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ON A POSSIBILITY OF ATTAINING SUPERHIGH ENERGY BY NEW METHODS OF CHARGED PARTICLE ACCELERATION

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О возможности достижения сверхвысоких энергий новыми методами ускорения заряженных частиц

Обсуждается возможность достижения сверхвысоких энергий при ускорении ионов в электронном релятивистском сильноточном пучке, в котором при резонансном взаимодействии пучка с E-волной возбуждается переменное электрическое поле /волна/ с амплитудой до 10^7 В/см. Рассмотрены физическое и математическое описание резонансного доплеровского взаимодействия электронного пучка с E-волной.

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On a Possibility of Attaining Superhigh Energy by New Methods of Charged Particle Acceleration

The physical and mathematic description of the process of resonance Doppler interaction (RDI) of relativistic electron beams with E-waves is considered. The possibility of solution of Veksler's problem is also discussed here on the ground of the RDI process, i.e., how to attain, at least from the principal point of view, the energy of accelerated particles of the order of 1000 GeV and above by new methods.

The investigation has been performed at the Laboratory of High Energies, JINR.

1. INTRODUCTION

In order to accelerate charged particles effectively, it is necessary either to maintain their multiple passage over a gap with an electric high intensity field (as in classical synchrotrons) or to move this field in space with variable speed as is in essence done in linear resonance and collective type accelerators.

V.I.Veksler's suggestion on the use of coherent (later on collective) principle of acceleration put forward in the fifties-sixties at JINR, Dubna refers just to the second type [1,2a]. In general, the coherent principle lies in that the electric field intensity is determind by the number of accelerated particles; the collective principle is characterized by the field depending on the number of charged particles of another type, e.g., electrons in a bunch accelerating ions. All these ideas of Veksler were aimed at attaining an accelerated particle energy of above 1000 GeV using new methods. Then this energy was considered to be the limiting one for classic type accelerators. The application of superconductivity and the method of colliding beams was only originating.

Let me formulate Veksler's problem as follows:

To find the mechanism of generation of large electric fields in an ensemble of charged particles for the acceleration of charged particles of the same or another type to energies of the order of 1000 GeV and above.

To solve Veksler's problem, it is obviously required to get an energy gain of no less than 100-1000 MeV/m and to keep it along a length of hundreds of metres.

The initial way of an experimental solution of this problem as a single bunch (ring) of circulating electrons with captured ions, which is moving in space, was put into realization at the beginning of the sixties [2a,b] and stimulated a great number of investigations and interesting results all over the world. However, it did not lead finally to the ultimate objective as it is extremely difficult to provide a stable motion of a bunch with great space charge ($Ne > 10^{13}$) at such large length. A very long stay of electrons in the bunch rest system as a whole makes this bunch, i.e., the ensemble of electrons

with captured ions, practically unprotected against static effects of space charge and against dynamic effects - numerous instabilities.

However, Veksler's formulation of new acceleration principles called into being quite a number of studies in the new field of accelerating physics.

As it turned out later on, Ya.B.Fainberg's ideas of the possibility of generation of very strong electric fields in dense plasma and high-current beams when exciting great amplitude waves in them (suggested practically at the same time) are of particular importance [3,4].

The known estimate for a maximum electric field in the ensemble of electrons with electron density n_e and wave $\exp ik_z z$ is written as

$$E_M \sim \sqrt{4\pi mc^2 n_e}. \qquad (1)$$

For density $n_e \simeq 10^{13}$ cm^{-3}, for example, this estimate yields $E_M \sim 3\cdot 10^6$ B/cm.

The basis of the beam-wave approach is the fundamental idea of creating density modulation due to transit electrons, i.e. such electrons which exist in the region of fluctuations over an extremely short period of time, $\sim 10^{-10}$ s. This removes most principal difficulties associated with the influence of large space charge. It is conceivable that the first clear formulation of this idea was given by M.S.Rabinovich in 1968 [5].

A rapid evolution of the conception of beam-wave collective acceleration methods has begun approximately since 1967 after developing the technique of high-current relativistic electron beams. Several tens of general modifications of the collective method have been put forward to date. However, the experimental achievements are limited by fields of $\sim 10^5$ V/cm and length of ~1 m which is much less than estimate (1), see, e.g., [6].

Correspondingly, in one of the recent reviews of Ya.B.Fainberg [7] it is stressed that the existing varieties of collective methods do not solve yet the problems of development of superhigh energy accelerators.

