объединенный
институт
ядерных
исследований
дубна

E9-93-73

E.L.Saldin*, E.A.Schneidmiller*, M.V.Yurkov

HIGH EFFICIENCY FEL OSCILLATOR WITH TIME-DEPENDENT UNDULATOR TAPERING

Submitted to «Physics Letters A»

*Automatic Systems Corporation, Samara

1993

1. Introduction

The problem to increase an efficiency of FEL devices is a central one in free electron laser physics. It is widely accepted now to solve this problem by adiabatic variation of undulator parameters along its axis [1,2]. In the case of a FEL amplifier this approach gives excellent results. At first this idea was confirmed by the results of numerical simulations [3,4] and later it was demonstrated experimentally, an efficiency $\eta \sim 34\%$ was achieved [5].

The situation with a FEL oscillator is a more complicated one. First, the efficiency of the FEL oscillator with a homogeneous undulator is rather small, $\eta \sim 0.29/N$, where N is number of undulator periods [6, 7]. Usually N is about several tens which results in the FEL efficiency η less than one percent. Second, undulator tapering does not give such excellent results with respect to the FEL amplifier, it enables one to increase the FEL oscillator efficiency by a factor of 2 or 3. Using additional possibilities, such as a prebuncher, one increases the efficiency additionally by a factor of two [8]. In any case the maximal FEL oscillator efficiency does not exceed a value of several percents.

So, it seems extremely difficult to achieve high efficiency in the FEL oscillator using conventional approach of undulator tapering. An alternative approach to the problem was proposed in ref. [9]. This approach is based on a natural feature of FEL oscillator, namely the dependence on time of the radiation field stored in the resonator. It was proposed to introduce time-dependent accelerating fields into interaction region (which is equivalent in its action to the undulator tapering). As a result, it makes possible to trap electrons into the ponderomotive well and perform the conversion of microwave energy to optical one. To increase a number of trapped electrons (which results in higher efficiency), the authors of ref. [9] proposed to use a prebuncher together with a homogeneous undulator. Numerical estimations, presented in ref. [9, 10] show that an efficiency about several tens of percent can be achieved in such a FEL oscillator.

In the present paper we consider another way to realize an idea of time-dependent variation of FEL parameters to increase the FEL oscillator efficiency. We propose to change in time the magnetic field of the undulator rather than to introduce accelerating fields into the interaction region. The scheme under consideration consists of a prebuncher, drift space and main undulator with time-dependent tapering. Results of numerical simulations have shown that a significant increase of the FEL oscillator efficiency can be achieved in such a modification of the FEL oscillator against traditional schemes.

It should be noted that there are some technical problems on the way to realize the proposed method: the undulator field should change significantly within a current macropulse duration of the accelerator. Nevertheless, it may be realized with driving beam from superconducting accelerator operating in the continuous or quasi-continuous regime. Nowadays there is a number of superconducting accelerators in the world, operating or under construction [11-16]. To demonstrate the possibility of the proposed method realization, in section 3 we present the results of numerical simulations of a concrete example which have shown that accompanying technical problems may be resolved at the present level of FEL technology.

2. Analysis of the time-dependent scheme

The present treatment is based on a one-dimensional FEL oscillator model. We assume an electron current pulse duration to be long enough and do not take into account longitudinal modes competition effects. Despite its simplicity such a model reveals the main features of the FEL oscillator concerning the problem to increase efficiency. Moreover, this model enables one to take into account many factors influencing the FEL oscillator operation and it will be illustrated in section 3.

