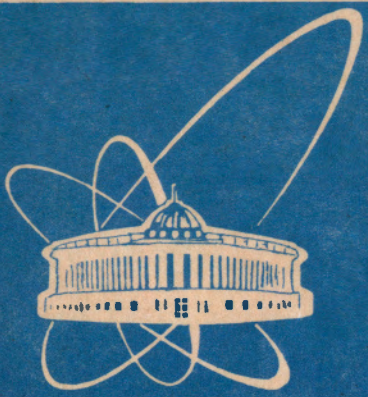

ОБЪЕДИНЕННЫЙ
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THE COMPUTATION OF ELECTRON COOLING PROCESS IN A STORAGE RING

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Лаврентьев А.Ю., Мешков И.Н. Е9-95-317

Расчет процесса электронного охлаждения в накопителе

Представлена программа расчета процесса электронного охлаждения в ионном накопителе. Программа позволяет учесть главные особенности динамики охлажденной частицы в накопителе: бетатронные колебания, дисперсию накопителя, влияние пространственного заряда электронного пучка, фазовое движение в присутствии ВЧ-напряжения (режим сгруппированного пучка).

Программа свободно распространяется авторами по просьбам пользователей.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Lavrentev A.Yu., Meshkov I.N. E9-95-317

The Computation of Electron Cooling Process in a Storage Ring

The program of computation of electron cooling process in a cooler-storage ring is presented. This program permits to take into account the main peculiarities of the particle dynamics in the storage ring: the particle betatron oscillations, the ring dispersion, the influence of electron beam space charge and phase space motion in presence of RF voltage (the bunched ion beam regime).

The program is available freely upon the user request to authors.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995

Introduction

The complicated dependence of electron cooling friction force on parameters brings many difficulties at an attempt of analytical calculation of the cooling process. And the problem becomes practically unresolvable when one tries to take into account the influence of such very important effects like particle betatron oscillations, storage ring dispersion, space charge of the cooling electron beam and electron energy variation. Only rough estimations can be done in such cases [1, 2]. However, a practical application of electron cooling methods needs knowledge of cooling time value and its dependence on parameters.

A certain attempt of numerical integration of cooling process was made in [3]. However the way used there does not permit to consider the effects mentioned above. The computation program, presented in this report, calculates the evolution of particle oscillation amplitude, using analytical formulae for friction force from [2], and takes into account the particle motion in focusing and radio-frequency (RF) systems of a storage ring and the influence of cooling electron beam space charge.

The principle of the program is described and a few examples are given for demonstration of the program efficiency. The method is applied to the case of coasting and bunched beam.

1 Principle of the particle motion computation

One considers the particle with charge Ze and mass $M = Am_p$ (e – the electron charge, m_p – the proton mass), which moves in a constant magnetic field of a storage ring of circumference C and interacts with electrons of an electron beam in the cooling section of the storage ring. One

can introduce dimensionless transverse particle coordinates x_α instead of dimensional ones X_α

$$x_\alpha = \frac{Q_\alpha}{R_s} X_\alpha, \quad \alpha = x, z, \tag{1.1}$$

where $Q_{x,z}$ – betatron numbers of the storage ring, $R_s = C/2\pi$ – its average radius. The transverse components P_α of the particle momentum relate to x_α by expressions

$$\theta_\alpha = \frac{P_\alpha}{P_0} = \frac{dX_\alpha}{ds}, \qquad \alpha = x, z, \tag{1.2}$$

here s – the coordinate along the particle trajectory, and P_0 is the average value of the particle momentum

$$P_0 = \beta\gamma Mc, \qquad \beta = \frac{v_0}{c}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}},$$

where v_0 is the average particle velocity in the particle beam to be cooled, c – the light velocity. Besides,

$$\theta_s \equiv \frac{\Delta P_s}{P_0}$$

is the relative difference between longitudinal particle momentum P_s and average momentum P_0. For convenience and uniform, hereinafter one uses $x_s \equiv 0$ and $Q_s \equiv 0$. Then, the change of the coordinates x_α and θ_α ($\alpha = x, z, s$) from exit of the cooling section to its entrance, after a turn in the storage ring, is given by the following matrix expression:

$$\begin{pmatrix} x_\alpha \\ \theta_\alpha \end{pmatrix}_{\text{entrance}} = \begin{pmatrix} \cos\psi_\alpha & \sin\psi_\alpha \\ -\sin\psi_\alpha & \cos\psi_\alpha \end{pmatrix} \begin{pmatrix} x_\alpha \\ \theta_\alpha \end{pmatrix}_{\text{exit}} \tag{1.3}$$

where

$$\psi_\alpha = 2\pi Q_\alpha.$$

Here one neglects the deviation of betatron phase along the cooling section. The exact formula is $\psi_\alpha = 2\pi Q_\alpha (1-\eta)$, $\eta = l/C$, where l is the cooling section length. The cooling friction force decreases the particle momentum components and does not change coordinates X_α of

the particle on each certain pass of the particle through the cooling section. Therefore,

$$(x_\alpha)_{\text{exit}} = (x_\alpha)_{\text{entrance}}, \qquad (\theta_\alpha)_{\text{exit}} = (\theta_\alpha)_{\text{entrance}} + F_\alpha \frac{l}{v_0}, \tag{1.4}$$

where F_α is α-component of the friction force in laboratory reference frame (LRF) (see (2.6) and (2.7) below). This approximate formula is used instead of exact integration of the particle motion equations in the cooling section to decrease the computation time, which arises drastically in opposite case, that makes an use of PC for such computations absolutely unrealistic. A special "trick" is used to reach necessary precision of calculations (see item 8 in section 2.2).

The next question is how to adapt the friction force formulae, written for a particle rest frame (PRF) [2], to LRF where equations (1.1) – (1.4) are given. The simplest way is the transformation of friction force and particle momentum components from one reference frame to another one. So, the friction force in PRF is described in [2] by the formulae (1.36), (1.37).

