces only (magnetic field defined by particle transverse motion is neglected), the matrices M^*, A^*, B^* are transposed matrices M, A, B. The matrices A and B for typical magnetic elements are given in Appendix.

Space charge forces in (3) are considered to be linear, that is correct for the beam distribution functions with a particle density constant on 4D ellipse in transverse beam phase space [10], [12]. In this case

$$B^s = \frac{I}{\beta^2\gamma^2 I_A} \; \frac{M_{xx}^{-1/2}}{Sp M_{xx}^{1/2}} \tag{4}$$

with Alfven current $I_A = \beta\gamma A m c^2/Ze, \gamma$ is relativistic factor, Am and Ze are ion mass and charge. Matrix $M_{xx}^{1/2}$ is defined by conditions $M_{xx}^{1/2}\, M_{xx}^{1/2} = M_{xx}$, $M_{xx}^{1/2}\, M_{xx}^{-1/2} = I$, I is the unit matrix. Direct calculation gives for the elements of the matrices $M^{1/2}$ and $M^{-1/2}$

$$M_{11}^{1/2} = \frac{M_{11} + (det M_{xx})^{1/2}}{[Sp\ M_{xx} + 2(det M_{xx})^{1/2}]^{1/2}}, \quad M_{12}^{1/2} = \frac{M_{12}}{[Sp\ M_{xx} + 2(det M_{xx})^{1/2}]^{1/2}},$$
$$M_{22}^{1/2} = \frac{M_{22} + (det M_{xx})^{1/2}}{[Sp M_{xx} + 2(det M_{xx})^{1/2}]^{1/2}}, \quad M_{21}^{1/2} = M_{12}^{1/2}, \tag{5}$$

$$M_{11}^{-1/2} = \frac{M_{22}^{1/2}}{det(M^{1/2})}, \quad M_{12}^{-1/2} = -\frac{M_{12}^{1/2}}{det(M^{1/2})}, \quad M_{21}^{-1/2} = M_{12}^{-1/2}, \quad M_{22}^{-1/2} = \frac{M_{11}^{-1/2}}{det(M^{1/2})}, \tag{6}$$

where M_{ij} are elements of the matrix M_{xx}.

3 Brief Description of Program

The program is based on solving of 10 differential equations (3) by 4^{th} order Runge-Kutta method. Moments of the fisrt order can be added easily. It has been written for IBM PC, and is used in interactive mode, with graphic presentation of results. File with beam parameters along a beamline is also available.

Along with magnetic elements listed in Appendix, arbitrary solenoidal magnetic field given by user, is accepted also. After the first beam transportation through a beamline program puts in the memory beam parameters at the entrance and exit of every element. For the next variant, user can change parameters in arbitrary element and start from it, keeping envelopes before untouched. The beamline parameters can be optimized in order to fit the M matrix elements.

4 Space Charge Effects in Axial Injection Line for U400 Cyclotron

The following parameters of the beam were used under calculations: both horizontal and vertical sizes are equal to 0.4cm, the emittance is 150πmm·mrad. Two variants of ion mass to charge ratio A/Z equal 5 and 10 have been used under investigation. The beam kinetic energy at the injection line entrance corresponds to the ECR source voltage of 26.5kV and 13kV for these cases respectively. The parameters of the axial injection line from the ECR ion source for the U400 cyclotron are summarized in Table 1.

Element	Magnetic field kGs	Length m	Aperture/Gap cm	Pole face angle degree
Magnetic field of ECR source	from 7.4 to 0	0.44		
Drift space		0.21		
Solenoid	≤ 6.0	0.134	8.5	
Drift space		0.821		
Analysis magnet, 102°	1.4998/1.4889	0.3115	7	33.5
Drift space		1.433		
Vertical bending magnet, 90°	1.3124/1.3024	0.31416	8	26.5
Drift space		1.10		
Axial magnetic field	0.2	0.1		
Drift space		0.25		
Solenoid	≤ 1.7	1.12	21.5	
Cyclotron magnetic field	up to 22.2	2.3875		

Table 1: Parameters of Axial Injection Line for Cyclotron U400

Two values of magnetic fields given for the analysis magnet and for the vertical bending magnet correspond to A/Z equal to 5 and 10 respectively. For the solenoids the maximum magnetic fields are given. It should be noted that second solenoid has three equal sections with independent power supply.