In high-current relativistic electron beams (REB) the density of electrons can reach 10^{13} cm^{-3} and above for a beam current up to 10^6 A at a beam "transporting" length of 1-10 m and more. Consequently, REBs are in principle an ideal basis for the development of the conception of moving density modulation on transit electrons.

One of the pioneer papers, in which a concrete mechanism of implementation of this idea has been suggested, is that of

Sloan and Drummond[8]. In this paper is was proposed to generate the moving modulation of beam density with the aid of slow cyclotron beam mode, i.e., when the electron velocity is larger than the phase wave one. The performed experiments[9] showed a field increase up to 100 kV/cm at a beam current of above 1 kA.

In the paper of Sprangle at al.[10] this idea was modified due to the excitation of a slow Langmuire mode in magnetized beam, and to the mechanism of phase wave velocity increasing by decreasing the external diameter of a waveguide.

Nation[11,12] carried out very important experimental investigations on the excitation of a slow Langmuire wave ($v/c=0.2$) in a relativistic electron beam (J = 200-500 A, We=200-400 KeV) at a radiation frequency of 1 GHz inserted into a diaphragming waveguide. The excitation of fast waves, nonlinear effects with increasing the field up to 300 kV/cm and a destructive action of electron capture by the wave field were observed.

Experiments on the acceleration of protons by a slow wave with an energy gain up to 4 MeV/m were also performed.

Later on the process of resonance Doppler interaction (RDI) of REB with longitudinal electromagnetic waves (E-waves) will de considered from a new physical point of view, a review of original mathematical results of the theory of this process will be given and the discussion of RDI applicability to the solution of Veksler's problem will be presented.

2. PHYSICAL BASES OF RDI OF RELATIVISTIC ELECTRON BEAMS WITH E-WAVES

The simplest possibility to create density fluctuation in an electron beam is realized when the beam meets the space region (gap) inside which a longitudinal (relative to the motion of electrons) electrical field is generated by an external source. As is known, if electrons are decelerated, density increase takes place, and if they are accelerated, density decreases. But this fluctuation is immovable in space (laboratory reference frame) as well as the gap itself.

So, it is necessary to move the external field in space along the beam. An electromagnetic wave with a variable longitudinal field running along the beam with phase velocity $v < C$, i.e.,

$$E_{||} = |E|\exp(-i\omega t + ikz), \quad v = \frac{\omega}{k}, \quad k \equiv k_z, \tag{2}$$

can serve as a natural solution of this task. Such a wave can be excited in special slowing-down structures. If the wave velocity v is not equal to that of beam electrons v_e, electrons are transit ones with respect to the wave. They pass periodically through the space region with decelerating and accelerating electrical fields.

In order to create a strong enough density modulation in the beam, the resonance condition should be fulfilled which will lead to increasing the density modulation in a beam and the summary variable field in the beam-wave system.

The characteristic frequency of longitudinal electron oscillations in the beam is the Langmuire frequency:

$$\omega_e = \sqrt{\frac{4\pi e^2 n_o}{m\gamma_o}}, \tag{3}$$

n_o is the density of electrons;and γ_o, the relativistic factor of longitudinal electron motion.

It is expected that the resonance will be realized if this frequency coincides with that of an external longitudinal electromagnetic wave taking into account the Doppler shift of frequency ω for transit electrons. Consequently, taking electron relativism into account, the expected resonance condition can be written as

$$\gamma_o(\omega - kv_o) = \pm\,\omega_e, \tag{4}$$

v_o is the velocity of electrons, $\gamma_o = (1 - v_o^2/c^2)^{-1/2}$, index "0" always denoted the corresponding quantities at the beginning of the process when the field is relatively small, $E = E_b \simeq 0$.

Formula (4) allows for the possibility of different relation between mutual directions and the values of velocities v and v_o. Sign (+) stands for a fast parallel wave ($v > v_o$, $k > 0$) and an opposite one ($v \uparrow\downarrow v_o$, $k < 0$), sign (-) for a slow parallel wave ($v < v_o$, $k > 0$).

The analysis performed in papers[13-15] shows that resonance (4) actually takes place in a not too strong field of the wave moving along a high-current electron beam; only taking into consideration the specific geometry of a slowing-down structure, the frequency is "reduced" by the geometric factor S: $\omega_e \to \omega_e/S$.

To prove the statement that such a resonance Doppler interaction (RDI) of a beam with a E-wave can result in the solution of the problem of producing a strong enough variable electric field moving in space, it is necessary to form the RDI theory within the frame of the relativistic model beam and wave in view of non-linear effects as soon as the aim is pursued to obtain large in amplitude electric fields.