To find out the friction force magnitude on each step of integration, one has to know the particle velocity components in PRF [2]

$$V_{x,z} = \gamma\beta c\theta_{x,z}, \qquad V_{\|} = \beta c\theta_s,$$

to insert them in formulae cited above, and to transform the friction force component from PRF to LRF:

$$F_{x,z} \equiv (F_{x,z})_{\text{LRF}} = \frac{1}{\gamma}(F_{x,z})_{\text{PRF}}, \qquad F_s \equiv (F_s)_{\text{LRF}} = (F_s)_{\text{PRF}}.$$

One needs also to use well-known relations between average (over a particle turn) electron density $\langle n_e \rangle$ in PRF and electron current density J in LRF:

$$\langle n_e \rangle = \eta n_e = \frac{\eta J}{e\gamma\beta c}.$$

After such a procedure one obtains the formula for friction force in LRF presented below (see (2.6) and (2.7)).

In presence of RF accelerating voltage (bunched beam) and cooling electron beam the computation of the particle dynamics is performed with the use of the equations of the particle phase motion. They give the

particle momentum change δP_s which occurs at radio-frequency cavity (RFC) crossing:

$$\delta P_s = \frac{ZeV\sin\varphi}{\beta c}, \qquad \delta\theta_s \equiv \frac{\delta P_s}{P_0} = \frac{Z}{A}\frac{eV\sin\varphi}{\beta^2\gamma m_p c^2}. \tag{1.5}$$

Here V, φ – RF voltage amplitude and phase (when the particle crosses RFC). This momentum change leads to the change the particle circulation frequency ω:

$$\frac{\delta\omega}{\omega} = \eta_\omega\frac{\delta P_s}{P_0} = \eta_\omega\delta\theta_s, \qquad \eta_\omega = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\rm tr}^2}, \tag{1.6}$$

where $\gamma_{\rm tr}$ is so-called transition energy factor, which corresponds to the ring transition energy $\mathcal{E}_{\rm tr} = (\gamma_{\rm tr} - 1)\,Am_pc^2$.

The equations (1.5) and (1.6) are used for calculation of the particle phase motion (see section 2.3).

2 Program scheme

The program consists of several parts, described below. The concrete units, which are used for all the parameters, are given in section 2.1 and the final formulae below are written in these units.

2.1 Input

The following parameters are to be introduced (default values are shown in the rightmost column, they are chosen for the case of $^{208}Pb^{53+}$ ion beam cooling in LEAR storage ring at CERN):

Ion beam characteristics		
$(X_{x,z})_0$, initial coordinates[1],	cm	0, 0
$(\theta_{x,z,s})_0$, initial angles,	mrad	3, 3, 3
Bunched beam		No
Ion characteristics		
$\mathcal{E}$, energy per nucleon,	MeV/amu	4.2327
A, atomic mass,	amu	208
Z, charge number,	—	53

(to be continued on the next page)

(continued)

φ_0, initial phase[2],	rad	π
Electron beam characteristics		
I, current,	Amp	0.5
a, beam radius,	cm	2.5
θ_e, angular spread,	mrad	3
B, magnetic field,	kG	0.6
η_n, neutralization coefficient,	—	0
$\theta_{\|\|}^0$, initial momentum shift,	mrad	0
Sweeping		No
$\tau_{\rm sweep}$, sweeping time[3],	ms	80
Ring characteristics		
C, circumference,	m	78
η, ratio of cooling section length to C,	—	0.02
D, dispersion in cooling section,	m	3.6
$Q_{x,z}$, betatron numbers[4],	—	2.305, 2.73
$\gamma_{\rm tr}$, transition energy factor[2],	—	10
V, RF voltage[2],	keV	1
Result representation		
$Y_{\rm max}$, maximal result value,	mrad	4
$t_{\rm max}$, maximal time,	ms	200

2.2 Procedure. Coasting beam

This part of the program calculates the cooling process. It starts with calculations of all the constants:

1. the relativistic factors:

$$\xi = \frac{\mathcal{E}}{m_pc^2} = \frac{\mathcal{E}_{\rm [MeV/amu]}}{938}, \qquad \gamma = 1 + \xi, \qquad \beta = \frac{\sqrt{\xi\,(\xi+2)}}{\gamma};$$

[1] $X_s \equiv 0$.
[2] The input is available only if beam is bunched.
[3] The input is available only if sweeping is on.
[4] $Q_s \equiv 0$.

2. the relative velocity spread of electrons (see [2]) and the lower limit of ion parameters $\theta_\alpha^{\text{ampl}}$ (see (2.10), (2.11)):

$$\theta_{\parallel} = \frac{1}{\beta}\sqrt{\frac{e^2}{mc^2}n_e^{1/3}} = \\ = 1.068 \cdot 10^{-2}\sqrt[6]{\frac{I_{[\text{Amp}]}}{\gamma\beta^7 a^2_{[\text{cm}]}}}\ [\text{mrad}], \qquad \theta_{\min} = 0.05\theta_{\parallel};\ (2.1)$$

3. the particle revolution period and the average ring radius:

$$T_0 = \frac{C}{\beta c} = 3.333 \cdot 10^{-3}\frac{C_{[\text{m}]}}{\beta}\ [\mu\text{s}], \qquad R_s = \frac{C_{[\text{m}]}}{2\pi}\ [\text{m}];$$

4. the constant in the friction force formula:

$$\Delta = \frac{2r_p}{\beta^5\gamma^5}\frac{Z^2}{A}\frac{\eta C}{a^2}\frac{eI}{mc^3} = 1.8 \cdot 10^{-9}\frac{Z^2}{A}\frac{I_{[\text{Amp}]}}{\beta^5\gamma^5}\frac{\eta C_{[\text{m}]}}{a^2_{[\text{cm}]}}\ [\text{mrad}], \quad (2.2)$$

where r_p – the proton classic radius, m – electron mass.