In the beginning of the study, optimized values of the beam sizes at the end of the beamline in the absence of space charge fources have been found. They are σ_x=5.4mm, σ_y=3.5 mm for A/Z=5 and σ_x=3.7 mm, σ_y=6.0 mm for A/Z=10. It is necessary to note that the strong axial magnetic field (more than 20 kGs) of the U400 cyclotron at the end of the injection line makes the control of the exit beam parameters to be very difficult. To improve it, the installation of two quadrupoles located after the vertical bending magnet is under consideration.

With taking into account space charge forces (and keeping magnetic fields in solenoids unchanged), the beam sizes at the end of the injection line are increased, as it is shown in Fig.1 and Fig.2. From these figures one concludes that the space charge effects are important when current $I > 500\mu$A for A/Z=5 and $I > 150\mu$A for A/Z=10.

The influence of space charge effects on beam envelopes in the injection line for A/Z=5 and current $I = 1000\mu$A is shown in Fig.3 and 4. For comparison the beam envelopes in the absence of space charge effects (I=0) are shown also. The beam sizes at the end of the injection line are increased to σ_x=5.9mm and σ_y=8.8mm compared with σ_x=5.4mm and σ_y=3.5mm for negligible space charge effects. To improve beam envelopes the solenoid strengths have been readjusted. As a result the beam sizes for I =1000μA have been made very close to those for $I = 0$ (Fig.5 and 6). After optimization beam sizes at the end of the injection line are σ_x=5.2mm and σ_y=4.8mm.

The beam envelopes in injection line for A/Z=10 variant without space charge forces are shown in Fig.7 and 8. The space charge effects in this case are much more stronger in accordance with $\beta\gamma$A/Z scaling (formula (4)). The beam sizes at the end of injection line are σ_x=13.1mm and σ_y=9.8mm. Readjusting of the solenoids strengths is not so effective as in the previous case, but keeps beam sizes tolerable (Fig.9 and 10). The beam sizes at the end of the beamline are σ_x=6.6mm, σ_y=7.2mm.

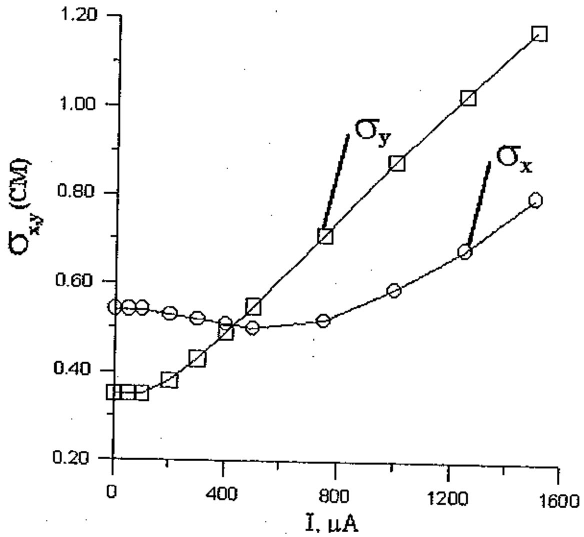

Figure 1: Dependences of the beam sizes at the end of injection line versus current for A/Z=5.