Conditions (4) or (18) coincide with resonance condition (sign(-1)) of the so-called collective Čerenkov effect ("Raman" instability) (see /16/). But, RDI and instability have quite different physical nature: RDI is the amplification of the external E-wave by beam charge modulation with an increment $\sim n_0$ and takes place for any types of wave - slow, fast and opposite ones if condition (4) is fulfilled.

"Raman" instability is connected with radiation of electron beams in a slowing-down structures, appears without external wave and is characterized by exponential increase of the slow wave field (the beam is always faster than the wave) with the increment $\sim n_0^{1/4}$.

This instability practically does not display in presence of the RDI process because of: a) large initial amplitude of the external wave, ~10 kV/cm, b) large increment of RDI process.

III. MATHEMATICAL BASES OF THE RDI THEORY

The mathematical RDI theory will be set forth in a condensed form below. We are talking about the formulation of the problem, the methods of its solution and main results, (details and proofs see in /13–15,17,18/).

1. Problem Statement, Model and RDI Basic Equation

The basis for the RDI mathematical theory is the hydrodynamic model of a high-current relativistic electron beam (REB) and its interaction with a longitudinal electromagnetic wave. This model* is characterized by the following principal assumptions:

1) H o m o g e n e i t y of an initial beam state, i.e., density n_0 does not depend on coordinates r, θ and z. In addition, an axial symmetry of all the system is anticipated: the beam plus the structure slowing down a wave.

2) M o n o e n e r g e t i c s of electrons. It is required that the spread in electron velocity should be less than the difference of beam and wave ones (exclusive of the possibility of electron-wave Cherenkov interaction):

$$\Delta v_e \ll |v_e - v| \text{ or } \frac{\Delta w_e}{w_e} \ll \gamma_e^2\beta_e^2|1-\frac{\beta}{\beta_e}|. \tag{5}$$

$$\beta = \frac{v}{c},\ \beta_e = \frac{v_e}{c},\ \gamma_e = (1-\beta_e^2)^{-1/2},\ w_e = mc^2(\gamma_e - 1).$$

This condition is not severe for present-day REBs (see, e.g., /7/). It should be noted that the conventional condition related to resonance distuning and a real value of attenuation in the structure can turn out to be stronger than (5) for an opposite wave ($\beta < 0$).

3) B e a m m a g n e t i z a t i o n, i.e., the one-dimensional motion of electrons in the longitudinal direction along the beam (further the z-axis). The corresponding condition is written in the form

$$\left(\frac{eH}{mc}\right)^2 \gg \omega_e^2, \tag{6}$$

H is the longitudinal magnetic field strength. This condition also "separates" strongly enough the frequencies of the beam cyclotron and Langmuire modes. Their excitation can be considered independently.

Let us introduce an eikonal (phase)

$$\psi = -\omega t + kz, \tag{7}$$

and the potential function of the field $\phi(r, \psi)$ related to the longitudinal electric field by

$$E = E_z = \text{Const}\frac{\partial\phi}{\partial\psi}. \tag{8}$$

Solving the hydrodynamic equation of electron motion

$$m\gamma_e^3\left(\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial z}\right) = eE \tag{9}$$

and choosing const in (8)

$$\text{Const} = -\frac{mc^2k}{e\gamma}, \tag{10}$$

we come to an important integral of motion

$$\gamma_e' + \phi = \text{Const} = \gamma_0'. \tag{11}$$

The relativistic factor of longitudinal electron motion in the wave rest system (WRS), i.e., in the reference frame, which is

*Interesting only for purposes of this paper.

moving with wave phase velocity v, is denoted by prime here. From relativistic energy-momentum transformations it is not difficult to obtain

$$\gamma_e' = \gamma\gamma_e(1-\vec{\beta}\vec{\beta}_e), \quad \gamma_o' = \gamma\gamma_o(1-\vec{\beta}\vec{\beta}_o). \tag{12}$$

For parallel waves $\vec{\beta}\vec{\beta}_e = +\beta\beta_e$ and for an opposite one $\vec{\beta}\vec{\beta}_e = -|\beta|\beta_e$. As β_e is chosen to be > 0, in writing without vectors we shall assume later on that $\beta > 0$ for parallel waves and $\beta < 0$ for an opposite one.