To obtain the approximate cooling time, which is widely used for estimation, the following formula is used (it's convenient for comparison with numerical integration results):

$$(\tau_{\text{cool}})_{\text{theor}} = \frac{\beta^4\gamma^5}{6cr_p}\frac{mc^3a^2}{\eta eI_e}\frac{A}{Z^2}\left(\theta_{x,0[\text{rad}]}\right)^3 = \\ = 615\, a^2\frac{\beta^4\gamma^5}{\eta I}\frac{A}{Z^2}\left(\theta_{x,0[\text{mrad}]}\right)^3\ [\text{ms}]. \quad (2.3)$$

The initial values of dimensionless coordinates are calculated:

$$x_{\alpha,0[\text{mrad}]} = 10\,\frac{Q_\alpha}{R_{s[\text{m}]}}X_{\alpha,0[\text{cm}]}, \qquad \alpha = x, z, s.$$

To simulate the cooling process occurring in time, on each step on $t_{[\text{ms}]}$ from 0 to $t_{\max}$ the following computations are taken place (the time t is stepped by T_0):

1. The square of the particle distance from the electron beam axis

$$r^2 = \frac{C^2}{4\pi^2}\left(\left(\frac{x_x}{Q_x} + \frac{2\pi D}{C}\theta_s\right)^2 + \left(\frac{x_z}{Q_z}\right)^2\right) \times 10^{-2}\ [\text{cm}]. \quad (2.4)$$

2. The shift in velocities of the particle and average electron velocity

Sweeping = Yes:

$$\theta_{\parallel}^* = \begin{cases} \theta_{\parallel}^0\left(1 - \dfrac{t}{\tau_{\text{sweep}}}\right), & 0 \le t \le \tau_{\text{sweep}} \\ \theta_{\parallel}^0\left(\dfrac{t}{\tau_{\text{sweep}}} - 2\right), & \tau_{\text{sweep}} < t \le 2\tau_{\text{sweep}} \\ 0, & \text{otherwise} \end{cases}; \quad (2.5)$$

Sweeping = No:

$$\theta_{\parallel}^* = \theta_{\parallel}^0.$$

3. The shift in electron velocity due to electron beam space charge

$$\delta\theta_{\parallel} = (1 - \eta_n)\frac{eI}{\beta^3\gamma mc^3}\frac{r^2}{a^2} = (1 - \eta_n)\frac{I}{17\beta^3\gamma}\frac{r^2}{a^2}\ [\text{mrad}].$$

4. The difference between particle velocity and electron one[5]

$$\theta_s^* = \theta_s - \left(\theta_{\parallel}^* + \delta\theta_{\parallel}\right).$$

5. The full particle velocity in PRF in units $\gamma\beta c$

$$\theta = \sqrt{\theta_x^2 + \theta_z^2 + \left(\frac{\theta_s^*}{\gamma}\right)^2}.$$

6. The friction force calculation. One introduces

$$\vec{\theta}^* = \{\theta_x, \theta_z, \theta_s^*\}.$$

The friction force $\vec{F} = \{F_x, F_z, F_s\}$ can be represented as following (see details in [2] and section 1).

$$\vec{F} = -\Delta\frac{\beta c}{\eta C}\vec{\mathcal{F}}, \qquad \vec{\mathcal{F}} = \{\mathcal{F}_x, \mathcal{F}_z, \mathcal{F}_s\}, \quad (2.6)$$

[5] One should mention that the worst case corresponds to the cooling of the particle with $\theta_s < 0$. Moreover, $\theta_{\parallel}^*$ relates to cathode potential shift ΔU by formula $\Delta U = \beta^2\gamma\theta_{\parallel}^* mc^2/e$.

where Δ is given by formula (2.2). If the condition $r > a$ (see (2.4)) is met i.e. the ion is out of the electron beam, then the friction force is equal to zero, and values of function $\vec{\mathcal{F}}$ is set to zero. Otherwise

$$\mathcal{F}_{x,z}\left(\vec{\theta}^*\right) = \begin{cases} (2L_F + k_\perp L_M)\dfrac{\theta_{x,z}}{\theta^3}, & \{\mathrm{I}\} \\ 2(L_F + N_{\mathrm{col}} L_A)\dfrac{\theta_{x,z}}{\theta_e^3} + k_\perp L_M \dfrac{\theta_{x,z}}{\theta^3}, & \{\mathrm{II}\} \\ 2(L_F + N_{\mathrm{col}} L_A)\dfrac{\theta_{x,z}}{\theta_e^3} + L_M \dfrac{\theta_{x,z}}{\left(\frac{\theta_\parallel}{\gamma}\right)^3}, & \{\mathrm{III}\} \end{cases}$$

$$\mathcal{F}_s\left(\vec{\theta}^*\right) = \begin{cases} (2L_F + k_s L_M + 2)\dfrac{\theta_s^*}{\theta^3}, & \{\mathrm{I}\} \\ 2\operatorname{sgn}\theta_s^*\,(L_F + N_{\mathrm{col}} L_A)\dfrac{\gamma}{\theta_e^2} + \\ \qquad + (k_s L_M + 2)\dfrac{\theta_s^*}{\theta^3}, & \{\mathrm{II_a}\} \\ 2(L_F + N_{\mathrm{col}} L_A)\dfrac{\theta_s^*}{\theta_e^2 \frac{\theta_\parallel}{\gamma}} + L_M \dfrac{\theta_s^*}{\left(\frac{\theta_\parallel}{\gamma}\right)^3}, & \{\mathrm{II_b},\ \mathrm{III}\} \end{cases} \tag{2.7}$$

Domains I, II = $\mathrm{II_a} \cup \mathrm{II_b}$, and III for $\vec{\theta}^*$ are shown in Fig. 1 which corresponds to the domain choice defined in [2].

Coulomb logarithms are defined by the formulae

$$L_M = \ln\frac{R}{2\langle\rho_\perp\rangle}, \quad L_A = \ln\frac{2\langle\rho_\perp\rangle}{\rho_F}, \quad L_F = \ln\frac{\rho_F}{\rho_{\min}}.$$

Note that if argument of a logarithm is less than 1, then the logarithm value is set to zero.

$$R = \max\left\{\left(V_x^2 + V_z^2 + V_\parallel^2\right)^{1/2}\sqrt{\frac{m}{4\pi n_e e^2}},\ \sqrt[3]{\frac{3Z}{n_e}}\right\} =$$

$$= \max\left\{0.06535\, a\theta\sqrt{\frac{\beta^3\gamma^3}{I}},\ 0.00355\sqrt[3]{\frac{\beta\gamma a^2 Z}{I}}\right\}$$

is the maximum impact parameter;

$$\langle\rho_\perp\rangle = \frac{\beta\gamma\theta_e mc^2}{eB} = \frac{1.7\cdot 10^{-3}\beta\gamma\theta_e}{B} \text{ – Larmor radius;}$$

$$\rho_F = \langle\rho_\perp\rangle\frac{\left|\theta + \frac{\theta_\parallel}{\gamma}\right|}{\theta_e} \text{ – the intermediate impact parameter;}$$

$$\rho_{\min} = \frac{Ze^2}{m}\frac{1}{\beta^2\gamma^2(\theta + \theta_e)^2} = 2.818\cdot 10^{-7}\frac{Z}{\beta^2\gamma^2(\theta + \theta_e)^2}$$

is the minimum impact parameter;

$$N_{\mathrm{col}} = 1 + \left[\frac{\theta_e}{\pi\left|\theta + \frac{\theta_\parallel}{\gamma}\right|}\right] \text{ – the number of adiabatic collisions}^6;$$

$$k_\perp = 1 - 3\left(\frac{\theta_s^*}{\gamma\theta}\right)^2, \qquad k_s = 2 + k_\perp.$$