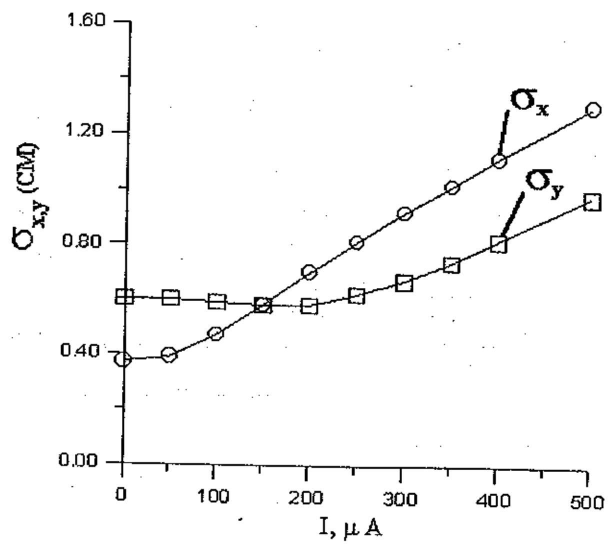

Figure 2: Dependences of the beam sizes at the end of injection line versus current for A/Z=10.

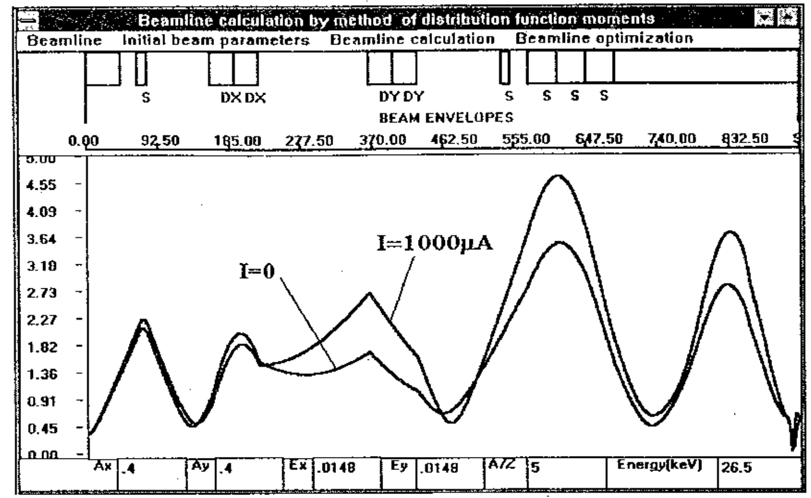

Figure 3: Horizontal beam envelopes for A/Z=5, I=0 and I=1000 μA. No optimization has been made in solenoid strenghs.

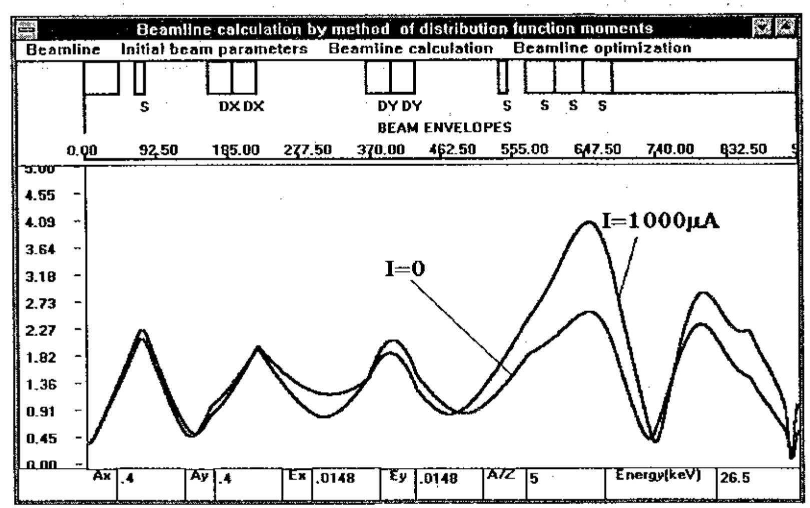

Figure 4: Vertical beam envelopes for A/Z=5, I=0 and I=1000 μA. No optimization has been made in solenoid strengths.

