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A.M.Taratin, E.N.Tsyganov, H.-J.Shih*

RESONANCE EXCITATION OF THE SSC BEAM HALO BY ELECTRIC PULSES SYNCHRONIZED WITH RF VOLTAGE

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*Superconducting Super Collider Laboratory, Dallas, Texas, USA

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1. Introduction

A resonance excitation of a longitudinal motion of beam halo particles by means of an amplitude or a phase perturbation of the main RF voltage can be used to bring them onto a crystal deflector for the extraction from the Superconducting Super Collider (SSC). There is a proposal [1] of such extraction for doing B physics in a fixed target configuration. At the amplitude perturbation considered here the additional electric pulses synchronized with the RF voltage are introduced. When their duration is mach smaller than the RF period, $T_p \ll T_{rf}$, we may eject from the bunch only the halo particles with $l_m > l_b$, where l_m is the longitudinal displacement amplitude of particles from the bunch center during synchrotron oscillations, l_b is a some boundary value of l_m. The beam core particles are not perturbed in this case. The corresponding phases of the pulse switching on φ_p with respect to the main RF voltage (see Fig.1) must satisfy by the condition

$$\varphi_b > \varphi_p > 2\pi - \varphi_b, \qquad (1)$$

$$\varphi_b = \pi \, \frac{l_b}{l_w}, \quad l_w = \frac{1}{2}\, \frac{L}{h},$$

where L is a length of a design orbit, h is a harmonic number, l_w is a bucket half-width. For the SSC parameters $T_{rf} \simeq 2.78$ nsec, therefore, for the differential action on the bunch halo the electric pulses with $T_p < 500$ psec are necessary.

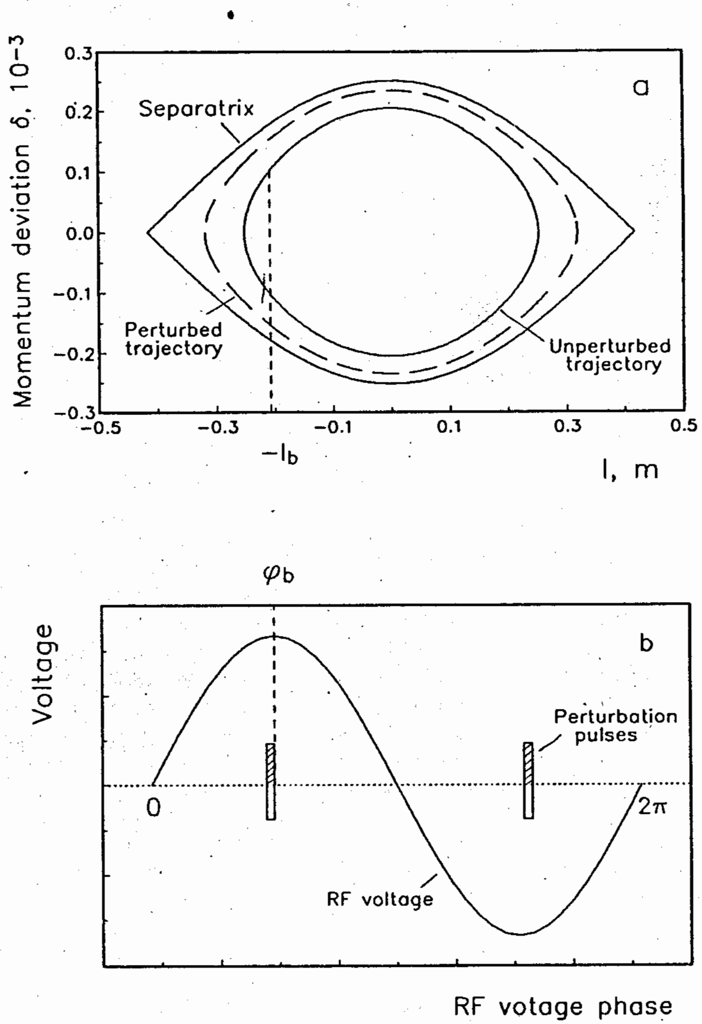

Fig.1. Schematic picture of the process of strengthening the synchrotron oscillations of the bunch halo partcles by means of the electric pulses synchronized with RF voltage.

a). Bunch halo region in the phase space (δ,φ).
b). A sinusoidal RF voltage and the possible positions for the perturbation pulses.