We assume the electron beam to move along the z axis in the magnetic field $\vec{B} = \vec{e}_x B\cos(\kappa_0 z)$ of a planar undulator. The field amplitude B varies along the undulator axis and the tapering depth slowly changes in time: $[B(z,t)-B_0]/B_0 = -\alpha(t)z$. The amplitude of the electron oscillation angle in the main undulator is equal to $\theta = Q/\gamma$, where $Q = eB/\kappa_0 mc^2$ is undulator parameter, $\gamma = \mathcal{E}/mc^2$, $\mathcal{E}$ is electron energy, and $-e$ and m are the charge and mass of electron, respectively. It is assumed that $\theta^2 \ll 1$ and the longitudinal electron velocity v_z is close to the velocity of light c. The amplitude of the electric field of the wave synchronous with the electron beam is written in the form: $\vec{E} = \vec{e}_y \tilde{E} \exp [\ i\omega(z/c-t)\ ] + C.C.$ The lasing frequency ω is defined by the condition of the maximum gain in the small-signal regime. In this section we shall consider the situation when the efficiency is increased by many times against the case of homogeneous undulator but still remains much less than unity. Therefore, we can neglect the changes of $\mathcal{E}$ and B in amplitude factors and keep these changes only in the detuning parameter of the particle with nominal energy $\mathcal{E}_o$ from resonance: $C(z,t) = \kappa_o - \omega[1+Q^2(z,t)/2]/2c\gamma_o^2$, where $\gamma_o = \mathcal{E}_o/mc^2$. Particle motion is described in "energy-phase" variables with energy $\mathcal{E}$ as canonical momentum and phase $\psi = \kappa_0 z + \omega(z/c-t)$ as a canonically conjugated coordinate. In this representation the longitudinal coordinate z is an independent variable (see, e.g., ref. [17] and [18]). To study the nonlinear mode of FEL oscillator operation we use the macroparticle method. We choose M macroparticles over modulation density period, i.e. in phase ψ interval from 0 to 2π. The equations of motion (averaged over undulator period) may be written in the following reduced form [18]:

$$d\hat{P}_{(i)}/d\hat{z} = \hat{\varphi}\ \cos(\psi_{(i)} + \psi_s), \tag{1}$$

$$d\psi_{(i)}/d\hat{z} = \hat{P}_{(i)} + \hat{C}\ , \tag{2}$$

where $i = 1,...M$, $\hat{z} = z/l_0$, l_0 is the length of the main undulator, $\hat{P} = \omega l_0 P/c\gamma_{z0}^2\mathcal{E}_0$, $P = \mathcal{E} - \mathcal{E}_0$ is energy deviation from the nominal value $\mathcal{E}_0$, $\varphi = [JJ]_0\omega l_0^2\varphi/2c\gamma_{z0}^2\mathcal{E}_0$ is the reduced amplitude of effective potential, $\varphi\exp(i\psi_*) = -e\theta_0\tilde{E}$, $\theta_0 = Q_0/\gamma$, $Q_0 =$

$eB_0/\kappa_0 mc^2$, $\gamma_{z0}^{-2} = \gamma_0^{-2} + \theta_0^2/2$, $[JJ]_0 = J_0(\nu_0)-J_1(\nu_0)$ and $\nu_0 = Q_0^2/4(1 + Q_0^2/2)$. The detuning parameter $\hat{C}$ is given with the expression $\hat{C} = \hat{C}_0 + \hat{\alpha}(t)\hat{z}$, where $\hat{\alpha} = \alpha l_0 Q_0^2/2g(1+Q_0^2/2)$, $g = \gamma_{z0}^2 c/\omega l_0 \simeq (4\pi N_0)^{-1}$ and N_0 is the number of periods of the main undulator. From Maxwell's wave equation we get the reduced equations for the amplitude $\hat{\varphi}$ and phase ψ_s of the effective potential of interaction:

$$d\hat{\varphi}/d\hat{z} = - \beta \frac{2}{M} \sum_{i=1}^{M} \cos(\psi_{(i)} + \psi_s) \quad , \tag{3}$$

$$d\psi_s/d\hat{z} = \beta \frac{2}{M} \sum_{i=1}^{M} \sin(\psi_{(i)} + \psi_s) \quad , \tag{4}$$

where $\beta = \pi\theta_0^2 j\omega l_0^3[JJ]_0^2(2c\gamma_{z0}^2\gamma_0 I_A)^{-1}$ is the gain parameter, j is the beam current density and $I_A = mc^3/e \simeq 17$ kA.

