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Объединенный  
Институт  
Ядерных  
Исследований  
Дубна

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A.M. Taratin

COMPUTER SIMULATION OF ACCELERATOR  
BEAM EXTRACTION WITH A BENT CRYSTAL

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# 1 Introduction

There are good perspectives of using bent crystals for particle extraction from the beam halo at the largest proton and ion colliders LHC, RHIC and UNC. The most attractive feature of such extraction is that the colliding mode of machine operation is not disturbed, and some fixed target experiments can be carried out simultaneously. The experimental study of appropriate extraction systems is being investigated at the Tevatron and the SPS [1, 2]. Important peculiarities of these experiments are that beam halo particles begin to hit the crystal very close to its edge and the possibility of multiple passages of circulating particles through the crystal.

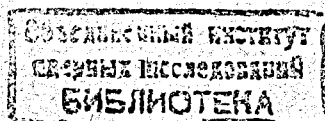
In the SPS beam extraction experiment [3] the reached extraction efficiency was about 10 % what is much larger than in the previous experiments [4, 5]. However, unexpected big widths of the extracted beam and extraction efficiency dependence upon crystal orientation were observed.

In this paper the multiturn extraction of the SPS beam with the injection of a white transverse noise to initiate particle diffusion onto the bent crystal deflector is considered by computer simulation. The dependence of the extraction efficiency and extracted beam characteristics upon crystal orientation and its transverse deformation have been studied for the case of large impact parameters of beam particles with the crystal, when the influence of its surface imperfections is negligible.

# 2 Simulation program

For the simulation of beam extraction from the SPS, two points in the accelerator ring were considered. The first one is in the bent crystal position (BC). Another one in the position of the deflector plates of the feedback system, where particles get uncorrelated kicks in the horizontal direction. This noise generator (NG) was used to initiate particle diffusion in the horizontal direction and bring them onto the crystal. Two transfer matrices were used to transport particles from the crystal to the noise generator and back. The main accelerator parameters for these two points are shown in table 1.

The initial values of  $X, X', Y, Y'$  are generated according to Gaussian distribution in the bent crystal position. The initial standard deviations are determined as



$$\sigma_{x,y} = \frac{1}{2} \sqrt{\frac{\epsilon_N \tilde{\beta}_{x,y}}{\gamma}},$$

$$\sigma_{x',y'} = \frac{1}{2} \sqrt{\frac{\epsilon_N}{\gamma \tilde{\beta}_{x,y}}},$$

where  $\gamma$  is the Lorentz factor of protons and  $\tilde{\beta}$  is the beta function in the principal frame. For the SPS the normalized emittance determined at a  $2\sigma$ -level,  $\epsilon_N$ , is  $10^{-5} m \cdot rad$ . It gives the following values of standard deviation:  $\sigma_x = 1.3$  mm,  $\sigma_{x'} = 14.6$   $\mu rad$ ,  $\sigma_y = 0.682$  mm,  $\sigma_{y'} = 28.6$   $\mu rad$ . So, at the considered distance of the bent crystal from the closed orbit, 10 cm, we have no particles which hit the crystal without diffusion.

Due to the injection of white noise into the transverse feedback system of the SPS, circulating particles begin to hit the crystal increasing their horizontal betatron oscillation amplitudes. Fig.1 shows the phase trajectories in the  $X-X'$  space for the SPS beam particles at the azimuth of the crystal location. The trajectory with the amplitude which equals the crystal distance from the closed orbit determines the horizontal angle of beam particles at the first hit stage with the crystal. This angle is called "the beam direction" below. It changes after particle passages through the crystal due to multiple scattering (see the dotted line). This circumstance can determine the orientational dependence width of extraction efficiency for the crystal without transverse deformations.

Fig.2 shows the calculated particle distributions in the impact parameters (a) and angles (b) for  $\sigma_{noi} = 0.05$   $\mu rad$ , where  $\sigma_{noi}$  is a standard deviation for noise. This noise is approximately maximum used in the experiment. The angular peak position  $\theta_{beam} = 239$   $\mu rad$ . The width of the impact parameter distribution is larger than  $10$   $\mu m$  what allows one to neglect the influence of the surface imperfections of the crystal on the beam extraction efficiency. This influence can be also reduced considerably for smaller amplitudes of noise by increasing the impact parameters of beam particles with the crystal by means of their prescattering on a thin foil [6].