The stop of electrons in the WRS ($\gamma_e' = 1$) corresponds to the capture potential of electrons by the wave field

$$\phi_s = \gamma_o' - 1. \tag{13}$$

The function $\phi(r, \psi)$ is related to Φ and $A = A_z$, the scalar and vector potentials as follows:

$$\phi = \frac{e\gamma}{mc^2}(\Phi - \beta A). \tag{14}$$

From Dalamber's equations one can get a basic equation of evolution for $\phi(r, \psi)$

$$\frac{\partial^2\phi}{\partial\psi^2} + \frac{1}{\kappa^2}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) = q - q\beta_o' \frac{\gamma_o' - \phi}{\sqrt{(\gamma_o' - \phi)^2 - 1}} + \phi_{ext}. \tag{15}$$

Here

$$\kappa^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}, \quad q = \frac{\omega_e^2}{\omega^2}\beta^2\gamma^2\gamma_o', \quad \phi_{ext} = \frac{4\pi e\beta^3\gamma^3}{mc\omega^2} j_{ext},$$

with j_{ext} the current density of a source creating an external (initial) wave: $E = E_b(r)\exp i\psi$.

Attention should be given to the second nonlinear term on the right of (15) ($\sim 1/\beta_e'$); it increases going to ∞ at the capture point with increasing the field and approaching the capture potential (13). Fundamental nonlinear effects in the RDI process are associated just with this term. In particular, the nonsymmetry of oscillations $\phi(\psi)$ at $|\phi|$ tending to $\gamma_o' - 1$ is clear at once.

The second characteristic property of eq.(15) is the presence of stimulating force ϕ_{ext} on its right which is proportional to the sinusoidal in t and z current of an external source.

This term leads to the appearance of a resonance of the (4) type as long as the influence of the nonlinear term is negligible.

After these quantitative remarks, let us turn to stringent results.

2. Linear Approximation

A linear stage of resonance Doppler interaction can be realized at the beginning of the RDI process when the field is relatively small. The following condition can serve as the criterion of linearity:

$$\phi \ll \gamma_o' - 1, \tag{16}$$

i.e., the field is much smaller than that of the wave when electrons capture occurs.

Expanding the nonlinear term on the right-hand side of (15) in a power series of $\phi/\gamma_o' - 1$ and restricting ourselves to the linear term, we can obtain the following linear equation for a resonance field harmonic (its index is not written down below) under sufficiently general assumptions of the form of exciting term ϕ_{ext}:

$$\frac{\partial^2\phi}{\partial\psi^2} + \Omega^2\phi = \frac{\omega_e^2}{\gamma_o^2(\omega - kv_o)^2}|\phi_{ext}|\,\mathrm{Sin}\,\psi. \tag{17}$$

Here

$$\Omega^2 = \frac{\omega_e^2}{\gamma_o^2(\omega - kv_o)^2} \mp \frac{k_\perp^2}{\kappa^2}, \quad \kappa^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}.$$

with $|\phi_{ext}|$ the amplitude of an initial field harmonic ϕ (external wave field). Obtaining (17), the radial field dependence was assumed to be proportional to the Bessel functions $J_o(k_\perp r)$ or $I_o(k_\perp r)$.

The resonance means that $\Omega^2 = 1$ or

$$\frac{\omega_e^2}{\gamma_o^2(\omega - kv_o)^2} = 1 \pm \frac{k_\perp^2}{\kappa^2} \equiv S^2. \tag{18}$$

This is just generalization of previously written condition (4).

If (18) holds, the solution of (17) can be written as $(|\phi|_{t=0} = \phi_{ext},\ \phi'|_{t=0} = 0)$:

$$\phi = |\phi_{ext}| \operatorname{Cos}\psi - \frac{|\phi_{ext}| S^2}{2} (\psi \operatorname{Cos}\psi - \operatorname{Sin}\psi). \qquad (19)$$

Thus, the field increases linearly with the time or distance of interaction as usual for the integral resonance. Let us introduce an "increment"

$$\Gamma = \frac{1}{|E_{ext}|} \frac{\partial E}{\partial z}, \qquad (20)$$

where $|E_{ext}|$ is the amplitude of the initial electric field of the wave. It is not difficult to calculate Γ:

$$\Gamma = \frac{kS^2}{2}. \qquad (21)$$

The following important circumstance should be stressed: the field always increases in the RDI process in the direction of wave propagation, i.e., for example, in the case of an opposite wave the field increases against the motion of beam electrons.

As is seen from the solution of (19), in a linear approximation field oscillations are symmetric about zero and the velocity of increasing the field can be very significant: by a factor of several times along the wave length.

