объединенный
институт
ядерных
исследований
дубна

E9-92-26

D.I.Kaltchev, E.A.Perelstein

# COHERENT CORRECTIONS TO THE SYNCHROTRON RADIATION SPECTRUM OF A RELATIVISTIC ELECTRON BEAM

Submitted to "Journal of Physics of Fluids B,
Plasma Physics"

1992

Калчев Д.И., Перельштейн Э.А. E9-92-26

Когерентные поправки к спектру синхротронного излучения релятивистского электронного пучка

Исследованы когерентные поправки к спектру излучения релятивистского продольно-замагниченного электронного пучка. В системе отсчета пучка вычисляется циклотронное излучение горячей магнетоплазмы при учёте электрон-электронных корреляций. Спектр флуктуаций плотности тока находится при помощи флуктуативно-диссипативного соотношения, записанного для случая равновесной плазмы. На гармониках гирочастоты больше второй коллективное искажение флуктуационного спектра уменьшает интенсивность излучения по сравнению со спонтанной. Когерентные эффекты проявляются на длинах волн, сравнимых с радиусом дебаевского экранирования. В лабораторной системе для типичных экспериментальных значений параметров они становятся значимыми, когда плотность электронов превышает $10^{13}$ см$^{-3}$.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

---

KaltchevD.I., Perelstein E.A. E9-92-26

Coherent Corrections to the Synchrotron Radiation Spectrum of a Relativistic Electron Beam

The coherent corrections to the radiation spectrum of a relativistic electron beam, propagating along an external magnetic field are investigated. In the beam reference frame the cyclotron radiation from a hot magnetoplasma is calculated taking into account the electron correlations. The fluctuation-dissipation theorem for the case of an equilibrium plasma is used to find the spectrum of current-density fluctuations. At harmonics of the cyclotron frequency $\geqslant 2$, the collective distortion of this spectrum decreases the radiation intensity compared to the spontaneous one. The coherent effects manifest themselves at wavelengths of the order of the Debye shielding length and become important when the electronic density in the laboratory frame is higher than $10^{13}$ cm$^{-3}$, for typical experimental beam parameters.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992

## I. Introduction

The adequacy of the assumption that the individual particles radiate independently, in relation to the problem of synchrotron radiation, was investigated initially by Pomerantchuck[1]. In doing this, a heuristic method was used - comparison of the maximal fluctuation potentials inside the beam with the energy spread. In later studies of radiation losses from a thermonuclear reactor[2-7] the correlation effects have been essentially neglected due to the fact, that the radiation wavelength is much shorter than the correlation radius (the Debye length). However, the possible importance of these effects was suggested[3,4] for high densities and at low harmonics of the cyclotron frequency. In the present paper we consider successively the influence of the two-particle correlations on the cyclotron radiation spectrum of an equilibrium hot plasma. We also use a transformation to a reference system, in which the plasma moves along the external magnetic field, to calculate the radiation of a relativistic electron beam.

The calculations will be done in the beam reference frame, where we assume the plasma to be limited, with dimensions much bigger than the radiated wavelength and in a state of equilibrium - having a relativistic electronic temperature T (in units of energy). The dielectric permitivity tensor for such plasma can be found using the well known Trubnikov's method[6,7].

According to the theory of fluctuations, the fields radiated at frequency $\omega$ can be described as a result of the action of some stohastic currents $\vec{j}_\omega$ distributed inside the beam[8]. At low densities the current fluctuations in different spatial points and times are statistically independent and the radiated intensity is proportional to the number of electrons. Intraparticle interactions generate collective fields, that introduce correlations between the electrons[6,8-11]. These correlations deform the current correlation spectrum and cause nonlocality of the corresponding spatial correlation functions, i.e. the latter have a nonzero correlation radius. One can expect, that in the beam reference frame deviations from the spontaneous radiation exist at wavelengths, comparable to this radius and the radiation is not materially affected at shorter wavelengths.

The spectrum of the interacting current-density fluctuations can be expressed in terms of the dielectric permitivity plasma tensor using the fluctuation-dissipation relation[12].