7. The deviation of the particle angular spread after crossing of the cooling section

$$\Delta\vec{\theta} = \Delta\cdot\vec{\mathcal{F}}\left(\vec{\theta}^*\right). \tag{2.8}$$

8. Here the "trick" mentioned above is used: if for any α the deviation is large, i.e. $|\Delta\theta_\alpha| > 0.05\,|\theta_\alpha|$, then the cooling section is divided into two equal segments, all the calculations (items 1 – 7) are repeated for the first segment and for this segment from (2.8) one obtains the partial deviation value, by means of which the value $\vec{\theta}$ can be revised and used on the rest of the cooling section. The partitioning procedure can be applied several times until all partial deviations become sufficiently small. Their sum gives $\Delta\vec{\theta}$ for the whole cooling section. The constant 0.05 is chosen experimentally.

   One has to stress that the use of the straightforward computation is impossible here: an application of Runge–Kutt method to the equation

$$\frac{d\vec{P}}{dt} = \vec{F}\left(\vec{\theta}^*\right),$$

   or its form

$$\frac{d\vec{\theta}}{ds} = \frac{\vec{F}\left(\vec{\theta}^*\right)}{P_0\beta c}, \qquad \text{where } 0 \le s \le \eta C,$$

   slows down the program significantly.

[6] [·] stands for the whole part.

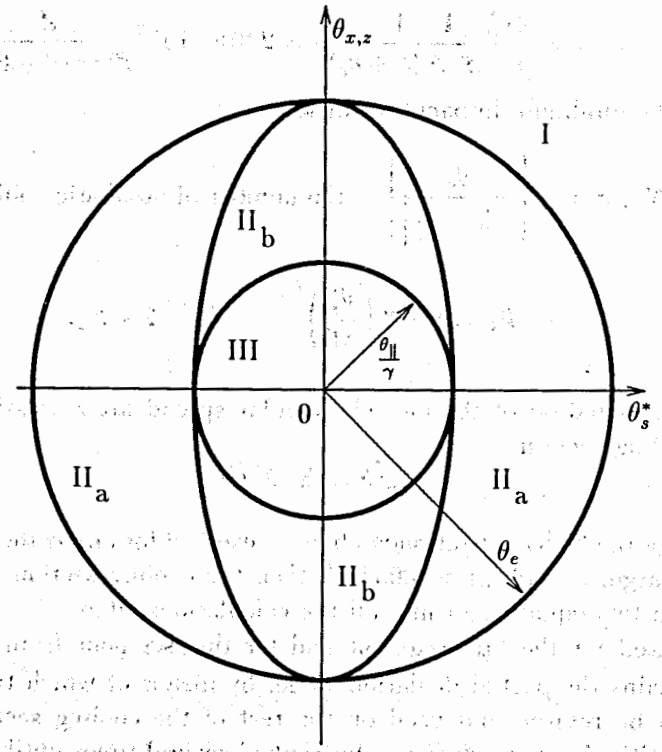

Figure 1: Domains in the velocity space for $\mathcal{F}\left(\vec{\theta}^*\right)$.
θ_e, $\theta_\parallel/\gamma$ – the electron velocity spread in units $\gamma\beta c$.

9. The calculation of new particle coordinates after crossing the cooling section, at its exit:

$$x'_\alpha = x_\alpha, \qquad \theta'_\alpha = \theta_\alpha - \Delta\theta_\alpha.$$

10. If the ion beam is not bunched, new values of x and θ, which are used on the next turn at the entrance of the cooling section, can be found:

$$\begin{pmatrix} x_\alpha \\ \theta_\alpha \end{pmatrix} = \begin{pmatrix} \cos\psi_\alpha & \sin\psi_\alpha \\ -\sin\psi_\alpha & \cos\psi_\alpha \end{pmatrix} \begin{pmatrix} x'_\alpha \\ \theta'_\alpha \end{pmatrix}. \qquad (2.9)$$

Here $\psi_\alpha = 2\pi Q_\alpha$.

2.3 Procedure. Bunched beam

When one wishes to compute a bunched beam regime, the answer 'Yes' has to be introduced in the line "Bunched beam?" and the program asks to input parameters φ_0, γ_{tr}, and V. Afterwards, the program takes into account phase motion of the particle.

The constant η_ω (1.6) is calculated as well as the parameters to be displayed on the screen:

$$\tau_{\text{phase}} = \frac{C}{c}\sqrt{2\pi\frac{A}{Z}\frac{m_p c^2}{eV}\gamma\eta_\omega} = 8.09\cdot 10^{-3} C_{[\text{m}]}\sqrt{\frac{A}{Z}\frac{\gamma\eta_\omega}{V_{[\text{keV}]}}}\ [\text{ms}]$$

– the period of small phase oscillations,

$$(\theta_s)_{\text{separatrix}} = \frac{1}{\beta}\sqrt{\frac{2}{\pi}\frac{eV}{\eta_\omega\gamma m_p c^2}}\frac{Z}{A} = \frac{1}{\beta}\sqrt{\frac{2}{\pi}\frac{V_{[\text{keV}]}}{0.938\eta_\omega\gamma}}\frac{Z}{A}\ [\text{mrad}]$$

– the separatrix value of momentum spread.

Additional computations are following:

11. The ring is divided into two parts:

(1) the half of the ring from the cooling section exit up to RFC;

(2) the half of the ring from RFC up to the cooling section entrance.

12. Values x_α, θ_α at RFC (after exit from the cooler) can be obtained from formula (2.9) when $\psi_\alpha = \pi Q_\alpha$.

13. The phase shift during the time, when particle travels along the part (1):

$$\varphi^{(1)} = \varphi^{(2)} + \pi\frac{\omega}{\omega - \Delta\omega} - \pi = \varphi^{(2)} + \frac{\pi}{1 - \eta_\omega\theta_s\cdot 10^{-3}} - \pi.$$

Note that θ_s is measured in mrad and the initial value of $\varphi^{(2)}$ is specified as φ_0 during the input.