3. Nonlinear Processes

As is seen from basic evolution equation (15), with increasing the field ϕ, the influence of the "singular" nonlinear term on the right begins to increase, and it can become predominant over the "external force" and the resonance effect. In the general case it is difficult to obtain analytical results for arbitrarily large nonlinearity; only numerical calculations can be carried out here. However, they make sense only far enough from the capture of electrons as the above hydrodynamic model itself is no longer applicable at $|\phi| \to (\gamma_0' - 1)$; the beam-wave system transforms to a kinetic and multiflux stage with an intensive generation of spurious harmonics, a sharp decrease of the increment of RDI development, a substantial change of velocities V and V_e and so on. They are precisely the effects that have been observed in the experiments [6,9,12]. Therefore, we lay down the following main condition (limitation) which is of importance for the physics of RDI development and for its analysis

$$|\phi_{max}| \le 0{,}5(\gamma_0' - 1). \qquad (22)$$

Under experimental conditions, limitation (22) can be satisfied by choosing the values of some parameters (see below).

In the mathematical analysis a condition of the (22) form allows one to use the method of weak nonlinearity, in particular the Krylov - Bogolubov asymptotic method.

Nonlinear terms, in particular in eq.(15), are therewith expanded into powers of the small parameter $\phi/\gamma_0'-1$; the determination of wave shape turns out to be possible taking into account nonlinearity, the change in field increase velocity, the presence of a quasi-stationary nonlinear regime and its characteristics.

The most important results are the following [16,17]:

As the field ϕ increases, the increment decreases and the quasi-stationary nonlinear regime is reached even before the capture of electrons by the wave. This regime is characterized by slow oscillations of the field wave envelope (in comparison with the wave period) nearly a maximum value of $|\phi| = a_{st}$:

$$\phi = -a_{st} \operatorname{Sin}\psi - c_2 \frac{a_{st}^2}{2} \operatorname{Cos} 2\psi + c_3 \frac{a_{st}^3}{32} \operatorname{Sin} 3\psi - c_2 \frac{a_{st}^2}{2},$$

$$a_{st} \simeq (\gamma_0'^2 - 1) \sqrt[3]{\frac{8}{3} \cdot \frac{|\phi_{ext}|}{1 + 4\gamma_0'^2}}, \qquad (23)$$

$$\omega_{env}^2 = \omega^2 \left(\frac{3}{4} c_3 a_{st} + |\phi_{ext}| \frac{S^2}{2a_{st}^2}\right) \ll \omega^2,$$

c_2, c_3 are the coefficients of expansion of the right part of (15) at terms ϕ^2 and ϕ^3 proportional to the beam current.

These approximate analytical results are completely confirmed by numerical calculations of evolution $\phi(\psi)$ according to basic equation (15) only with insignificant quantitative specifications (see results and diagrams for a slow wave in [14], for fast and opposite ones in [15,17]).

Frome the results of (23) the following conclusions can be drawn: with increasing the field there arise additional harmonics which are small under the quasi-stationary nonlinear regime ($c_2\phi^2$, $c_3\phi^3 \sim \epsilon \ll 1$), the shape of the field wave envelope becomes nonsymmetric ($\phi \neq 0$, $\phi^+ < |\phi^-|$), the maximum amplitude of oscillations is determined by γ_0' and $|\phi_{ext}|$ (initial amplitude of the wave field), the period of oscillations is much larger than that of the wave (slow oscillations of the envelope in the quasi-stationary nonlinear regime).

Some other results are discussed in [14,15,18]: decrease of the increment, change of the wave velocity, increase of the modulation of electron beam density (at the stage of (22) it reaches tens of percent and exceeds $\Delta n_e/n_0$ in the linear case by an order of magnitude). These characteristics are identical for different types of waves; the main features of the RDI process are practically the same for parallel and opposite waves taking into account nonlinear effects.

Of principal interest is

4. RDI Energetics, i.e. the character of energy exchange between transit relativistic electrons and E-waves including those with large amplitude, change of the kinetic energy of electrons and RDI energy balance taking into account the work of the source of an external (initial) electromagnetic field.

Let us calculate the work of the field above the beam:

$$\mathcal{P} = \int \vec{j}\vec{E}\,dv = e\int n_e v_e E\,dv. \tag{24}$$

This quantity determined the change of the total kinetic energy of electrons in the volume V per time unit. Using a number of transformations, taking into consideration (8), (10) and the integral of the continuity equation

$$n_e(\beta_e - \beta) = n_0(\beta_0 - \beta) \tag{25}$$

the quantity $\mathcal{P}$ can be presented as

$$\mathcal{P} = \pm\, mc^2 n_0 \pi a^2 \gamma^3 (v_0 - v) \int_{\phi_{ext}}^{\phi} \frac{(\beta_e' + \beta)^2}{\beta_e'^2}\, d\phi. \tag{26}$$

Here a is the beam radius; and $\beta_e' c$, the velocity of electrons in the WRS:

$$\beta_e' = \frac{\beta_e - \beta}{1 - \beta\beta_e}. \tag{27}$$

In (26) (+) is for an opposite wave; and (-), for parallel (fast and slow) waves.