In the plasmas of interest the density parameter $q \equiv \omega_p^2/\omega_0^2$ is usually less than 1. Here the usual meaning of symbols is used in the beam reference frame: $\omega_p=(4\pi N_e e^2/m_0)^{1/2}$ - plasma frequency; $\omega_0=eB_0/m_0c$ - electron cyclotron frequency; $N_e$-number of electrons/volume; $B_0$-static magnetic field; $e, m_0$- electronic charge and mass, respectively. The emission coefficient of a noninteracting plasma, $\eta_\omega^0(\theta)$ (the energy/sec, radiated per unit solid angle, per unit frequency and per unit volume) is defined as the energy radiated by a test charge averaged over the momentum distribution of test charges. For the considered case of an equilibrium Maxwellian distribution an equivalent procedure may be used[2,7]. It consists first in calculation of the absorption coefficients of two eigenmodes for propagation at the same angle $\theta$, and



second, in use of Kirchoff's formula:

$$\eta_\omega^0 = I_{RJ}(\omega/c)\Big(n''_+(\omega,\theta)+n''_-(\omega,\theta)\Big), \qquad (1)$$

where $I_{RJ}=\omega^2 T/8\pi^3c^2$ is the Rayleigh-Jeans distribution; $\omega$ is the radiated frequency; $\theta$ is the angle of the emission relative to the dc magnetic field; $n''_\pm=\mathrm{Im}(n_\pm)$- absorption coefficients; $n_\pm$-the refraction indexes, obtained from the dispersion relation. It can be proved[2,6], that if $n''_\pm$ are calculated to an accuracy of first order in $q$, then Eq. (1) describes the radiation of noninteracting electrons. It is well known, however[5], that Kirchoff's formula is valid always when the refractive properties of the medium are neglected and one could expect, that higher order in $q$ terms in Eq. (1) describe the correlation effects. We obtain exactly this result in Sec.II, using the free-space field propagators and the interacting current fluctuation spectrum. We find that the terms of second order in $q$ in $n''_\pm$ give the coherent corrections to the spontaneous emission.

We do not consider the effects of refraction and absorption of the radiation, due to its propagation trough the beam plasma, that are well studied in Ref. 6. That is why in Sec.III we compare the independent actions of the correlation effects and the absorption on the radiated spectrum for various electronic temperatures. In doing this we roughly separate these two effects, which in the real case should be considered together. We conclude that the absorption dominates at frequencies close to the first harmonic ($\omega \approx \omega_0$) except in the case of very low temperatures.

The transparency of the plasma in combination with the condition of macroscopic dimensions with respect to the radiated wavelength, leads to the following restriction on the maximal absorption coefficient under consideration:

$$2\pi n''_{+}(\theta) \ll \lambda/L(\theta) \ll 1 \; ; \tag{2}$$

Here $\lambda = 2\pi c/\omega$; $L(\theta)$ is the propagation length inside the plasma.

## II. Correlation corrections to the radiation spectrum

In the beam reference frame we evaluate the radiation intensity of a region having volume V and dimensions much greater than $\lambda$. We use a rectangular coordinate system $\vec{x}_1, \vec{x}_2, \vec{x}_3$, centered in the middle of the region V; the $\vec{x}_3$-axis is taken to coincide with the propagation vector, $\vec{e}$, and the magnetic field vector $\vec{B}_0$ lies in the $\vec{x}_1$-$\vec{x}_3$ plane: $\vec{B}_0 = (B_0 \sin(\theta), 0, B_0 \cos(\theta))$.