So, equations (1)-(4) enable one to calculate the field amplification $G = \Delta\hat{\varphi}/\hat{\varphi}$ per one undulator pass (in this section we assume $G \ll 1$). To take into account the resonator losses we use a phenomenological approach introducing the field damping factor K per one resonator round-trip (K is approximately equal to one half of the resonator power relative losses). The lasing takes place when amplification in the small-signal mode of operation is greater than the damping factor, i.e at $G_s > K$ and saturation takes place when G becomes equal to K. FEL efficiency is given with the expression $\eta = ce|\tilde{E}\Delta\tilde{E}|/\pi j\mathcal{E}_0$. It is convenient for the further representation to introduce the reduced efficiency $\hat{\eta} = \eta/g = \hat{G}\hat{\varphi}^2/2$, where $\hat{G} = G/\beta$. One can easily show that the conservation energy law takes place and the reduced FEL efficiency is equal to the averaged value of the reduced electron energy losses: $\hat{\eta} = - \langle\hat{P}\rangle$. At the saturation we obtain: $\hat{\eta} = \hat{K}\hat{\varphi}^2/2$, where $\hat{K} = K/\beta$. It is useful to note that for a conventional scheme of the FEL oscillator with the homogeneous undulator, maximal reduced FEL efficiency $\hat{\eta}_h = 3.62$ is achieved at the optimal value of resonator losses $\hat{K} = \hat{K}_{opt} = 0.028$ [6,7].

Prior to a detailed consideration we should perform a brief qualitative analysis of the proposed FEL oscillator scheme with time-dependent tapering. Let us consider a model situation when the electron beam density after the prebuncher can be approximated by a sequence of the δ - functions and the detuning parameter is given by expression: $\hat{C} = \hat{\alpha}(t)\hat{z}$ (for simplicity we have set here the initial detuning $\hat{C} \simeq 0$). The bunches are fed into the main undulator in an optimal decelerating phase of effective potential (i.e. in equations (1)-(4) we set $M = 1$, $\psi_{(1)} \simeq \pi$ and $\psi_s \simeq 0$ at $\hat{z} = 0$). Parameter $\hat{\alpha}$ is assumed to be a slowly changing function of time: the characteristic time of its change must be much less than a rise time of the radiation field in the resonator. The process of field amplification is developed as follows. At first, when the maximum field is not achieved and $\hat{\alpha}$ grows in time, the radiation field $\hat{\varphi}$ is increasing and $\hat{\varphi}$ is greater than $\hat{\alpha}$. At the same time the electron bunch shifts away from the optimal phase $\psi_{(1)} \simeq \pi$ to provide an equilibrium between the field amplification and resonator losses. As $\hat{\alpha}$ is increasing, the value of $\hat{\varphi}$ approaches the value of $\hat{\alpha}$ and the phase motion of the electron bunch becomes slower. Finally, the regime of the maximum radiation field is settled when there is no phase motion of the electron bunch. At this moment of time the growth of $\hat{\alpha}$ is stopped and further the FEL oscillator operates in stationary regime. In this case $\hat{\alpha} \simeq \hat{\varphi}$ and the losses of the energy of the electron bunch in accordance with eq. (1) are given with the expression $\hat{P} \simeq - \hat{\varphi}\hat{z} \simeq - \hat{\alpha}\hat{z}$. It follows from eq. (3) that the radiation field increment is equal to $\Delta\hat{\varphi} \simeq 2\beta$ and the field amplification coefficient is equal to $G \simeq 2\beta/\hat{\varphi}$. From the saturation condition $G = K$ we get $\hat{\varphi} \simeq 2/\hat{K}$ which corresponds to the reduced FEL efficiency $\hat{\eta}_{max} \simeq 2/\hat{K}$.

To verify the qualitative results obtained above, we have performed a set of numerical simulations with a large number of macroparticles. The parameters of the whole system (prebuncher, drift space and main undulator) have been optimized. As a result, these simulations have confirmed the validity of the parametric dependence $\hat{\eta} \propto \hat{K}^{-1}$ but resulted in the smaller proportionality factor:

$$\hat{\eta}_{max} \simeq 0.8/\hat{K} . \qquad (5)$$

Simulations have shown that in the real situation the electron bunches have finite phase extent, their optimal phase at the entrance into the main undulator is less than π and at saturation the value of parameter $\hat{\alpha}$ is less than $\hat{\varphi}$. So as usually the value of $\hat{K}$ is rather small, $\hat{K} \ll 1$, one can conclude from this simple consideration that a considerable growth of FEL efficiency can be achieved in the FEL oscillator with the prebuncher and the main undulator with time-dependent tapering.