14. The deviation of θ_s due to crossing of RFC is given by formula

$$\delta\theta_s^{\text{RFC}} \doteq \frac{Z}{A}\frac{eV}{m_p c^2\beta^2\gamma}\sin\varphi^{(1)} = 1.066\cdot 10^{-3}\frac{Z}{A}\frac{V_{[\text{keV}]}}{\beta^2\gamma}\sin\varphi^{(1)}\ [\text{mrad}].$$

15. The value of θ_s after RFC equals

$$\theta_s^{\mathrm{RFC}} = \theta_s + \delta\theta_s^{\mathrm{RFC}},$$

the other x_α, θ_α remain unchanged:

$$x_\alpha^{\mathrm{RFC}} = x_\alpha, \quad \alpha = x, z, s; \qquad \theta_\alpha^{\mathrm{RFC}} = \theta_\alpha, \quad \alpha = x, z.$$

16. The phase shift during the particle travel along the part (2) (the value is used on the next turn):

$$\varphi^{(2)} = \varphi^{(1)} + \frac{\pi}{1 - \eta_\omega \theta_s^{\mathrm{RFC}} \cdot 10^{-3}} - \pi.$$

17. The new values of x_α, θ_α which are used on the next turn at the entrance of the cooling section can be found:

$$\begin{pmatrix} x_\alpha \\ \theta_\alpha \end{pmatrix} = \begin{pmatrix} \cos\psi_\alpha & \sin\psi_\alpha \\ -\sin\psi_\alpha & \cos\psi_\alpha \end{pmatrix} \begin{pmatrix} x_\alpha^{\mathrm{RFC}} \\ \theta_\alpha^{\mathrm{RFC}} \end{pmatrix},$$

where $\psi_\alpha = \pi Q_\alpha$.

2.4 Output

The magnitudes

$$\theta_x^{\mathrm{ampl}} = \sqrt{\theta_x^2 + x_x^2}\ [\mathrm{mrad}], \qquad \theta_z^{\mathrm{ampl}} = \sqrt{\theta_z^2 + x_z^2}\ [\mathrm{mrad}] \tag{2.10}$$

and θ_s in mrad as functions of time $t_{[\mathrm{ms}]}$ taken at the entrance of the cooling section are plotted on the screen. The program computes automatically two cooling processes (see section 2.5): the first one with $\theta_{s,0}$, and another one with $-\theta_{s,0}$.

Additionally the program traces the condition

$$\theta_x^{\mathrm{ampl}} \le \theta_{\mathrm{min}} \tag{2.11}$$

and the minimal $t_{[\mathrm{ms}]}$, at which the condition is held, is logged as τ_{cool} for either process.

2.5 Usage

The program[7] is easy to use, but without perfect user-friendly interface, so the basic principles of input and output are discussed here.

To start the program a user should supply a data file name that can be the name of a new file or the name of an existing file created by the program past time. This file is used to keep all the parameters and supplemental information after computation. The command line looks like

```
BOSCOOL data_file_name
```

After initialization (including file reading), the user is in the input parameter phase. Here, any parameter has a default value (read from file or kept in the program core) and the user can change the parameter, putting a new value, or left the parameter unchanged, just pressing the ENTER key. The program asks the next parameter and so on.

All the parameters are divided into several groups and the header of the each group appears first. In the most cases the user can skip the rest of parameters within the group, pressing the ESC key. Doing so, the program enters the current parameter (either explicitly typed by the user or implicitly entered by default), assigns default values to the rest of parameters in the group and goes to the next group. This fast input is unavailable when the input file was empty, bad or damaged, and to ensure that all the numbers are correct user should supply all of them.

After all the parameters are accepted, the program carries out computations stated in the previous sections, and draws curves of $\theta_{x,z}^{\mathrm{ampl}}$ and θ_s in two passes. The first pass takes initial values $X_{\alpha,0}$ and $\theta_{\alpha,0}$ as they are entered during the input, and the second pass just reverses the sign of $\theta_{s,0}$. Graphics screen consists of two plots and some useful information. The first plot contains two curves:

- θ_x^{ampl} [mrad] over t [ms], shown in magenta for the first pass, and shown in green for the second pass;
- θ_z^{ampl} [mrad] over t [ms], shown in yellow for the first pass, and shown in cyan for the second pass.

[7] The program name BOSCOOL stands for "Betatron OScillations with electron COOLing".

The second plot contains one curve of θ_s [mrad] over t [ms]. The curve is shown in magenta for the first pass, or in green for the second pass.

The red dashed line drawn on either plot denotes the calculated τ_{theor} (2.3). The textual information on the screen shows some input parameters and other ones calculated from them.

An example of the parameters displayed on the screen is given here:

`Z = 53, A = 208`	– the ion charge Z and atomic mass A numbers
`E = 4.2327 MeV/amu`	– the energy per nucleon $\mathcal{E}$
`I = 0.5000 Amp`	– the electron beam current I
`B = 0.6000 kG`	– the magnetic field B
`Theta_long = 0.1096 mrad`	– the relative longitudinal spread of electron velocities spread $\theta_{\parallel}$ (see (2.1))
`Theta_e = 3.0000 mrad`	– the electron beam angular spread, θ_e
`Theta_min = 0.0055 mrad`	– the low level of the ion angular spread, θ_{min}, which fixes the cooling time (see (2.1) and (2.11))
`Tau_theor = 292.4010 ms`	– the "theoretical" value of cooling time, $(\tau_{\text{cool}})_{\text{theor}}$, see (2.3)
`Tau_sweep = 67.0000 ms`	– the sweeping time τ_{sweep} if sweeping is on
`Tau_phase = 1.2523 ms`	– the period of small phase oscillations, τ_{phase}, if beam is bunched
`Th_s_separ = 4.3824 mrad`	– the separatrix value of momentum spread, $(\theta_s)_{\text{separatrix}}$, if beam is bunched
`Tau_cool = 142.3767 ms`	– the cooling time τ_{cool} (see (2.11)) for the first pass
`Tau_cool = 112.4472 ms`	– the cooling time τ_{cool} for the second pass

To be sure that program works the "propeller" bar is shown on the right side in the middle of the screen.

During the drawing, the user can terminate calculations by pressing the `ESC` key.