Some of the particles which hit the crystal can be captured into the channeling states by bent planar channels. These particles are deflected at a bending angle  $\alpha$  if they are not dechanneled due to multiple scattering by crystal electrons and nuclei. Another part will be scattered by the crystal or lost due to inelastic nuclear interactions. We assume that a particle will be extracted if it exits from the crystal with an angle  $X' > \alpha - \theta_{col}$ ,

where  $\theta_{col}$  is a collimation angle. When a particle exits from the crystal at an angle  $\theta_{ac} < X' < \alpha - \theta_{col}$ , it is lost downstream at the wall of the beam pipe. For the SPS the acceptance angle in the crystal position is about 1 mrad. It is determined as

$$\theta_{ac} = \frac{r_p}{\sqrt{\beta_m \beta_x}},$$

where  $r_p$  is the radius of the beam pipe and  $\beta_m$  the maximum horizontal beta function. Otherwise the particle remains in the beam.

The particle trajectories in bent crystal were calculated by a numerical solution of the equations of motion in the potential of bent atomic planes. After passing through the crystal layer  $\Delta S = 0.5$   $\mu m$ , which thickness is much smaller than the wavelength of particle oscillations in the channel, the change of transverse velocity due to multiple scattering is computed. More details of these simulations can be found in ref.[7].

### 3 Perfect deflector

In this chapter we consider a perfect crystal deflector that is the crystal with a constant longitudinal curvature and without surface imperfections. The crystal deflector has to be aligned with the beam to get an optimum condition for particle capture by the bent crystal channels. Fig.3a shows the dependence of extraction efficiency on crystal orientation angle  $\theta$ , which is the angle between the plane direction at the crystal entrance and the closed orbit of the circulating beam.

The central maximum is stipulated by the particles extracted at the first hits with the deflector, that is in a single passage through it. The angular size of this region is determined by beam divergence and the critical channeling angle of particles. The extraction for crystal orientations which are far from  $\theta_{beam}$  occurs due to multiple passages of particles through the deflector. One can clearly see this in fig.4 where the distributions of extracted particles in the number of passages through the deflector are shown. Fig.5 shows the corresponding impact parameter distributions. They become approximately uniform along the crystal thickness for tilt angles outside the single passage region.

The extraction efficiency due to multiple passages is asymmetric with the crystal orientation about the beam direction. It happens because a crystal bending breaks symmetry with respect to the plane direction at the crystal entrance. A schematic picture in the  $X'$ -space for two different crystal orientations is shown in fig.6.

For positive tilt angles  $\theta_{\text{tilt}} = \theta - \theta_{\text{beam}}$ , that is for orientation 1, the particles move off the bent plane directions crossing the crystal. Their directions change due to multiple scattering. For the considered Si crystal the rms multiple scattering angle  $\theta_{m,s} = 66.7 \mu\text{rad}$ . So, the crystal will be a source of a strong additional diffusion of circulating particles in the horizontal and vertical directions.

For negative tilt angles of the crystal, orientation 2, the particles move to the side of crystal bending. They are deflected at the angle which is about the critical channeling angle to the opposite side due to volume reflection [8]. These asymmetric deflections in multiple passages of particles through the crystal increase monotonously their amplitudes of horizontal betatron oscillations, that is the drift of particles in the horizontal angular space will be observed in addition to diffusion due to multiple scattering. This drift reduces the number of passages through the crystal which are necessary for particles to come into the angular region where the surface capture by the bent planar channels occurs (Capture Region). Besides, the probability for the volume capture of particles by the bent channels exists for these crystal orientations. Both these circumstances decrease the loss of particles in inelastic nuclear interactions with crystal atoms for negative tilt angles (see fig.3b). This causes the asymmetry observed in the orientational dependence for extraction efficiency.

#### 4 Deflector with transverse deformation

The crystal deflector used in the SPS beam extraction experiment had different directions of the bent planes along its height at the entrance and exit ends. It happened because the longitudinal curvature, studied in the special measurements, was much smaller than the average one near the middle line of both crystal ends.