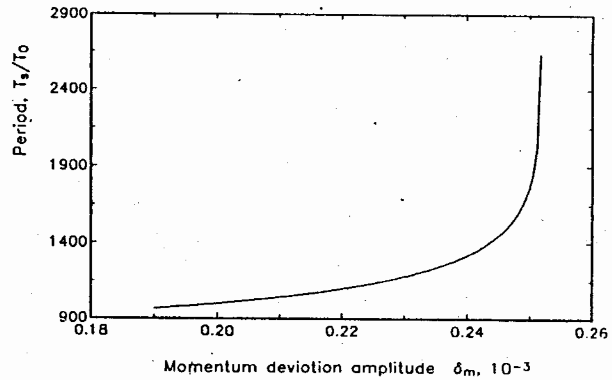

Fig.2. The synchrotron oscillation period for 20 TeV protons of the SSC as a function of the amplitude of relative momentum deviation δ_m.

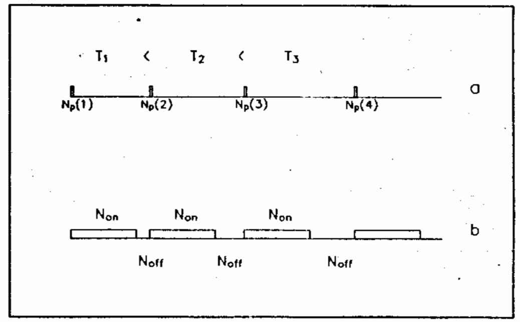

Fig.3. Schematic representation of the initial part for the resonance pulse sequence (a) and the repeating one (b).

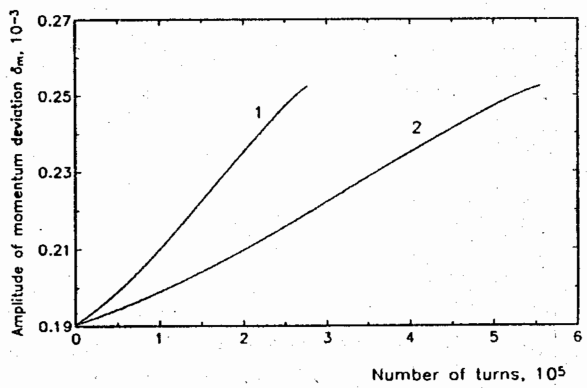

Fig.4. The amplitude of the relative momentum deviation δ_m as a fuction of the particle turn number after the resonance sequence switching on - "the resonance growth of δ_m ". The pulse amplitude $V_p = 10$ MV (1) and 5 MV (2).