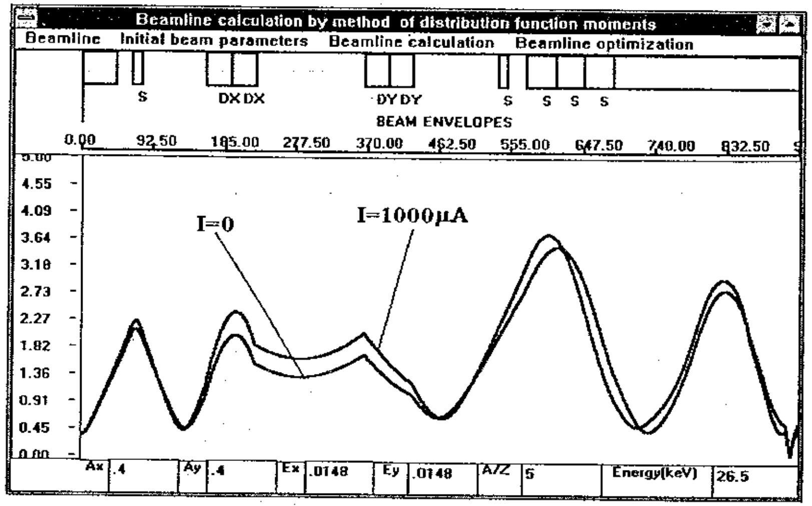

Figure 5: Horizontal beam envelopes for A/Z=5 in the absence of space charge effects (curve marked by I=0), and for current I=1000 μA. For the second case injection line has been optimized.

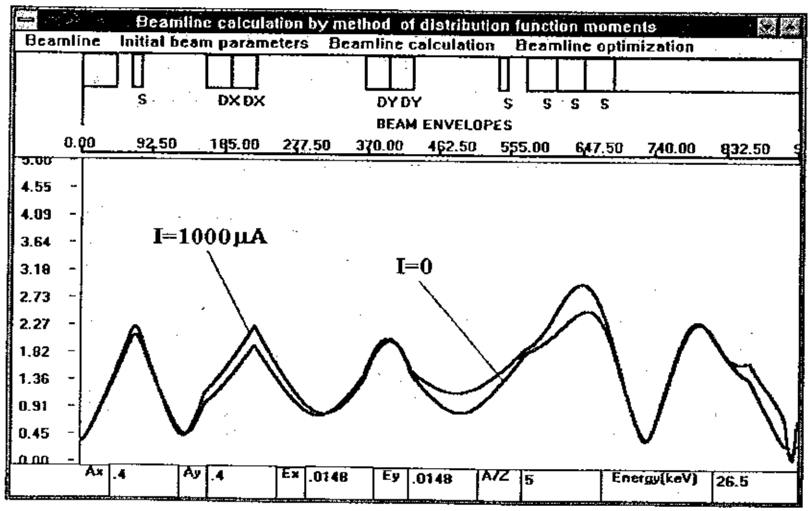

Figure 6: Vertical beam envelopes for A/Z=5 in the absence of space charge effects (curve marked by I=0), and for current I=1000 μA. For the second case injection line has been optimized.