Consider a current density fluctuation $\vec{j}(\vec{r}', t')$ at time $t'$ and in a point $\vec{r}'$ within the region V. We assume that the corresponding Fourier component $\vec{j}_\omega(\vec{r}')$ and the field observed in a point $\vec{R}$, $\vec{E}_\omega(\vec{R})$, obey the Maxwell equations for the free space. The total field radiated from the sources $\vec{j}_\omega(\vec{r}')$ at large distances R is[6]

$$\vec{E}_\omega(\vec{R}) = \int_V d^3r' \; \hat{W}(\vec{R}, \vec{r}') . \vec{j}_\omega(\vec{r}') \; , \tag{3a}$$

where $\quad \vec{j}_\omega(\vec{r}') = \int_{-t_1}^{t_1} dt' e^{i\omega t'} \vec{j}(\vec{r}', t')$

and similar for $\vec{E}_\omega$ (here $t_1$ tends to infinity). The vacuum propagator is taken above to be

$$\hat{W}(\vec{R}, \vec{r}') = \frac{i\omega}{c^2 \rho'} \; \hat{I} \; e^{i \frac{\omega}{c} \rho'} \simeq \frac{i\omega}{c^2 R} \; \hat{I} \; e^{i \frac{\omega}{c} (R - \vec{e}.\vec{r}')} \; , \tag{3b}$$



where $\hat{I} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; $\rho' \equiv |\vec{R} - \vec{r}'| \cong R - \vec{e}.\vec{r}'$.

Since we assume a homogeneous medium and stationary processes the cross-correlation function of current density fluctuations depends only on the differences $\vec{r} = \vec{r}' - \vec{r}''$, $\tau = t' - t''$:

$$\langle \vec{j}(\vec{r}', t') \; \vec{j}(\vec{r}'', t'') \rangle \equiv (\vec{j}\vec{j})_{\vec{r}, \tau} \; ,$$

where the symbol $\langle \rangle$ means statistical average. The spectral densities of current fluctuations are given by

$$(\vec{j}\vec{j})_{\vec{r}, \omega} \equiv \int_{-\infty}^{+\infty} d\tau \; e^{i\omega\tau} (\vec{j}\vec{j})_{\vec{r}, \tau} = \frac{1}{2t_1} \langle \vec{j}_\omega(\vec{r}') \; \vec{j}^{*}_\omega(\vec{r}'') \rangle \; ; \tag{4a}$$

$$(\vec{j}\vec{j})_{\vec{k}, \omega} \equiv \int d^3r \; e^{-i\vec{k}.\vec{r}} (\vec{j}\vec{j})_{\vec{r}, \omega} \; . \tag{4b}$$

The power radiated in direction $\vec{e}$ per unit area and per unit frequency interval is[6]

$$S_\omega = \frac{c}{8\pi^2} \lim_{t_1 \to \infty} \frac{1}{2t_1} \int_{-t_1}^{t_1} dt \int_{-\infty}^{\infty} d\tau \; e^{i\omega\tau} \langle E(\vec{R}, t) \; E(\vec{R}, t+\tau) \rangle =$$

$$= \frac{c}{8\pi^2} \lim_{t_1 \to \infty} \frac{1}{2t_1} \langle |\vec{E}_\omega(\vec{R})|^2 \rangle .$$

Using (3) and (4) we express $S_\omega$ in terms of the current fluctuations spectrum:

$$S_\omega = \frac{c}{8\pi^2} \, Sp \int_V d^3r' \int_V d^3r'' \; \hat{W}(\vec{R}, \vec{r}') . (\vec{j}\vec{j})_{\vec{r}' - \vec{r}'', \omega} . \hat{W}^{+}(\vec{r}'', \vec{R}) =$$

$$= \frac{\omega^2}{8\pi^2 c^3 R^2} \, Sp \int_V d^3r' \int_V d^3r'' \; \exp\left(-i\frac{\omega}{c} \vec{e}.(\vec{r}' - \vec{r}'')\right) \hat{I} . (\vec{j}\vec{j})_{\vec{r}' - \vec{r}'', \omega} =$$

$$= \frac{\omega^2}{8\pi^2 c^3 R^2} \, Sp \int \frac{d^3k}{(2\pi)^3} \left| \int_V d^3r' \; \exp\left(i(\vec{k} - \frac{\omega}{c} \vec{e}).\vec{r}'\right) \right|^2 \hat{I} . (\vec{j}\vec{j})_{\vec{k}, \omega} =$$

$$= \frac{\omega^5}{8\pi^2 c^6 R^2} \, Sp \int \frac{d^3n}{(2\pi)^3} \; F(\vec{n}) \; \hat{I} . (\vec{j}\vec{j})_{\vec{n}, \omega} \; , \tag{5}$$

here $\vec{n}=\vec{k}c/\omega$, $\vec{e}=(0,0,1)$, $F(\vec{n})\equiv \left|\int_V d^3r\, exp\left(i\frac{\omega}{c}(\vec{n}-\vec{e}).\vec{r}\right)\right|^2$. (6)