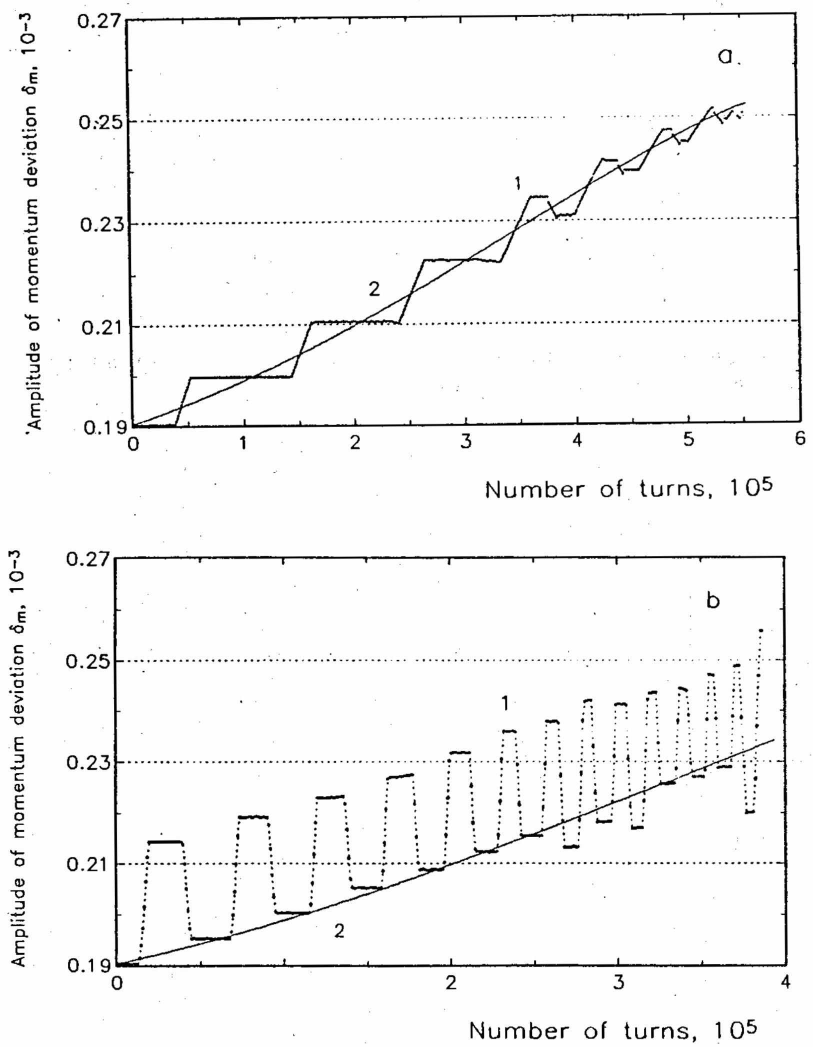

Fig.5. The same as Fig.4 for the repeating sequence switching on (curves 1). The pulse amplitude $V_p = 5$ MV. The initial amplitude value $\delta_{mo} = \delta^r_{mo}$. The pulse duration $T_p \simeq 15$ psec - "short pulses" (a) and 270 psec - "long pulses" (b). Curve 2 is the resonance growth of δ_m.

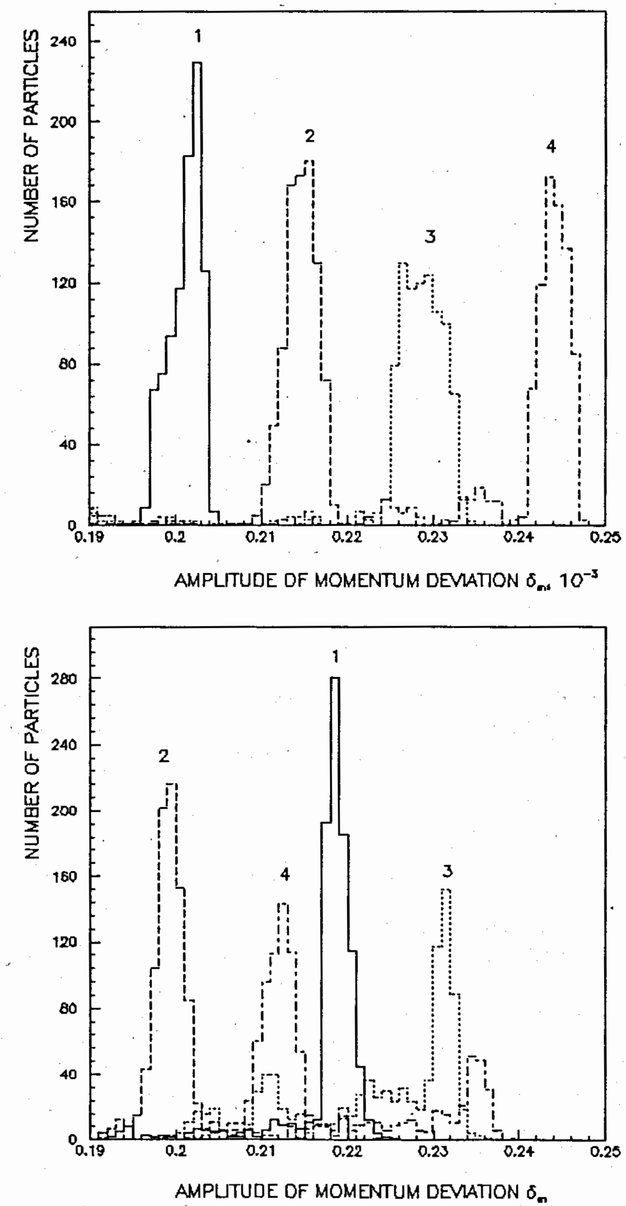

Fig.6. The distribution of the particle momentum deviation amplitudes for different turn number after the repeating sequence switching on. The initial distribution are uniform in the interval of $(\delta^r_{mo} - \Delta, \delta^r_{mo} + \Delta)$ with $\Delta = 10^{-6}$. (a) - for short pulses, the number of turns : 1.2 (1), 2.4 (2), 3.6 (3), 4.8 (4) $\times 10^5$. (b) - for long pulses, the number of turns : 0.8 (1), 1.0 (2), 2.0 (3), 2.2 (4) $\times 10^5$.