When drawing is completed, the additional possibilities appear as to print the screen contents (the 'P' key) and to revert to the parameter input (the `ENTER` key) and then to perform computations with a new set of parameters again.

The file created (or updated) by the program is a plain text file, which contains all the input parameters specified last time and some calculated values: τ_{theor} (2.3), τ_{cool} (2.11), and others worthless to mention.

2.6 Troubleshooting

Any error/warning condition is signaled by double beep sound, and the correspondent message (if appropriate).

The `CTRL/BREAK` combination can be used to abort the program in the most cases. However, it may hang your computer. Be careful.

Illegal parameter input can be intercepted prior the computations with banner message on the graphics screen. In this case you should reenter parameters or quit the program. But the program is not completely fool-proven, so any input that can cause the computation to over/underflow or to divide by zero, aborts the program immediately (see the message on the screen, it will appear without beeping).

While drawing if the "propeller" bar stops to twist, then probably the program locks, try to use the `CTRL/BREAK` or `CTRL/C` command.

2.7 Copyright. Comments and suggestions

Comments and suggestions are welcomed and should be sent to the authors An. Lavrentev (e-mail: anton@sunvas.jinr.dubna.su) or I. Meshkov (e-mail: meshkov@nusun.jinr.dubna.su).

3 Some results of computations

The computation, performed with the described program, demonstrates the peculiarities of electron cooling process very well (Fig. 2 –

8 and Table 2). As an example, the cooling of $^{208}Pb^{53+}$ ions in LEAR storage ring at CERN is chosen (see Table 1).

3.1 Particle dynamics

The nonlinear character of the cooling (friction) force as a function of particle velocity (see formula (2.7)) influences significantly on the behaviour of parameters θ_x^{ampl}, θ_z^{ampl} and $\theta_s \equiv \Delta P_s/P_0$ in time (Fig. 3). Particularly the cooling rate increases drastically and these parameters decrease very fast, when they reach sufficiently small values.

3.2 Betatron oscillations and electron beam space charge

The influence of betatron oscillations and electron beam space charge appears in some averaging process, which smoothes the gaps in the friction force function, described by formulae (2.6) and (2.7). On the other hand the betatron oscillations bring the particle motion across the electron beam, so it interacts with electrons, which have different velocity due to the beam space charge. Such an effect is taken into account and neutralization factor η_n let us to choose different states of the e-beam neutralization. Comparing Fig. 2 and Fig. 3 one can see that cooling time shortens very essentially, when the space charge is neutralized ($\eta_n = 1$).

The cooling time enlarges even more, if dispersion in cooling section presents and $\eta_n = 0$ (do compare Fig. 2 and Fig. 4). Meanwhile the computations demonstrate that dispersion influence is negligible when the space charge is neutralized (almost obvious result!).

Also one should mention the next well-understandable result of the computations: the cooling processes do not differ if angular or equivalent coordinate amplitudes are chosen as initial parameter values. In other words, the value $\theta_{\alpha,0}$ is absolutely equivalent to $x_{\alpha,0}$ (see (1.1) – (1.3)).

3.3 Cooling time and initial conditions

The estimation cooling time value (2.3) differs significantly with computed magnitudes – about 1.5 times for the cases, presented in Fig. 2 – 4. The dependence of cooling time on $\vec{\theta}_0$ is also of practical interest (Table 2 and Fig. 9).

3.4 "Sweeping" process

The problem of special interest is a possibility to cool fast a particle beam with large momentum spread. The method, discussed since very beginning of electron cooling method development (see [1]) but never tested, is so called sweeping – the variation of electron energy during cooling process. This method is demonstrated by computations presented here. The curves "a" (Fig. 5) correspond to the particle with positive momentum difference $\theta_{s,0} = 5$ mrad. The sweeping time chosen in this case is equal to 5 ms, and one can see that the decrease of θ_α decelerates at $t > \tau_{sweep}$ when the electron velocity shift changes its sing (see (2.5)). The curves "b", which correspond to $\theta_{s,0} = -5$ mrad, demonstrate just opposite behaviour. If sweeping is off, the cooling process is much slower (Fig. 6).

One should mention that dispersion function does not influence significantly even in the sweeping regime, if the electron beam space charge is neutralized.

3.5 Bunched beam

The cooling of a bunched particle beam has own peculiarities. One can say that the influence of the electron beam space charge and the ring dispersion is much more significant, then for a coasting beam (see Fig. 7 – 8). Even one can get regimes, when the particle beam does not reach the cooled state at all. And on the contrary, if electron beam is neutralized, the cooling of the bunched beam completed much more faster, and the effect of the fast cooling takes place in the final part of the process (see Fig. 8). The influence of dispersion is also unessential, when $\eta_n = 1$.

Acknowledgements

The authors thank Dr J.Bosser for his permanent interest to this work and inspiring discussions.

Parameters	Fig						
	2	3	4	5	6	7	8
$X_{x,0}$, cm	0						
$X_{z,0}$, cm	0						
$\theta_{x,0}$, mrad	3						
$\theta_{z,0}$, mrad	3						
$\theta_{s,0}$, mrad	3			8		3	
Bunched beam	No					Yes	
$\mathcal{E}$, MeV/amu	4.2327						
A, amu	208						
Z	53						
φ_0, rad	—					π	
I, Amp	0.5						
a, cm	2.5						
θ_e, mrad	3						
B, kG	0.6						
η_n	0	1	0	1		0	1
$\theta_{\|\|}^0$, mrad	0			8		0	
Sweeping	No			Yes		No	
τ_{sweep}, ms	—			10		—	
C, m	78						
η	0.02						
D, m	0	3.6		0		3.6	
Q_x	2.305						
Q_z	2.73						
γ_{tr}	—					10	
V, keV	—					1	
$Y_{\max}$, mrad	4			10		4	
$t_{\max}$, ms	250	40	250	50	150	200	40

Table 1: Input parameters for Fig. 2 – 8.