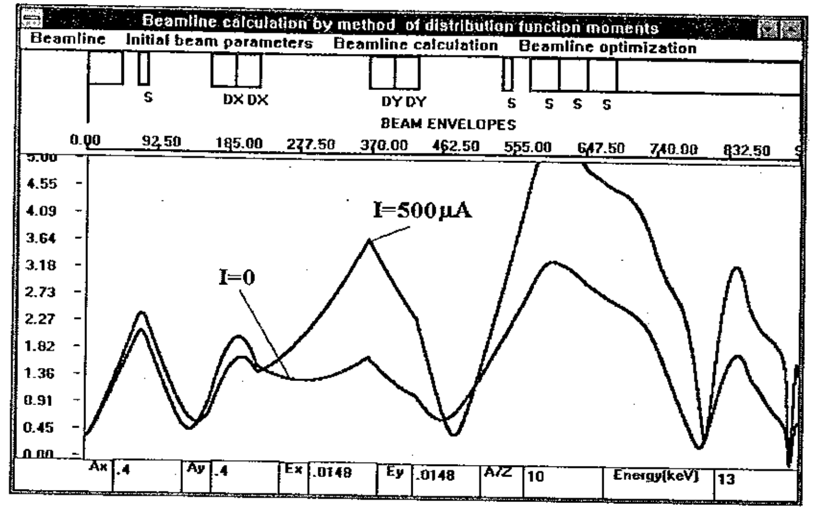

Figure 7: Horizontal beam envelopes for A/Z=10,I=0 and I=500μA. No optimization has been made in solenoid strengths.

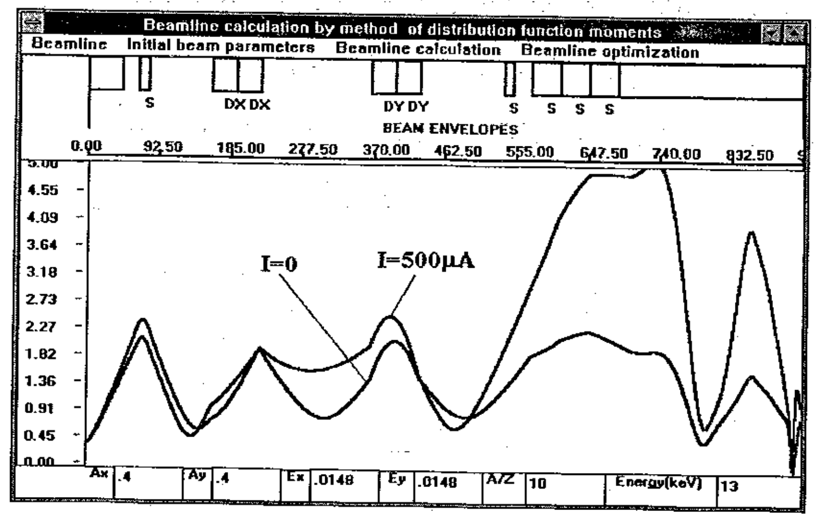

Figure 8: Vertical beam envelopes for A/Z=10, I=0 and I=500μA. No optimization has been made in solenoid strengths.

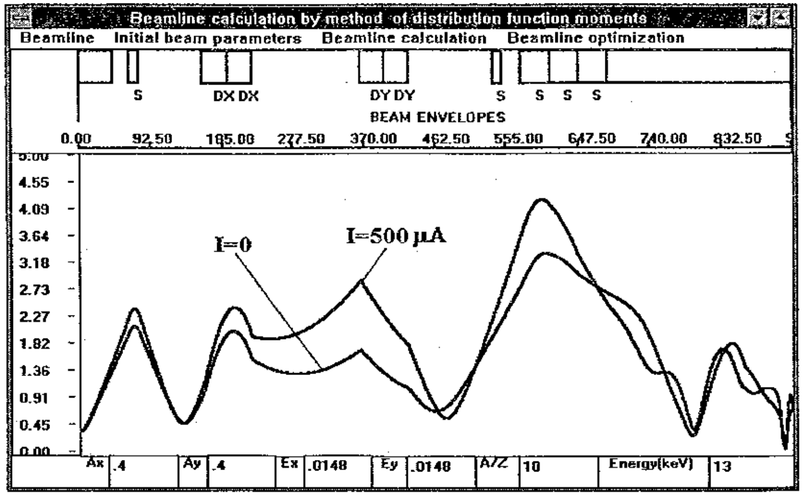

Figure 9: Horizontal beam envelopes for A/Z=10 in the absence of space charge effects (curve marked by I=0), and for current I=500 μA. For the second case injection line has been optimized.