The bucket half-width for the SSC $l_w \simeq 0.42$ m. Let us take as a boundary value for the longitudinal displacement from the bunch center $l_b = 0.21$ m, that is $\varphi_b \simeq \pi/2$. The corresponding synchrotron oscillation frecuency $f_b = f_s(l_b) \simeq 0.85 f_{so}$, where f_{so} is a small-oscillation frecuency, and the amplitude of a relative momentum deviation from the ideal one $\delta_b \simeq 0.18 \times 10^{-3}$, the bucket half-height $\delta_h = 0.252 \times 10^{-3}$.

For increase of the amplitude of particle synchrotron oscillations the pulses have to be accelerating at the half-period with $\delta > 0$ and/or decelerating in opposite case. This does not fulfil when the pulses switch on at every turn. In this case their resulting action during a long time is considerably compensated. Here we consider a resonance exitation when the pulses act on the bunch halo particles in a resonance with their synchrotron oscillations to eject them from the bucket.

2. Resonance pulse sequence

Large synchrotron oscillations are nonlinear, because they are governed by an electric RF field which departs significantly from a linear dependence upon a time. Their period T_s depends on the oscillation amplitude ψ of the particle phase φ near a synchronous phase φ_s. For a stationary bucket, when the beam is not accelerated, but is only kept bunched at a fixed energy, and for a sinusoidal RF voltage, $\varphi_s = \pi$ and for T_s we have [2]

$$T_s(\psi) = T_{so}\ \frac{2}{\pi}\ K\left(\sin\frac{\psi}{2}\right), \qquad (2)$$

$$T_{so} = T_o\ \left(\frac{2\pi}{\alpha h}\ \frac{E}{eV}\right)^{1/2},$$

where T_o is a revolution period of a synchronous particle, T_{so} is a small-oscillation period, K is the complete elleptic integral of the first kind. The phase oscillation amplitude is connected with the amplitude of the relative momentum deviation

$$\delta_m(\psi) = \left(\frac{1}{\pi\alpha h}\ \frac{eV}{E}\right)^{1/2} (1 - \cos\psi)^{1/2}. \qquad (3)$$

Fig.2 shows the dependence of the synchrotron oscillation period for protons 20 TeV on the amplitude δ_m calculated for the SSC parameters. The period increase is most intensive for the amplitudes near the bucket half-height δ_h .

The pulse frequency has to follow the period change $T_s(\delta_m)$ to intensify the synchrotron oscillations of the bunch halo particles. The resonance sequence of the pulses with the amplitude V_p—$N_p(n; \delta^r_{mo}, \varphi_p, V_p)$, where N_p is a turn number, n is a pulse number, which leads to a monotonous increase of the relative momentum deviation for the particle with a given initial δ^r_{mo} , when the pulses act one time during the synchrotron oscillation period, was found by computer simulation. Unperturbed particle trajectories in a phase spase (δ, φ) were calculated by a numeric solution of the finite difference equations for a stationary bucket with a sinusoidal RF voltage

$$\begin{aligned} \delta_{n+1} - \delta_n &= \tfrac{eV}{E}\ \sin\varphi_n, \\ \varphi_{n+1} - \varphi_n &= 2\pi\alpha h \cdot \delta_{n+1}. \end{aligned} \qquad (4)$$