Let the longitudinal curvature for the crystal edges in the vertical direction be uniform,  $R_{||}(z, y = \pm h) = R$ . Whereas there is a straight part with length  $S_o$  along the middle line near both ends (see fig.7), that is

$$R_{||}(z, y = 0) = \begin{cases} R, & S_o < z < S_{cr} - S_o, \\ \infty, & S_o > z > S_{cr} - S_o. \end{cases}$$

And let the straight part length change for the intermediate values of Y as

$$S_{st}(Y) = S_o(1 - (Y/h)^2).$$

The bent plane directions at both ends change along Y in the same way, that is for the capture region centre in the X'-Y space we have

$$\theta_{CR}(Y) = \theta - \theta_m(Y/h)^2,$$

where  $\theta_m = S_o/R$ . Let  $S_o = 3 \text{ mm}$ , then  $\theta_{m,s} = 850 \mu\text{rad}$ .

In fact, there is a more complicated law of longitudinal curvature change at the crystal ends. Moreover, the maximum difference between the plane directions is probably smaller than  $850 \mu\text{rad}$ . But this simple model allows one to estimate the effect of the transverse crystal deformation on beam extraction efficiency.

Fig.8 shows the capture region locations for different crystal orientations with respect to the beam. The dependence of the extraction efficiency for the considered crystal upon its orientation with the beam is shown in fig.9. For an optimum crystal alignment, when the (110) plane direction in the middle part of the crystal coincides with the beam direction, the intersection of the beam with the capture region occurs only for the central part of the crystal, fig.8a. It decreases the maximum extraction efficiency. For negative tilt angles the situation is approximately the same as for the perfect deflector, but the efficiency decreases faster with removing from the optimum alignment because the central part of the crystal works basically.

For positive tilt angles (fig.8b) the situation is quite different. The beam direction can intersect the capture region over a wide region of crystal tilt angles ( $0 - \theta_m$ ). It makes the extraction efficiency considerably higher for the considered crystal than for the perfect one for the orientations which are far from perfect alignment.

There are two intersections of the capture region with the beam direction that can lead to the separation of extracted particles in the vertical plane. These intersections are far from the crystal centre for large tilt angles. Whereas the vertical beam dimension at the first hit stage is small. Therefore, only after a few passages through the deflector the particles come into the capture region due to beam broadening by multiple scattering. Fig.10 shows the distributions of extracted particles in the vertical coordinate Y at the crystal entrance. The maximum distance between two peaks can achieve the crystal height. This structure with two maxima in the vertical and horizontal profile of the extracted beam was observed in the experiment [3]. The appearance of two peaks for the horizontal profile can occur when we have different bending angles for the upper

and lower edges of the crystal. That is when the channeling region is different and asymmetric with respect to the middle line near the crystal ends.

Fig.11 shows the distributions of extracted particles in horizontal angles. With crystal rotation to the bending side, the distribution becomes wider and shifts to bigger angles. It occurs because for the considered crystal the bending angles are different along the crystal height. They change from 6.8 mrad for the middle line up to 8.5 mrad for the crystal edges in the vertical direction. The width of the extracted beam can increase up to the angular spread of the bent planes across the crystal height at its exit end.

## 5 Conclusion

The performed simulation shows that there is an asymmetry of extraction efficiency with crystal orientation relative to the beam. For the perfect deflector it is stipulated by the volume reflection of particles by the planar crystal potential and their volume capture by the planar channels. For the deflector with the transverse deformation it occurs due to the asymmetric transformation of the angular region for particle capture into the channeling states.

The spread of the atomic plane directions along the crystal height at its entrance and exit ends for crystal deformation which appears with the bending device used in the experiment [3] increases the region of crystal orientations for particle capture and the interval of crystal bending angles. This leads to a considerable broadening of the extracted beam and the orientational dependence of the extraction efficiency in comparison with the critical channeling angle and beam divergence. Moreover, the possibility appears to observe two maxima in the vertical and horizontal profiles of the extracted beam. Both these peculiarities have been observed in the experiment.