Assuming that approximation (22) is valid, let us perform expansion accurate to ϕ^2 inclusive ($\phi_{ext} \ll \phi$)

$$\mathcal{P} = \pi a^2 n_0 mc^2 \left[\pm \frac{\beta_0 v_0}{\gamma(\beta_0 - \beta)}\phi \pm \frac{\beta v_0}{\gamma^2\gamma_0^3(\beta_0 - \beta)^3}\phi^2\right]. \tag{28}$$

At the linear stage, when $\phi(\psi)$ is a function of the form (19), i.e., $\phi = \phi_{ext}\,\mathrm{Cos}\,\psi - \frac{\phi_{ext}S^2}{2}(\psi\,\mathrm{Cos}\,\psi - \mathrm{Sin}\,\psi)$, the second term on the right of (28) makes a contribution to $\bar{\mathcal{P}}$ as $\bar{\phi} = 0$. In this case we get the partially known result: the beam is decelerated in slow and opposite waves ($\bar{\mathcal{P}} < 0$) transmitting its kinetic energy to the wave, and it is accelerated in a fast wave ($\bar{\mathcal{P}} > 0$). It is not difficult to show that a slow wave therewith possesses a negative energy ($W < 0$ is the density of the sum of the energy of electron oscillations and the energy of the wave electric field, see [19]), and the fast and opposite waves have a positive energy ($W > 0$).

As the field ϕ increases the situation changes: due to the nonlinear nonsymmetry of the wave envelope, the first term on the right of eq.(28) begins to contribute as we have now (see (23)):

$$\bar{\phi} = -\frac{c_2 a_{st}^2}{2}, \quad c_2 = \frac{3S^2}{2}\cdot\frac{1 - \beta\beta_0}{\gamma\gamma_0(\beta - \beta_0)^2}. \tag{29}$$

The analysis and numerical calculations show that for some field $|\phi| < a_{st}$ the beam in a slow wave transits from the stage of deceleration to that of acceleration, the total energy of electron oscillations and the wave field changes its sign, i.e. it becomes positive.

For a fast parallel wave the effect of changing the sign (transition from beam acceleration to its deceleration) is possible in a narrower interval of parameters.

In the most interesting case of an opposite wave the sign is not changed, the nonlinear effect of nonsymmetry strengthens the effect of beam deceleration ($\bar{\mathcal{P}} < 0$) only significantly.

Thus, we get an important result: the character of longitudinal waves in an electron beam, their energy sign and the change of the kinetic energy of electrons depend on the level of an excited field; standard assertions of the energy sign of parallel waves are not valid at a nonlinear level.

It is quite probable that this result takes place also for other types of dispersing media (some examples see in [20]).

The estimates show that energy transfer from beam to wave can be significant in a very large field of an opposite wave

($\sim 10^6$ V/cm) and a beam current of >10 kA; the corresponding power reaches 10^3 MW and more. In this case the relativistic factor of electrons changes relatively insufficiently ($\Delta\gamma_e/\gamma_o \ll \ll 1$); a pronounced effect of energy exchange is related to a large density of electrons in the beam ($n_e \simeq 10^{13}$ cm^{-3}).

5. Limiting Fields in the RDI Process

A standard estimate of a maximum electric field in beam-wave versions of the collective methods (e.g., /7,10/) follows from formulae of the type (8), (16) and is practically only the condition of applicability of a linear approximation

$$|E| \ll \frac{mc^2 k}{e\gamma}(\gamma_o' - 1). \qquad (30)$$

The results of the nonlinear theory and numerical calculations allow us to estimate more precisely a maximum electric field using condition (22)

$$|E| \lesssim \frac{mc^2 k}{2e\gamma}(\gamma_o' - 1) = \frac{\pi mc^2}{e\lambda\gamma}(\gamma_o' - 1). \qquad (31)$$

From here it is seen that the limiting values of the field are in large part determined by γ_o' (this is an initial value of the relativistic factor of electrons in WRS for the RDI process).