It should be noted that though the electron beam bunching in the prebuncher and drift space depends on the amplitude of the radiation field stored in the resonator, the prebuncher parameters, fixed in time may be chosen. They should be chosen to provide optimal bunching in the strong optical field corresponding to the regime of the maximum field. Of course, at a small field amplitude in the resonator such a choice leads to nonoptimal bunching but this does not stop the field rise. We should note that the presented scheme is rather stable with respect to the initial energy spread in the beam. In this case the prebuncher parameters should be chosen to provide the energy modulation which is much more than the initial energy spread but small enough with respect to the ponderomotive well depth of the main undulator. The results of numerical simulations have shown that in real situations, when energy spread is about some fraction of a percent and parameter $\hat{K}$ is rather small, the efficiency decreases not very significantly with respect to the case of the "cold" electron beam.

3. Numerical example

To realize the proposed scheme it seems realistic enough to use the driving electron beam generated by a superconducting accelerator operating in the continuous mode. The main undulator is a planar electromagnetic one with independently energized windings to provide the time-dependent tapering. The prebuncher (which is made of permanent magnets) is separated from the main undulator by drift space (or equivalent dispersion section). The magnetic system is placed inside the symmetric optical resonator. To calculate a numerical example and make it more realistic, we shall introduce some complications into the model. We assume the Gaussian mode TEM_{00} to be excited in the resonator. The Gaussian mode waist is placed in the middle of the main undulator and the transverse field distribution is given by the expression $|\tilde{E}(r)| \propto \exp\{-(r/w_0)^2\}$, where w_0 is the mode waist size. To describe more rigorously the interaction of the electron beam with the radiation field, we should modify equations (1)-(4) in the following way (for more details see, for example, ref. [18]):

$$d\hat{P}_{(i)}/d\hat{z} = TA_{(i)}^{-1}\hat{\varphi}\cos(\psi_{(i)} + \psi_s), \tag{6}$$

$$d\hat{\psi}_{(i)}/d\hat{z} = \hat{C}_o + 2^{-1}A_{(i)}^{-2}[\hat{P}_{(i)}(A_{(i)} + 1) + \hat{\alpha}\hat{z}(T + 1)], \tag{7}$$

$$d\hat{\varphi}/d\hat{z} = -\beta T\ (2/M)\sum_{i=1}^{M} A_{(i)}^{-1}\cos(\psi_{(i)} + \psi_s), \tag{8}$$

$$d\hat{\psi}_s/d\hat{z} = \beta T\ \hat{\varphi}^{-1}(2/M)\sum_{i=1}^{M} A_{(i)}^{-1}\sin(\psi_{(i)} + \psi_s). \tag{9}$$

Here factor $A_{(i)} = 1 + g\hat{P}_{(i)}$ reflects the dependence of the electron oscillation angle on energy and factor T refers to the undulator parameters tapering: $T = 1 - b_1\hat{\alpha}\hat{z}$, where $b_1 = 2g(1 + Q_0^2/2)/Q_0^2$. All the other notations are the same as in section 2. It should be noted that in the case of small energy deviation $(\mathcal{E}_{(i)}-\mathcal{E}_o)/\mathcal{E}_o = g\hat{P}_{(i)} \ll 1$ and at a small undulator field tapering, $(B_o-B)/B_o = b_1\hat{\alpha} \ll 1$, equations (6)-(9) transform to equations (1)-(4). When writing down equations (6)-(9) we have accepted some assumptions. First, we have neglected the change of factor $[JJ]$ and Gaussian mode parameters along the main undulator. Second, to calculate the beam current density j entering into the expression for the gain parameter β, we assume the electron beam transverse size to be much less than the Gaussian beam waist size w_o. As a result, in the one-dimensional equations we replace the beam current density j by the effective value $j = 2I/\pi^{3/2}w_0^2$, where I is the peak beam current.