## 6 Acknowledgements

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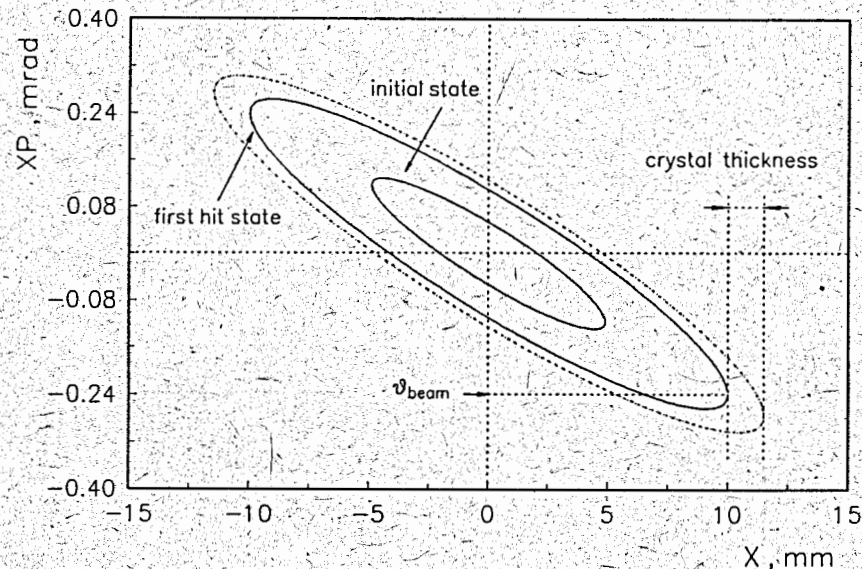


Fig.1. The phase trajectories in the  $X-X'$  space for the SPS beam particles at the azimuth of crystal location. The initial state trajectory is for the  $3\sigma$ -level before diffusion. The trajectory of the first hit state is determined by the crystal distance from the closed orbit. Here  $\theta_{beam}$  is the beam direction angle. The dotted line shows the trajectory for the particle after one or a few passages through the crystal with the amplitude increase which equals the crystal thickness.

Table 1. Some Relevant Accelerator Parameters

Parameter	Bent crystal	Noise generator
$\alpha_x$	2.182	2.042
$\alpha_y$	-0.708	-0.846
$\beta_x$	91.191 m	81.837 m
$\beta_y$	23.776 m	27.17 m
	BC $\Rightarrow$ NG	NG $\Rightarrow$ BC
$Q_x$	12.821	13.799
$Q_y$	12.811	13.769

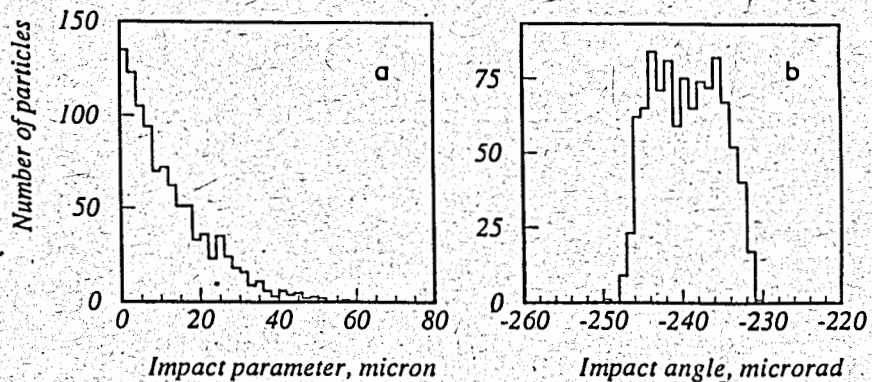


Fig.2. The particle distributions in the impact parameters (a) and angles (b) with the crystal deflector after the injection of white transverse noise with a standard deviation of  $\sigma_{noi} = 0.05 \mu\text{rad}$ . The deflector distance from the closed orbit  $d = 10 \text{ mm}$ .