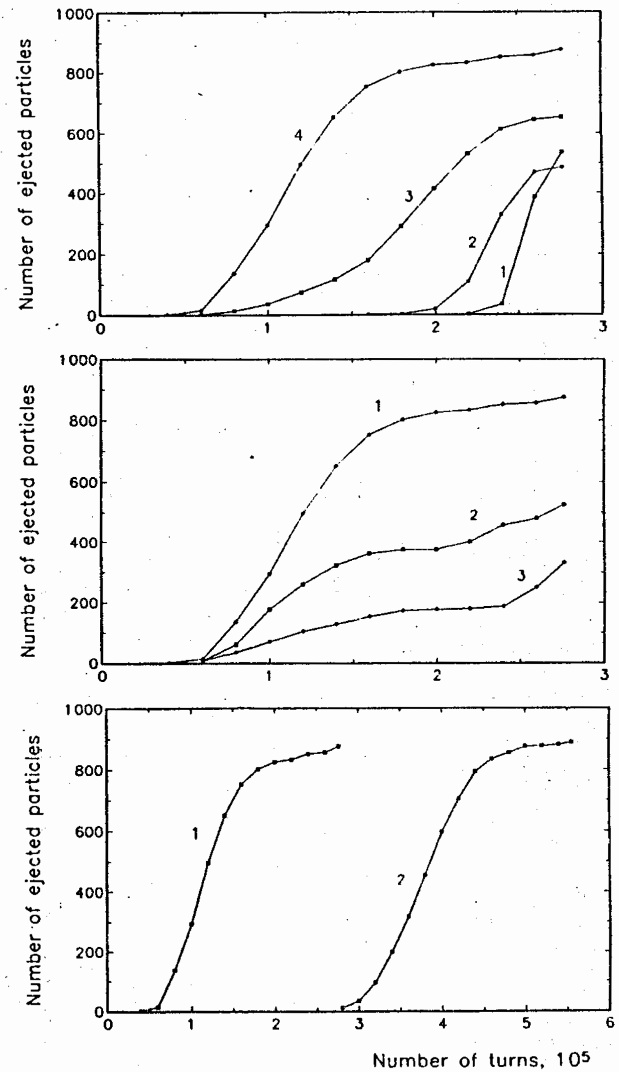

Fig.7. The number of particles left the bucket as a function of the turn number after the repeating sequence switching on. The initial particle amplitudes are near the resonance value δ^r_{mo}, as for Fig.6.

a). the different pulse durations : 33.3 psec (1), 66.6 psec (2), 133.2 psec (3), 270 psec (4). $V_p = 10$ MV, $N_{on} = 840$.

b). the different repetition numbers of the resonance sequence N_{on} : 840 (1), 400 (2), 200 (3). $V_p = 10$ MV, $T_p = 270$ psec.

c). the different pulse amplitudes V_p : 10 MV (1), 5 MV (2). $T_p = 270$ psec, $N_{on} = 840$.

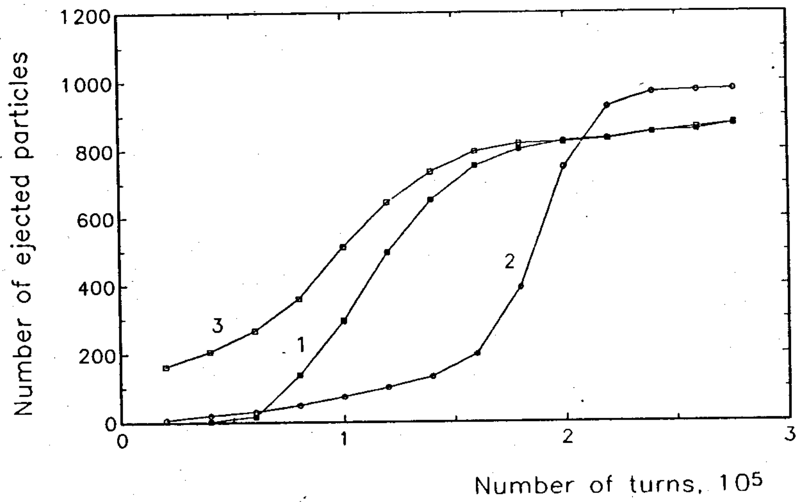

Fig.8. The same as in Fig.7. The initial particle amplitudes δ_{mo} are far from δ^r_{mo} : (2) 0.21 $\times10^{-3}$, (3) 0.23 $\times10^{-3}$. Curve 1 is for $\delta_{mo} \simeq \delta^r_{mo}$. $T_p = 270$ psec, $V_p = 10$ MV, $N_{on} = 840$.