We choose the geometry of the radiating region to be rectangular with dimensions $L_1, L_2, L_3$ and formally assume infinite transversal dimensions: $L_1/\lambda$, $L_2/\lambda\to\infty$. When substituting in (6)

$$\int_V d^3r=\int_{-L_1/2}^{L_1/2} dx_1 \int_{-L_2/2}^{L_2/2} dx_2 \int_{-L_3/2}^{L_3/2} dx_3$$

and using the δ-function definition:

$$\delta(x)=\lim_{A\to\infty} sin^2(Ax)/(\pi Ax^2)$$

then $F(\vec{n})$ is found to be

$$F(\vec{n})=(\frac{2c}{\omega})^4\pi^2L_1L_2\delta(n_1)\delta(n_2)\frac{sin^2\left[(n_3-1)\tilde{L}_3/2\right]}{(n_3-1)^2}. \quad (7)$$

We substitute (7) into (5) to obtain

$$S_\omega=\frac{\omega L_1L_2}{4\pi^3c^2R^2}\int_{-\infty}^{+\infty}dn\frac{sin^2((n-1)\tilde{L}_3/2)}{(n-1)^2}J^2_\omega(n), \quad (8)$$

where the spectral density of the transversal current fluctuations at a frequency ω is denoted by

$$J^2_\omega(n)\equiv Sp\, \hat{I}_2\, (\vec{j}\vec{j})_{n,\omega};\quad (\vec{j}\vec{j})_{n,\omega}\equiv(\vec{j}\vec{j})_{\vec{n}=n\vec{e},\omega}. \quad (9)$$

Now $J^2_\omega(n)$ should be expressed in terms of the dielectric permitivity tensor using the fluctuation dissipation theorem. For an equilibrium plasma the latter is given by[12]

$$(\vec{j}\vec{j})_{n,\omega}=\frac{i\omega T}{4\pi}\hat{\Lambda}^0.(\hat{\Lambda}^{-1}-\hat{\Lambda}^{-1^+}).\hat{\Lambda}^0. \quad (10)$$

For our choice of coordinate system the matrices $\hat{\Lambda}^0,\hat{\Lambda}$ are

$$\hat{\Lambda}=\hat{\Lambda}(n,\omega)=\hat{\Lambda}^0(n)+\hat{\kappa}(n,\omega),\quad \hat{\Lambda}^0=\hat{\Lambda}^0(n)=\begin{bmatrix}1-n^2 & 0 & 0\\ 0 & 1-n^2 & 0\\ 0 & 0 & 1\end{bmatrix}. \quad (11)$$

Using the symmetry properties of the electromagnetic susceptibility tensor: $\kappa_{13}=\kappa_{31}$, $\kappa_{21}=-\kappa_{12}$, $\kappa_{32}=-\kappa_{23}$, the determinant of $\hat{\Lambda}$ may be written in the form

$$D(n,\omega)\equiv det\,\hat{\Lambda}=(1+\kappa_{33})\left(n^2-1-f_+(n)\right)\left(n^2-1-f_-(n)\right), \quad (12)$$

where $$f_\pm(n)=1/2\left[\kappa_{11}+\kappa_{22}+\frac{\kappa^2_{23}-\kappa^2_{13}}{1+\kappa_{33}}\right]\pm$$
$$\pm 1/2\left\{\left[\kappa_{11}-\kappa_{22}+\frac{\kappa^2_{23}-\kappa^2_{13}}{1+\kappa_{33}}\right]^2-4\left[\kappa_{12}+\frac{\kappa_{13}\kappa_{23}}{1+\kappa_{33}}\right]^2\right\}^{1/2}. \quad (13)$$