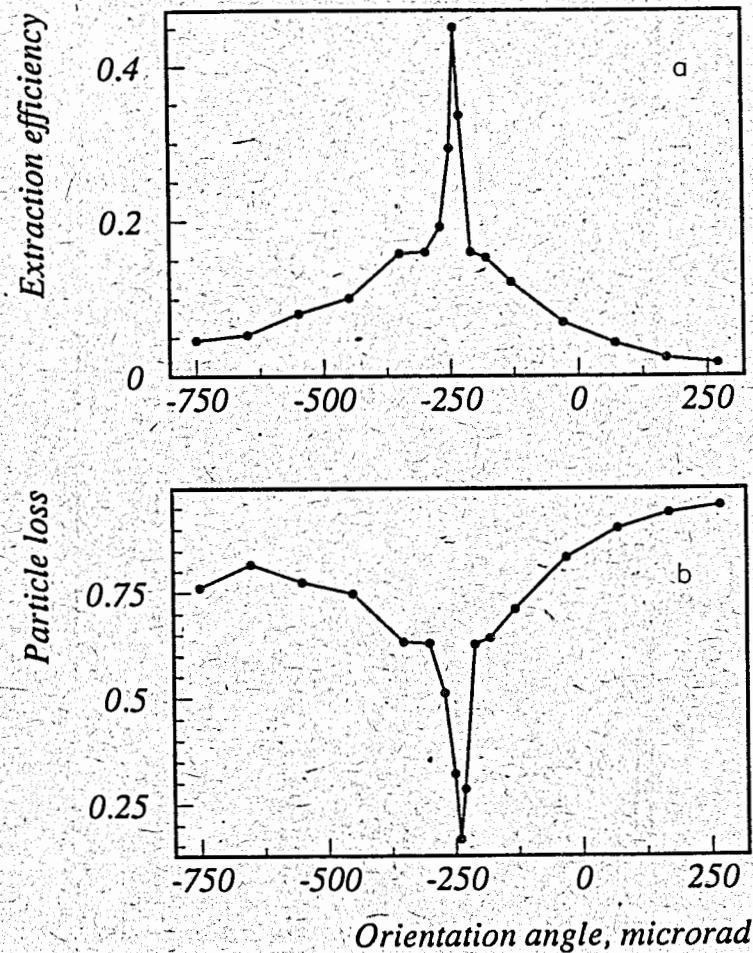


Fig.3. The dependence upon the crystal orientation angle  $\theta$  for the perfect deflector: (a) extraction efficiency, (b) the number fraction of particles lost due to inelastic nuclear interactions in the crystal.