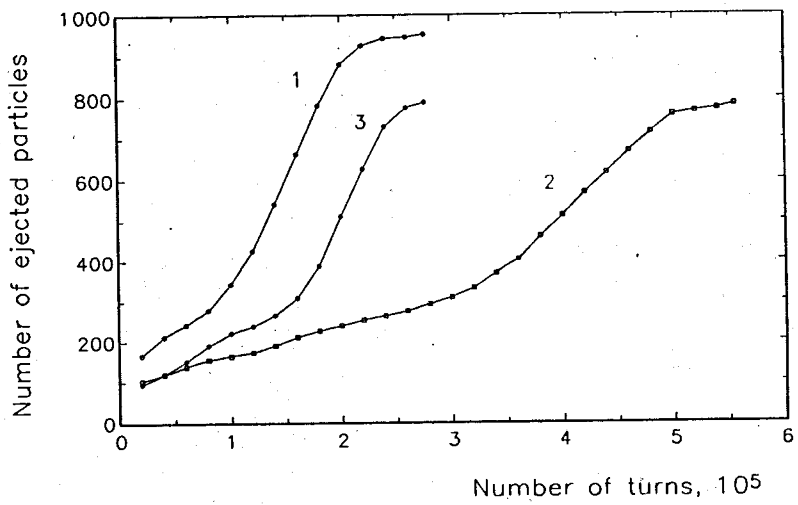

Fig.9. The same as in Fig.7 for all bunch halo particles: $\delta_{mo} = (0.19-0.25)\times10^{-3}$. Curve (1) for $T_p = 270$ psec, $V_p = 10$MV; (2) - $T_p = 270$ psec, $V_p = 5$ MV; (3) - $T_p = 133.2$ psec, $V_P = 10$ MV.

When the particle phase found itself in a small region near φ_p at $\delta > 0$ the additional increase of the particle energy occured, $\Delta\delta_p = eV_p/E$. At this moment the turn number and the particle amplitude deviation δ_m were registred. Fig.3a shows shematically the initial part of the resonance sequence.

The increase of the particle momentum deviation amplitude δ_m at one pulse action is defined by

$$\Delta\delta_m = (\Delta\delta_p^2 + 2\Delta\delta_p\delta + \delta_m^2)^{1/2} \simeq \frac{\delta}{\delta_m}\Delta\delta_p. \tag{5}$$

Here $\delta(\varphi_p, \delta_m)$ is the particle momentum deviation at the action moment.

Fig.4 shows the growth of the particle momentum deviation amplitude δ_m with the turn number after the resonance sequence switching on. The dependence is close to a linear one. It forms by the growth of $\Delta\delta_m$ and T_s with δ_m. The second circumstance decreases a relative number of the pulse actions at one particle turn.

3. Repeating pulse sequence

At a single switching on the resonance sequence captures and leads out through the separatrix a very small fraction of particles, which are near $(\delta^r_{mo}, \varphi_p)$ at the initial moment. With the growth of the pulse duration this fraction increases, but still remains an insignificant part of all particles with the amplitudes δ_{mo} near the resonanse one δ^r_{mo}.

The resonance sequence may be repeated through every turn to act on the halo particles of the considered bunch. The sequence got by this method may be considered as a new one, where the interval of the pulse switching on at every period of synchrotron oscillations has a constant duration, N_{on} of turns, and the interval, at which the pulses switch off, increases with the growth of the resonance sequence period. At n-th oscillation period $N_{off}(n) = N_p(n+1) - N_p(n) - N_{on}$. Fig.3b shows schematically this repeating sequece. A maximal number of the sequence repetitions is determined by the condition of the particle return to the initial phase, but with opposite sign of δ, $N_b < N_p(n=2)$. When the repetition number N_{on} is larger than N_b the compensation of the pulse actions will be occur. The value of N_b depends on δ^r_{mo}, but it is always larger than half the particle turn number, which occur for the period of synchrotron oscillations $N_b > 0.5T_s(\delta^r_{mo})/T_o$. So, it should seems we can lead out a bigger part of the bunch halo particles with the amplitudes δ_{mo} near δ^r_{mo} by means of the repetitions of the resonance sequence.

However, there is one circumstance connected with the pulse duration, which makes more complicated the picture observed with the repeating sequence. The change of the time displacement of the particle from the bunch center during one turn is defined as $\Delta t_r = hT_{rf}\alpha\delta$. That is its value changes with the relative momentum deviation δ. In the considered interval of the momentum deviation amplitudes $\delta_m = (0.19-0.25)\times10^{-3}$ and at the given phase of the pulse action φ_p we have $\Delta t_r = (1.5-5)$ psec. Therefore, for the repeating sequence the action by the pulses cannot be single at every synchrotron oscillation. Even at very short pulses, $T_p \ll T_{rf}/4$, they act repeatedly on the particle, $T_p/\Delta t_r$ of times, for one synchrotron oscillation. In this connection it should waits the synchronism violation for the particles with the resonance sequence when it repeats.