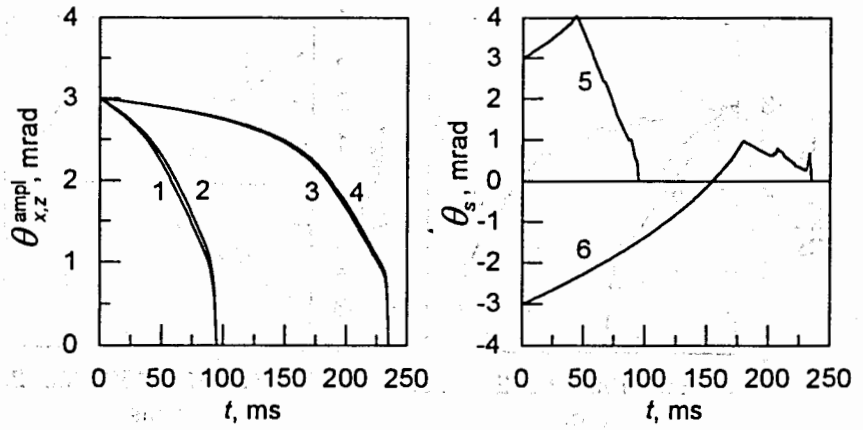

Figure 2: The cooling with non-neutralized electron beam, $D = 0$ m.
$(\tau_{\text{cool}})_{\text{theor}} = 63.1586$ ms

$1 - \theta_x^{\text{ampl}}$, $2 - \theta_z^{\text{ampl}}$, $5 - \theta_s$ when $\theta_{s,0} > 0$
$3 - \theta_x^{\text{ampl}}$, $4 - \theta_z^{\text{ampl}}$, $6 - \theta_s$ when $\theta_{s,0} < 0$

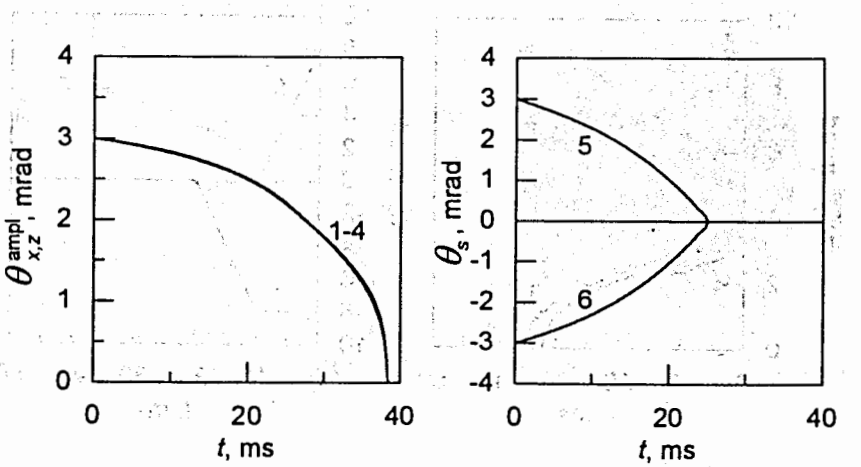

Figure 3: The cooling with neutralized electron beam, $D = 3.6$ m.
$(\tau_{\text{cool}})_{\text{theor}} = 63.1586$ ms
1 – 6 — see Fig. 2.

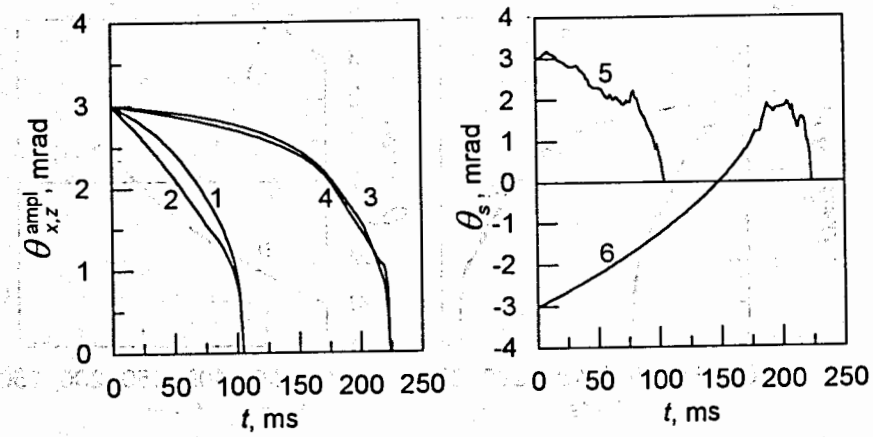

Figure 4: The cooling with non-neutralized electron beam, $D = 3.6$ m.
$(\tau_{cool})_{theor} = 63.1586$ ms
1 – 6 — see Fig. 2.

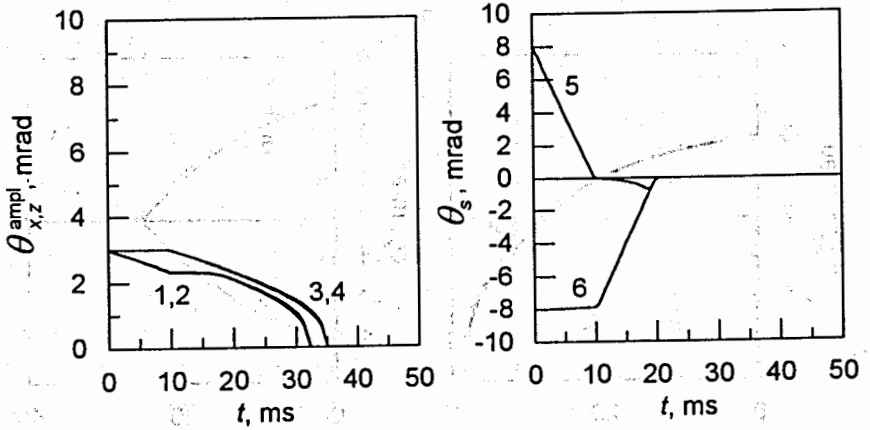

Figure 5: The cooling of large momentum spread in "sweeping" regime.
$(\tau_{cool})_{theor} = 63.1586$ ms
1 – 6 — see Fig. 2.