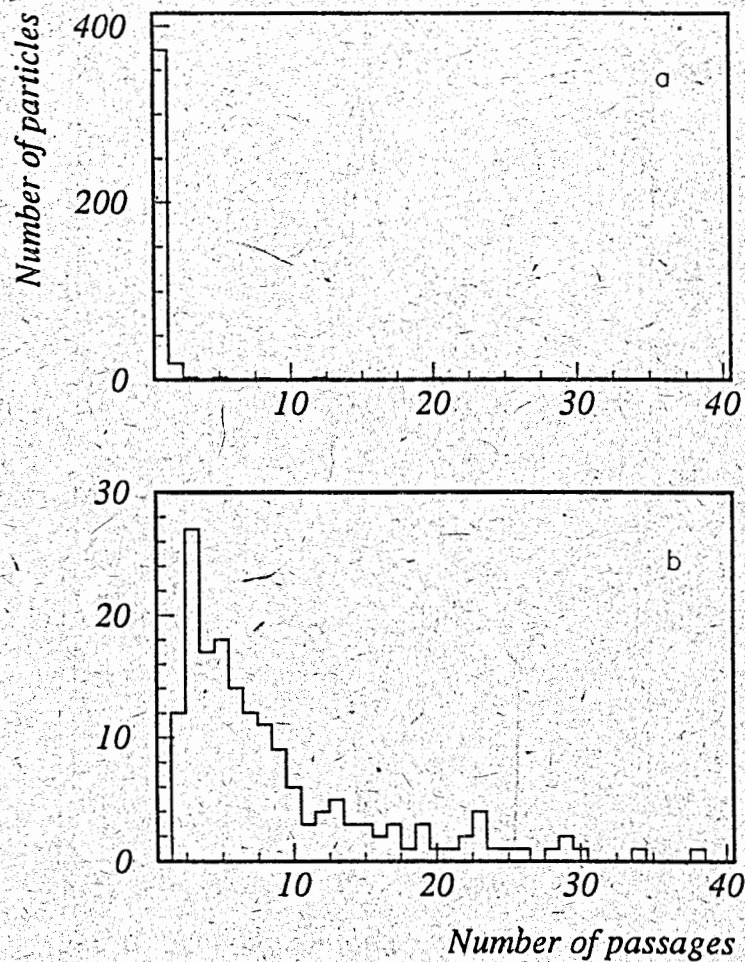


Fig. 4. The distribution of extracted particles in the number of passages through the perfect deflector. The crystal orientation angles  $\theta$ ,  $\mu\text{rad}$ : -240 (a), -210 (b).

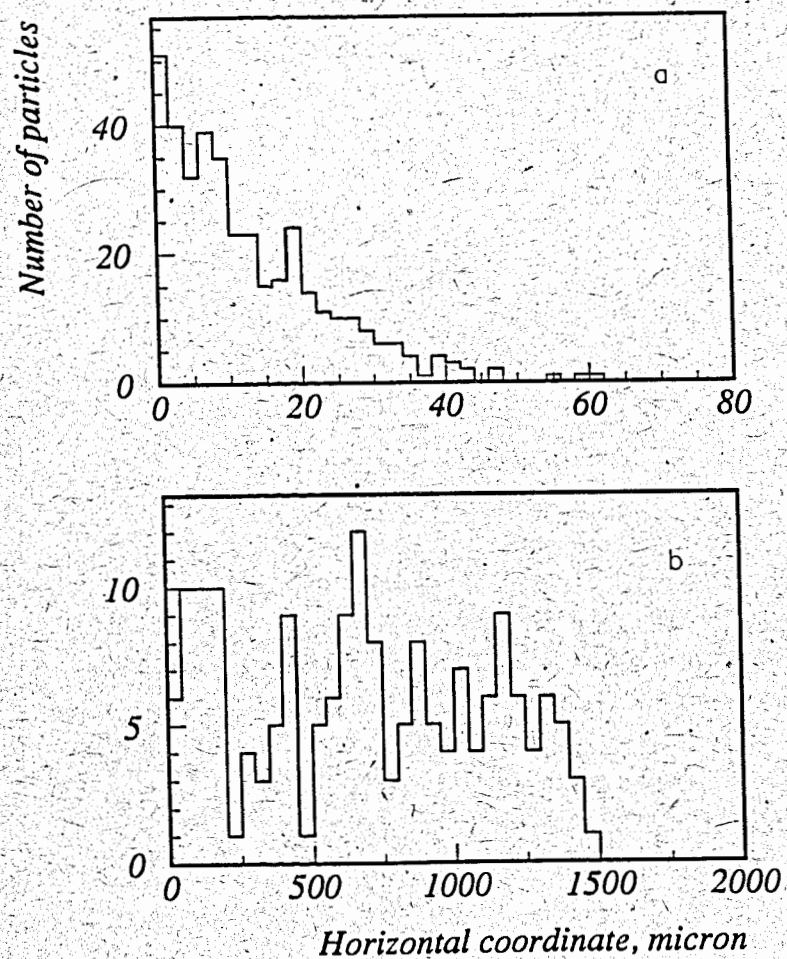


Fig. 5. The distributions of extracted particles in the impact parameters with the perfect deflector. The orientation angles are the same as in fig.4.