When writing down the reduced equations for the prebuncher, we perform their normalization to the parameters of the main undulator (the prebuncher reduced length is equal to $\hat{l}_p = l_p/l_0$). So as the magnetic field of the prebuncher is fixed and energy modulation in the prebuncher is small, these equations are similar to equations (1)-(4) but the right-hand sides of equations (1), (3) and (4) should be multiplied by factor $b_2 = Q_p[JJ]_p w_0/Q_0[JJ]_0 w_p$, where Q_p and $[JJ]_p$ are the undulator parameter and $[JJ]$ factor of the prebuncher, respectively. Introducing the factor w_0/w_p we have taken into account variation of the optical mode (w_p is the transverse size of the Gaussian mode in the prebuncher). The right-hand side of equation (2) should be multiplied by factor $b_3 = (1+Q_p^2/2)/(1+Q_0^2/2)$ and detuning parameter $\hat{C}$ should be set equal to $\hat{C}_0$. The prebuncher parameters must satisfy the condition $\lambda_p(1+Q_p^2/2) = \lambda_0(1+Q_0^2/2)$, where λ_p is the prebuncher period.

The change of electron phase in the drift space with the length D is given with the equation: $\Delta\psi_{(1)} = (\hat{C}_0 + \hat{P}_{(1)})\hat{D} + \delta\psi$, where $\hat{D} = D[(1+Q_0^2/2)l_0]^{-1}$. The term $\delta\psi = -2\pi N_0\hat{D} - \delta\psi'$ takes into account the fact that the electron beam slippage length is not multiple to the radiation wavelength ($\delta\psi'$ is the change of radiation mode phase in the drift space).

The initial energy spread of the electrons is assumed to be the Gaussian with the distribution function $F(P) = (2\pi\sigma^2)^{-1/2}\exp\{-P^2/2\sigma^2\}$ which corresponds to the distribution function of the reduced energy deviation $\hat{F}(\hat{P}) = (2\pi\hat{\Lambda}_T^2)^{-1/2}\exp\{-\hat{P}^2/2\hat{\Lambda}_T^2\}$, where $\hat{\Lambda}_T^2 = (4\pi N_0\sigma/\mathcal{E}_0)^2$ is the energy spread parameter. Simulation begins at a small amplitude $\hat{\varphi}$ and at the fixed main undulator parameters ($\hat{\alpha} = 0$). The value of initial detuning $\hat{C}_0$ corresponds to the maximum gain in the small-signal mode of operation. When the saturation regime is settled, we begin a slow increase of the tapering parameter $\hat{\alpha}$ and continue this process until the cessation of the field amplitude growth. Then the FEL oscillator operates in the stationary regime. It is assumed that the rate of parameter $\hat{\alpha}$ growth is much less than the rate of the radiation field change. In this case the field amplitude $\hat{\varphi}$ and the FEL efficiency $\hat{\eta}$ are functions of the tapering parameter $\hat{\alpha}$ and are growing in time while $\hat{\alpha}$ is increasing.

Using the algorithm described above we have calculated numerical example for parameters of the electron beam, magnetic system and optical resonator which are summarized in Table 1:

Table 1

Electron beam	
Energy, $\mathcal{E}_0$	30 MeV
Peak current, I	22 A
Energy spread, $\sigma/\mathcal{E}_0$	2 10^{-3}
Macropulse duration	> 10 ms
Magnetic system	
Prebuncher period, λ_p	12 cm
Prebuncher magnetic field, B_p	570 G
Number of prebuncher periods, N_p	3
Length of drift space, D	130 cm
Main undulator period, λ_0	6 cm
Main undulator magnetic field, B_0	3 kG
Number of main undulator periods, N_0	60
Maximal taper, $\Delta B/B_0$	51 %
Optical resonator	
Wavelength	20 μm
Resonator length	9 m
Curvature radius of mirrors	5.5 m
Total power losses	2 %

The corresponding reduced parameters are as follows: $\beta = 5$, $\hat{K} = 2\circ10^{-3}$, $g = 1.33\circ10^{-3}$, $\hat{\Lambda}_T^2 = 2.3$, $\hat{I}_p = 0.1$, $\hat{D} = 0.15$, $\delta\psi = 0.4$, $\hat{C}_0 = 2.5$, $b_1 = 2.27\circ10^{-3}$, $b_2 = 0.23$ and $b_3 = 0.5$. Fig.1 presents the time dependencies of the undulator tapering depth and FEL efficiency. At the end of the time-dependent tapering process (τ = 10 ms) about 74 % of the particles are trapped in the regime of coherent deceleration and FEL efficiency $\eta \simeq 20\%$ is achieved which is more than 40 times larger than the maximum efficiency of the FEL oscillator with the homogeneous undulator. Analyzing technical parameters presented in Table 1, we may conclude that they are achievable at the present level of the accelerator and FEL technology.