According to (12), for parallel waves (fast and slow) γ_o' is equal to

$$\gamma_o' = \gamma\gamma_o(1 - \beta\beta_o), \quad \beta > 0. \qquad (32)$$

It is not difficult to make sure that in this case γ_o' is not too large even for relativistic velocities β, β_o:

$$\gamma_o' \simeq \frac{\gamma^2 + \gamma_o^2}{2\gamma\gamma_o}.$$

and, for example, for a fast wave at $\gamma^2 \gg \gamma_o^2$

$$\gamma_o' \simeq \frac{\gamma}{2\gamma_o}.$$

The situation is different for an opposite motion of the beam and the wave ($\beta < 0$)

$$\gamma_o' = \gamma\gamma_o(1 + |\beta|\beta_o) \simeq 2\gamma_o\gamma. \qquad (33)$$

This value can be by an order of magnitude (and more) greater than the value of γ_o' for parallel waves.

Now

$$|E_{max}| \le \frac{2\pi mc^2}{c\lambda}\gamma_o. \qquad (34)$$

For centimeter waves and $\gamma_o \simeq 10$, E_{max} can reach 10^7 V/cm while for parallel waves $E_{max} \simeq 10^5 - 10^6$ V/cm in accord with the experiment /9,12/ .

In order to reach a field of $\sim 10^7$ V/cm, from Maxwell's equations and resonance condition (18) it is not difficult to show that a beam current of > 10 kA, or otherwise approximately 10% of density modulation $n_e \simeq 10^{13}$ cm^{-3}, is required.

From comparison of estimates (1) and (34) one can conclude that potential possibilities of a dense ensemble of electrons are in principle realized completely enough by RDI on an opposite wave for the generation of powerful electric fields.

IV. CONCLUSION: A POSSIBLE APPLICATION OF THE RDI PRINCIPLE TO THE SOLUTION OF VEKSLER'S PROBLEM

As analytical and numerical calculations show, resonance conditions can be fulfilled in the interaction of a high-current relativistic beam with a longitudinal wave in the slowing-down structure at $v_e \neq v$, and a variable electric field in the system increases fastly beginning from some initial value given by an external microwave frequency generator. A maximum amplitude of the E-field is determined at the stage of the nonlinear quasi-stationary regime (before the capture of electrons by a wave) and can reach very large values at the corresponding choice of initial conditions (ϕ_{ext}, γ_o).

In this case two circumstances are of fundamental significance:

1) The RDI process under condition (22) has a regular and controlled character with increasing the field for a short time $T_{env}/2 \simeq 10 \div 100$ nsec and thus can play a dominating role in a real picture of waves excitation in the structure with a beam.

2) An electron beam in this mechanism has several substantial functions: it amplifies an initial E-wave due to arising density modulation produced by transit electrons, transmits part of its enormous kinetic energy to excite an opposite wave (and a slow one at the linear stage) in a strong field due to

nonlinear effects thus unloading the source of an external (initial) wave and redistributes the field in the transverse plane; in particular the beam forms a deep potential well for positive charges in their motion along the beam thus providing their effective focusing.

These conclusions are reasons to assume that the resonance Doppler interaction can serve as a physical foundation for the solution of Veksler's problem.

When the RDI principle is being realized, there arise numerous experimental and technical problems which solution, in the author's opinion, is not beyond the level achieved my modern high-current beam technique, accelerating and SHF technique.

Some moments should be pointed out which are characteristic of the version of collective acceleration considered in this paper.

1. The main task which is common for all varieties of beam collective acceleration methods lies in the formation of a high-current relativistic electron beam (J = 1 - 10 kA and more, w_e = 1 - 10 MeV) with a small energy spread. There are many experimental results on the formation and passage of such beams thought the structure along the length L (see, e.g.,[21]).

The transportation of an electron beam with a high current along L > 100 m is not so clear to date if the achievement of superhigh energies of accelerated ions is kept in mind.

2. As the wave should have a phase velocity variable in length to accelerate ions at least at the initial stage, the structure geometry, in which the E-wave is excited and amplified, must be changeable. For example, the thickness of an internal dielectric coat of the wave-guide or the diaphragm geometry can be changed. Acceleration at the final stage is possible with the "asymptotic phase" when the phase velocity of the wave is practically constant ($V \lesssim c$) and the amplitude of the E-field is large.

3. The source of the external (initial) field should have a power of approximately 100-500 kW at a wave length λ=3-10 cm and more. Present-day generators, SHF-magnetrons, klystrons and gyrotrons satisfy these requirements.

4. When fields of the order of 10^6-10^7 V/cm are excited, the problem of electrical breakdown in proximity to an internal surface of the waveguide structure can arise. In connection with this, we should like to note important experimental results on eliminating of the breakdown due to a special technology of surface treatment[22] and also the results of discussion of this problem in Frascati[12], in particular a growth in the breakdown field with increasing frequency which allows one to count on a significant facilitation of solving this problem at λ = 3 cm (f = 10GHz).