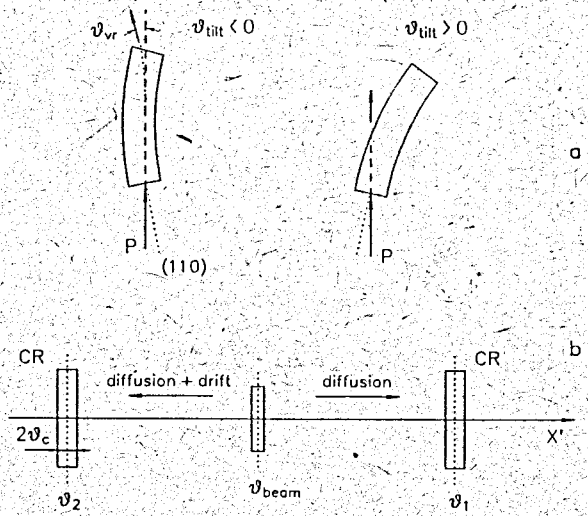


Fig. 6. A schematic picture of two crystal orientations with  $\theta_{tilt} > 0$  and  $\theta_{tilt} < 0$  near the beam direction (a) and the corresponding locations of the capture regions (CR) in  $X'$ -space:  $\theta_1, \theta_2$  (b). For crystal orientation 1 the particles come into the capture region from the angular region of the beam only due to diffusion (multiple scattering in the crystal and noise) in multiple passages through the crystal. For orientation 2 there is additionally a drift due to the volume reflection of particles by the bent plane potential. Here  $\theta_c$  is the critical channeling angle and  $\theta_{vr}$  the volume reflection angle.

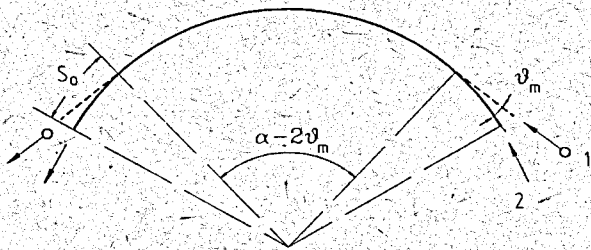


Fig. 7. A schematic picture of the bent crystal with straight parts near the middle line, which length is  $S_0$ , at both ends,  $\theta_m = S_0/R$ . Particle 1 moving through the deflector along the middle line is deflected at angle  $\alpha - 2\theta_m$ . Particle 2 moving along the deflector edge is deflected at bending angle  $\alpha$ .

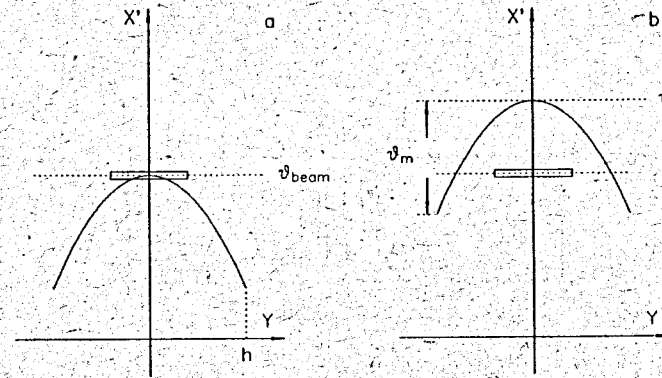


Fig. 8. A schematic picture of the capture region locations in the  $(X', Y)$ -space for different crystal orientations near the beam for the deflector with transverse deformation. The parabolic curves show the crystal plane directions at different positions along the crystal height. The rectangle shows the angular region of the beam at the first hit stage. (a) "perfect" alignment,  $\theta_{tilt} = 0$ , (b)  $\theta_{tilt} > 0, \theta > \theta_{beam}$ . Here  $\theta_{beam}$  is the beam direction angle,  $\theta_m$  the angular acceptance of the deflector and  $h$  the half-height of the crystal.

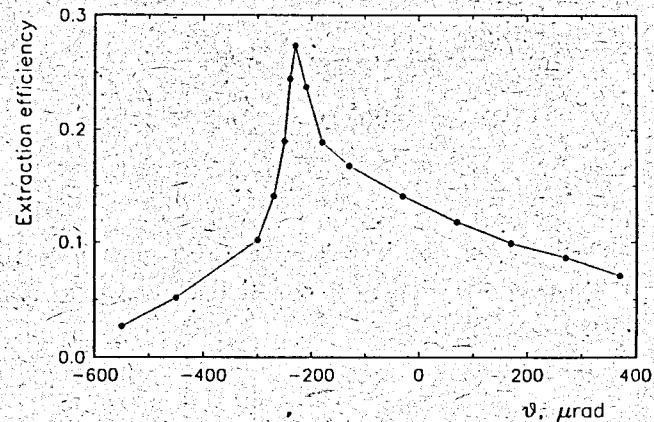


Fig. 9. The dependence of the extraction efficiency on the crystal orientation angle for the crystal with transverse deformation.