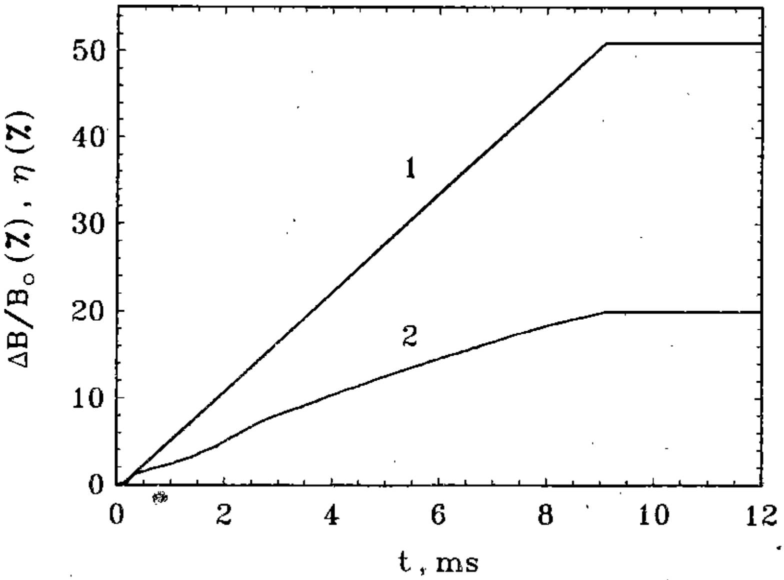

Fig.1. The depth of the magnetic field tapering in the main undulator (1) and FEL efficiency (2) as functions of time

4. Conclusion

In the presented paper we have considered a new method to increase the FEL oscillator efficiency. The magnetic system of the proposed device consists of the prebuncher with fixed parameters, drift space and main undulator with time-dependent tapering. The feasibility of the method has been confirmed by the results of numerical simulations in the framework of the one-dimensional model. An example of the FEL oscillator with realistic parameters operating in the continuous or quasi-continuous mode with an efficiency about 20 %, is presented.

References

[1] P.Sprangle, C.M.Tang and W.M.Manheimer, Phys. Rev. Lett. **43**(1979)1932

[2] N.M. Kroll, P.L. Morton and M.N. Rosenbluth, IEEE J. Ouantum Electron. **QE-17**(1981)1436

[3] P.Sprangle, C.M. Tang and W.M.Manheimer, Phys. Rev. **A21**(1980)302

[4] D.Prosnitz, A.Szoke and V.K.Neil, Phys. Rev. **A21**(1981)1436

[5] T.J. Orzechowski et al., Phys. Rev. Lett. **57**(1986)2172

[6] B.W.J.McNeil, Nucl. Instrum. and Methods **A296**(1990)388

[7] E.L Saldin, E.A.Schneidmiller and M.V. Yurkov, "Similarity techniques in a one-dimensional theory of a FEL oscillator", Optics Communications, in press

[8] D.W.Feldman et al., Nucl. Instrum. and Methods **A285**(1989)11

[9] A.H.Ho, J.Feinstein and R.H.Pantell, IEEE J. Quantum Electron. **QE-23**(1987)1545

[10] A.H.Ho, R.H.Pantell and J.Feinstein, Nucl. Instrum. and Methods **A318**(1991)758

[11] R.Rohatgi et al., Nucl. Instrum. and Methods **A272**(1988)32

[12] K.Alrutz-Ziemssen et al., Nucl. Instrum. and Methods **A304**(1991)159

[13] I. Ben-Zvi et al., Nucl. Instrum. and Methods **A304**(1991)181

[14] G.R.Neil et al., Nucl. Instrum. and Methods **A318**(1991)212

[15] Y.Kawarasaki et al., Nucl. Instrum. and Methods **A272**(1988)206

[16] M.Castellano et al., Nucl. Instrum. and Methods **A304**(1991)204

[17] E.L.Saldin, E.A. Schneidmiller and M.V.Yurkov, Nucl. Instrum. and Methods **A313**(1992)555

[18] E.L.Saldin, E.A.Schneidmiller and M.V.Yurkov, Sov. J. Particles & Nuclei **23**(1992)104

Received by Publishing Department
on March 9, 1993.