5. From the viewpoint of accelerating applications, two stages of experimental and technical investigations should be separated.

1) Low and medium energy accelerators (to 1 GeV) with a high ion current ($J_i \lesssim 1$ A) on slow and, may be, fast waves at L = = 1-10 m, a beam current of ~1 kA and an acceleration rate, say, of protons g = 10-100 MeV/m. At this stage all basic technical characteristic properties of using the RDI process should be worked out and the results which can allow one to go to the second stage should be obtained.

2) The stage of a step-by-step growth of the length L when passing to a section with an opposite wave at large fields and g ≃ 1 GeV/m. The main problems here are the transportation of a beam at L > 100 m in the field H ≥ 10 kOe, the protection from breakdown and acceleration working out near the asymptotic phase.

Note that the transverse size of an ion beam decreases due to an adiabatic decrease of transverse oscillations with increasing the energy of ions when they are moving in a deep Coulomb well of the beam; this effect can be of interest for the formation of colliding beams in future.

In conclusion it should be noted that at the present time it is impossible to confirm with full certainty that the above considerations are the only or a more true way to the solution of Veksler's problem. Only comprehensive experimental investigations can corroborate the validity of this or close conception or to indicate its weak aspects.

REFERENCES

1. Veksler V.I. - Proc.CERN Symp., 1956, v.1, p.80-83.

2a. Veksler V.I. et al. - Proc. VI Intern. Conf. on High En. Accel., Cambridge, 1967, p.289-292.

2b. Sarantsev V.P., Perel'shtein E.A. - Collective Acceleration of Ions by Electron Rings, Atomizdat, Moscou, 1979.

3. Fainberg Ya.B. - Proc. CERN Symp., 1956, v.1, p.84-90.

4. Fainberg Ya.B. - Sov.Phys.Plasma, 1977, v.3, p.442-449.

5. Rabinovich M.S. - Proc. All-Union Accel.Conf.,Moscow,VINITT 1970, p.473.

6. Keefe D. - Part.Accel., 1981, No.4.

7. Fainberg Ya.B. - Sov.Plasma Phys., 1985, v.11, p.1398.
8. Sloan M., Drummond W. - Phys. Rev. Lett., 1973, v.31, p.1234.
9. Cornet E. et al. - Phys. Rev. Lett., 1981, v.46, p.181.
10. Sprangle P. et al. - Phys. Rev. Lett., 1976, v.36, p.1180
11. Gammel G., Nation J.A., Read M.E. - J. Appl., Phys., 1979 v.50, p.5603.
12. Nation J.A. - In: The Generation of High Fields for Particle Acceleration to Very High Energies, Proc. of Gas-ECFA-JNFN Work-shop, Frascati, 1984; CERN 85-07, 1985, p.223.
13. Bonch-Osmolovsky A.G., Dolya S.N., Reshetnikova K.A. - JINR Preprint P9-82-113, Dubna, 1982; Journ. Techn. Phys. 1983, v.53, No.6, p.1055.
14. Bonch-Osmolovsky A.G., Reshetnikova K.A. - Jour. Techn. Phys., 1986, v.56, Issue 9, p.1666; JINR Preprint P9-84-270, Dubna, 1984.
15. Bonch-Osmolovsky A.G., Reshetnikova K.A. - Proc. XIII Int Conf. on High En. Accel., Nauka, Novosibirsk, 1987, p.190 JINR Preprint P9-84-270, Dubna, 1984.
16. Kuzelev M.V.,Ruchadze A.A.- Phys. Sci. Progr. (UFN), 1987 v.152, No.2, p.285.
17. Bonch-Osmolovsky A.G. - JINR Preprint P2-89-83, Dubna, 1989; Journ. Techn.Phys., 1992, v.62, No.2.
18. Bonch-Osmolovsky A.G., Reshetnikova K.A. - JINR Rapid Commun., No.1(47)-91, p.35, Dubna, 1991; Journ. Techn. Phys., No.9, 1991.
19. Nezlin M.V. - Phys. Sci. Progr. (UFN), 1976, v.120, No.3, p.481.
20. Kadomtsev B.B., Mikhailovsky A.B., Timofeev A.V. - J. Exp Theor. Phys., 1964, v.47, p.2265.
21. Miller R.B. - An Introduction to the Physics of Intense Charged Particle Beams, Plenum Press., New York, 1982.
22. Balakin V.E. et al. - Proc IX All-Union Conf. on Accel., JINR, Dubna, 1985, v.II, p.401.

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