Fig.5 shows the calculated dependence of the momentum deviation amplitude δ_m on the turn number after the repeating sequence switching on for the particles with $\delta_{mo} = \delta^r_{mo}$. The phase trajectories were calculated according to the equations (4). The additional action by the perturbation pulse occured when the particle phase turned out to be near φ_p in the interval of $\Delta\varphi$ and the particle turn number — in the switching on interval of the repeating sequence. The size of $\Delta\varphi$ is determined by the pulse duration $\Delta\varphi = 2\pi T_p/T_{rf}$.

In fact, the increase of the particle amplitude δ_m is not monotonous for the repeating sequence. At a small pulse duration the short intervals of the amplitude growth alternate with more long ones, where δ_m does not change practically, Fig.5a. In the case of the large pulse duration they are changed by the intervals of a sharp pulse decrease, fig.5b. The oscillations of the momentum deviation amplitude δ_m occurs near the growing amplitude of the resonance sequence. That is the average rate of the amplitude growth for the repeating sequence is close to the resonance one. The oscillations of δ_m occur due to the period change of the particle synchrotron oscillations with respect to the interval between the pulses of the resonance sequence.

4. Ejection efficiency of bunch halo particles

The distributions of the particle momentum deviation amplitudes for the different turn number after the repeating sequence switching on are shown in Fig.6. The initial particle distribution was uniform in the phase region with the amplitudes $\delta^r_{mo} \pm \Delta$, where $\Delta = 10^{-6}$. For the small pulse duration the distribution maximum shifts successively to the extreme value δ_h, Fig.6a. That is the particle transfer in the δ_m-space takes place. For the long pulses the maximum oscillates back and forth shifting to δ_h, Fig.6b.

Fig.7 shows the particle number left the bucket as a function of the turn number after the repeating sequence switching on for the different pulse duration. For the short pulses the particles leave the bucket at the very end of the sequence (curve 1 at Fig.7a). When the pulse duration is large the ejection of the bunch halo particles through the separatrix occurs due to the sharp growth of the amplitude δ_m at its oscillations near the average resonance value. It begins earlier and is more effective, although its duration is larger (curve 4). The decrease of the resonance sequence repetition number N_{on} leads to the decrease of the ejection efficiency of the bunch halo particles, Fig.7b. On the other hand when the repetition number equals the number of turns fulfiling for the period of synchrotron oscillations at $\delta^r_{mo}, N_{on} = 965$, a slow broadening of the initial δ_m-distribution of particles takes place only, as in the case when the pulses act on the bunch at every turn. The duration of the resonance pulse sequence increases with the pulse amplitude decrease (see Fig.4). Therefore, at $V_P = 5$ MV the ejection time of the particles from the halo depth is twice as larger than at $V_P = 10$ MV, Fig.7c. Although the ejection efficincy is approximately the same, it is close to 90 %.

Until now we considered only the particle fraction with the amplitudes near the resonance one δ^r_{mo}. However, the particle ejection efficiency from the bunch halo for the repeating sequence is determined by the behaviour of the particles with the amplitudes $\delta_{mo} \neq \delta^r_{mo}$. Our computer simulation results shows that these particles are not captured by the resonance sequence when the perturbation pulses are short. A broadening of their δ_m -distribution occurs only. For the long pulses the repeating sequence action is effective for these particles also. The corresponding results with the pulse duration $T_p = 270$ psec for the particle fractions with the initial amplitudes near $\delta_{mo} = 0.21 \times 10^{-3}$ (curve 2) and $\delta_{mo} = 0.23 \times 10^{-3}$ (curve 3) are represented in Fig.8. The results for all considered bunch halo are shown in

Fig.9. The initial distribution of the halo particles in the phase region with the amplitudes $\delta_{mo} = (0.19 - 0.25) \times 10^{-3}$ was uniform. The ejection efficiency of the bunch halo particles from the bucket by means of the repeating pulse sequence is high for the pulse amplitudes considered here. It reduces with the decrease of the pulse amplitude (curve 2) and their duration (curve 3).

5. Conclusion

Our computer simulation results show that the resonance exitation of longitudinal motion of the SSC beam halo particles by the electric pulses synchronized with the RF voltage is effective to eject them from the bucket onto a crystal deflector for the extraction from the accelerator. The ejection rate may be regulated by the pulse amplitude and duration, and also by the repetition number of the resonance sequence of the pulses. Moreover, it is possible to act on either a single bunch or on the few ones, simultaneously.

Acknowledgements

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