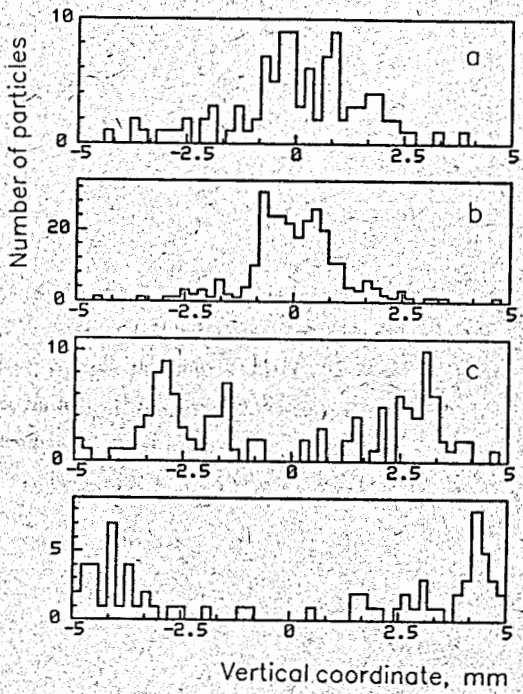


Fig.10. The distributions of extracted particles in vertical coordinates at the crystal entrance for the crystal with transverse deformation. The crystal orientation angles  $\theta$ ,  $\mu\text{rad}$ : -350 (a), -230 (b), 70 (c), 370 (d).

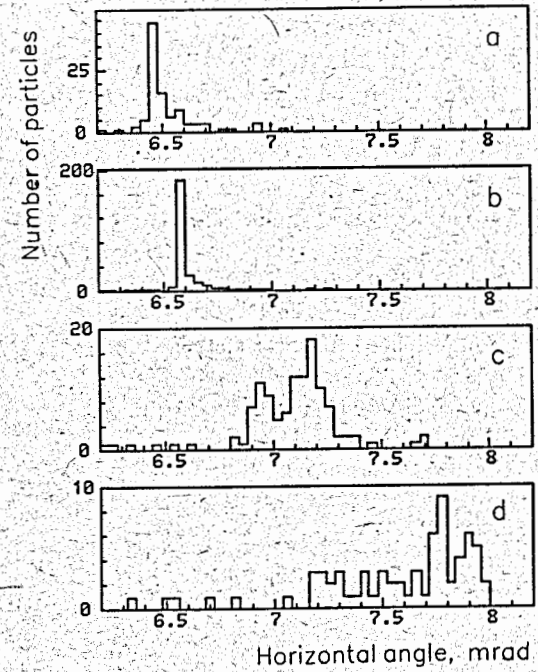


Fig.11. The distribution of extracted particles in horizontal angles for the crystal with transverse deformation. The orientation angles are the same as in fig.10.

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Таратин А.М.

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Компьютерное моделирование вывода пучка  
из ускорителя изогнутым кристаллом

С помощью компьютерного моделирования рассмотрен вывод пучка протонов с энергией 120 ГэВ из ускорителя SPS при наведении частиц на изогнутый кристалл возбуждением поперечной диффузии частиц. Обнаружена асимметрия эффективности вывода с ориентацией кристалла относительно орбиты. Показано, что нежелательная деформация кристалла в вертикальной плоскости приводит к значительному уширению ориентационной зависимости, а также к уширению и расщеплению выведенного пучка.

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Taratin A.M.

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Computer Simulation of Accelerator Beam Extraction  
with a Bent Crystal

The extraction of a 120 GeV proton beam from the SPS with transverse particle-diffusion onto a bent crystal deflector has been considered by computer simulation. The asymmetry of extraction efficiency with the crystal orientation relative to the beam direction was observed. It was shown that the crystal deformation in the vertical plane led to a considerable broadening of orientational dependence and also a broadening of the extracted beam and its split.

The investigation has been performed at the Laboratory of High Energies, JINR and CERN.

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