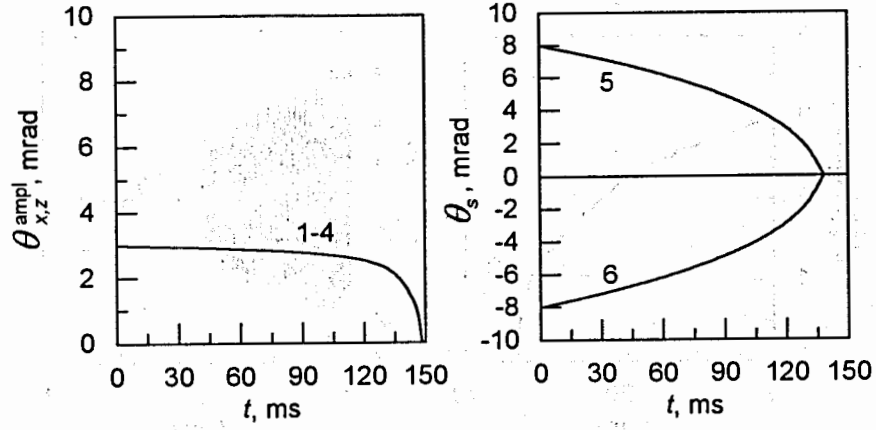

Figure 6: The cooling of the same momentum spread as in Fig. 5 without "sweeping".
$(\tau_{cool})_{theor} = 63.1586$ ms
1 – 6 — see Fig. 2.

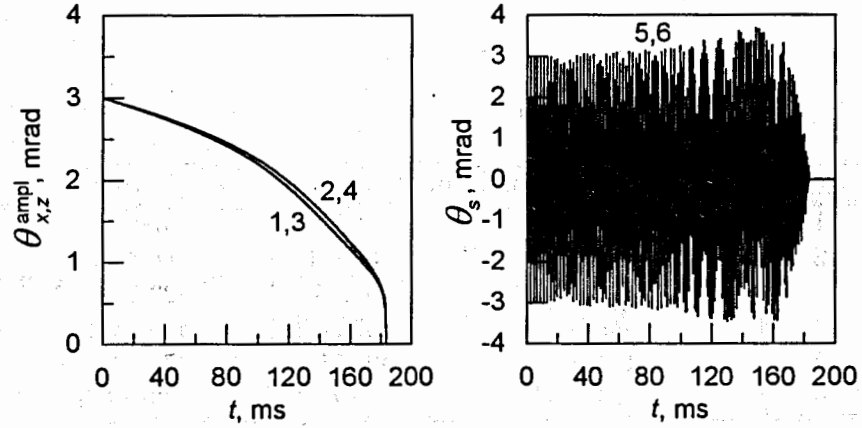

Figure 7: The cooling of the bunched ion beam with non-neutralized electron beam.
$(\tau_{cool})_{theor} = 63.1586$ ms
1 – 6 — see Fig. 2.

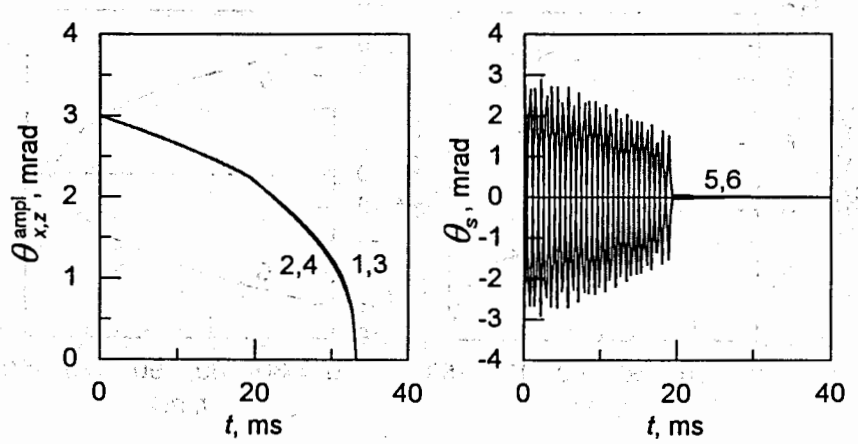

Figure 8: The cooling of the bunched ion beam with neutralized electron beam.

$(\tau_{cool})_{theor} = 63.1586$ ms
1 – 6 — see Fig. 2.

$(\theta_{x,z})_0$	Neutralized electron beam	Non-neutralized electron beam	
1	13.5726	18.8968	46.2315
2	21.6262	48.6643	92.2214
3	38.2604	103.1526	223.7412
4	66.7182	237.2479	489.7351
5	114.9679	492.7583	970.2306
6	224.1311	950.7242	1795.8905

Table 2: The dependence of transverse cooling time τ_{cool} on ion beam angular spread (used for Fig. 9).
Other parameters are the same as for Fig. 3 and Fig. 4 respectively. In the second column the values are given for two passes.

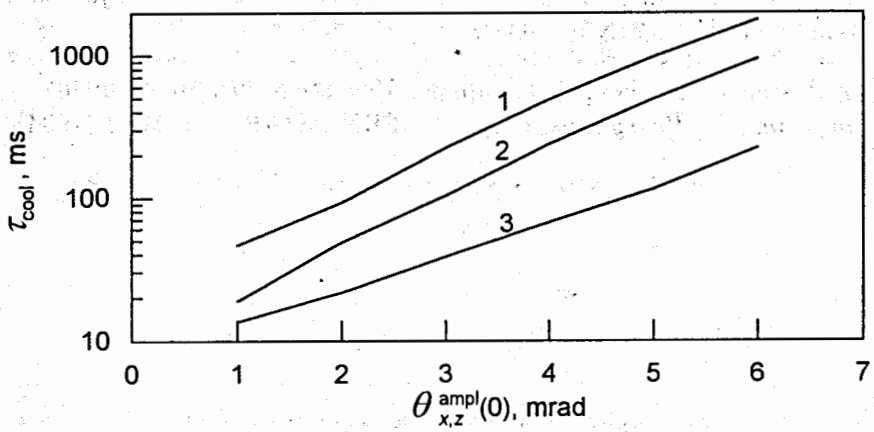

Figure 9: The dependence of transverse cooling time on ion beam angular spread, see Table 2.

1: $\eta_n = 0$ (space charge presents), $\theta_{s,0} < 0$;
2: $\eta_n = 0$ (space charge presents), $\theta_{s,0} > 0$;
3: $\eta_n = 1$ (neutralized electron beam).

The asymptotic behaviour of these curves is $\tau_{cool} = \text{Const}\ (\theta_{x,z})_0^{\varepsilon}$, where

$$\varepsilon = \begin{cases} 3.2 \div 3.4 & \text{– curves 1 and 2} \\ 3 & \text{– curve 3} \end{cases}$$

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[1] Ya. S. Derbenev, A. N. Skrinsky, *The Physics of Electron Cooling*, Physics Reviews, ser. Sov. Phys. Reviews, Vol **3**, p. 165 (1981).

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