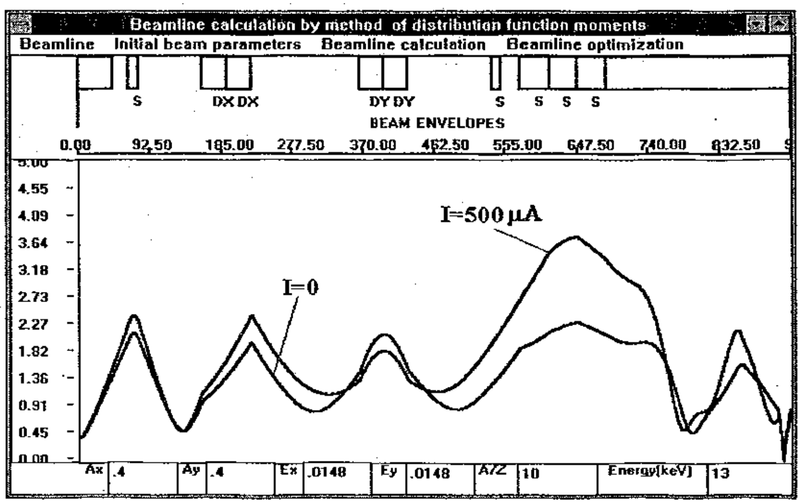

Figure 10: Vertical beam envelopes for A/Z=10 in the absence of space charge effects (curve marked by I=0), and for current I=500 μA. For the second case injection line has been optimized.

Appendix: matrices A and B for magnetic elements

Solenoid with axial magnetic field B_z

$$B^{ext} = \frac{e}{2\beta\gamma mc^2}\frac{\partial B_z(s)}{\partial z}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad A^{ext} = \frac{eB_z(s)}{\beta\gamma mc^2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{A.1}$$

quadrupole with gradient G

$$B^{ext} = \frac{eG}{\beta\gamma mc^2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_{ext} = 0, \tag{A.2}$$

horizontal bending magnet (sector type)

$$B^{ext} = -\left(\frac{eB_y}{\beta\gamma mc^2}\right)^2 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_{ext} = 0, \tag{A.3}$$

vertical bending magnet (sector type)

$$B^{ext} = -\left(\frac{eB_x}{\beta\gamma mc^2}\right)^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_{ext} = 0, \tag{A.4}$$

accelerating cavity

$$A^{ext} = \frac{1}{\gamma}\frac{d\gamma}{ds} I. \tag{A.5}$$

To take into account pole face rotation, we use well known formulae for beam coordinates and velocities transformation for a particle passing through edge lens [13]. If pole face rotation angle is α and bending radius is ρ, then new coordinares and velocities are expressed through old ones in a horizontal bending magnets by

$$\begin{pmatrix} x'_1 \\ x'_2 \\ v'_1 \\ v'_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\tan\alpha}{\rho} & 0 & 1 & 0 \\ 0 & -\frac{\tan\alpha}{\rho} & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0_1 \\ x^0_2 \\ v^0_1 \\ v^0_2 \end{pmatrix} = W \begin{pmatrix} x^0_1 \\ x^0_2 \\ v^0_1 \\ v^0_2 \end{pmatrix}. \tag{A.6}$$

For a vertical bending magnet α is changed to $-\alpha$. Then elements of moment matrix M are transformed accordingly

$$M_{ik} = z'_i z'^*_k = (Wz^0)_i (Wz^0)^*_k = W z^0_i (z^0_k)^* W^* = WMW^*, \tag{A.7}$$

where $\vec{z} = (x_1, x_2, v_1, v_2)$ and W^* is transposed matrix W.

5 Conclusion

The results of calculations show, that the space charge influences essentially on the ion beam dynamics in the axial injection line, when the beam current exeeds 500μA for A/Z=5 and 150μA for A/Z=10. The space charge effects can be corrected by proper readjusting of solenoid fields for currents up to 1500μA (A/Z=5) and 500μA (A/Z=10). The next step to take into account ions with different charges which influence strongly on the ion beam with selected charge through the injection line up to analysis magnet is now under consideration.

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