Here and bellow $\kappa_{ij}=\kappa_{ij}(n,\omega)$. However, we shall omit the argument ω from now on. The following relation can be obtained using (11) and (13):

$$\lambda_{11}(n)+\lambda_{22}(n)=\left[1+\kappa_{33}(n)\right]\left[2-2n^2+f_+(n)+f_-(n)\right]; \quad (14)$$

where $\lambda_{ij}$ is the cofactor of $\Lambda_{ij}$. Now we use that $\Lambda^{-1}_{ij}=\lambda_{ij}/D$ and also (10), (12) and (14) to transform (9):

$$J^2_\omega(n)=\frac{i\omega T}{4\pi}\quad Sp\,\hat{I}_2.\hat{\Lambda}^0.(\hat{\Lambda}^{-1}-\hat{\Lambda}^{-1^+}).\hat{\Lambda}^0=$$
$$=\frac{i\omega T}{4\pi}(n^2-1)^2\left(\frac{\lambda_{11}(n)+\lambda_{22}(n)}{D(n)}-c.c.\right)=$$
$$=\frac{i\omega T}{4\pi}(n^2-1)^2\left(\frac{1}{n^2-1-f_+(n)}+\frac{1}{n^2-1-f_-(n)}-c.c.\right)=$$
$$=\frac{\omega T}{2\pi}(n^2-1)^2\left(\frac{Im\,f_+(n)}{|n^2-1-f_+(n)|^2}+\frac{Im\,f_-(n)}{|n^2-1-f_-(n)|^2}\right). \quad (15)$$

Since we have taken into account the dispersive properties of the medium (dependence on $n$ in (15)), the spatial correlation function $J^2_\omega(\vec{r})$ is not local as in the case of noninteracting electrons. The corresponding expression for noninteracting electrons is:

$$J^{2(0)}_\omega=\frac{\omega T}{2\pi}\left(\kappa_{11}(1)+\kappa_{22}(1)\right)\text{ and, thus } J^{2(0)}_\omega(\vec{r})\sim\delta(\vec{r}).$$

By substituting (15) into (8) one finds

$$S_\omega = \frac{\omega^2 T L_1 L_2}{8\pi^4 c^2 R^2} (S^+_\omega + S^-_\omega) , \qquad (16a)$$

where $$S^{\pm}_\omega = \int_{-\infty}^{+\infty} dn \frac{(n+1)^2 \sin^2\left[(n-1)\tilde{L}_3/2\right]}{|n^2-1-f_\pm(n)|^2} \; Im\, f_\pm(n) . \qquad (16b)$$

Since $f_\pm(n)$ are small ($\sim q$), we can simplify the calculations by taking approximately $n=1$ in the arguments. In the same approximation one has for the high-frequency solutions $n_\pm$ of the dispersion relation $D(n,\omega)=0$ :

$$n^2_\pm = 1+f_\pm(1) \qquad (17)$$

and therefore (16b) may be written in the form:

$$S^+_\omega = 1/2 \; Im f_+(1) \; Re \int_{-\infty}^{+\infty} dn \frac{1-\exp\left[i\tilde{L}_3(n-1)\right]}{(n^2-n^2_+)(n^2-\bar{n}^2_+)} (n+1)^2$$

and similar for $S^-_\omega$. We solve the integral by closing the contour with an infinite semicircle in the upper half of the complex $n$-plane and calculating the residues in the poles, having a positive imaginary part: $n_+ = n'_+ + in''_+$, $-\bar{n}_+ = -n'_+ + in''_+$ ($n''_+ > 0$). The result is

$$S^+_\omega = \frac{\pi}{8n'_+ n''_+} Im\, f_+(1) \; Re \left\{ \left[1-\exp\left(i\tilde{L}_3(n_+-1)\right)\right] \frac{(n_+ +1)^2}{n_+} + \right.$$
$$\left. + \left[1-\exp\left(i\tilde{L}_3(-\bar{n}_+-1)\right)\right] \frac{(-\bar{n}_+ +1)^2}{\bar{n}_+} \right\} .$$

We neglect the small second term in the braces, which corresponds to backward propagating waves. After substituting into the first term $n'_+ = 1+a'_1 q + a'_2 q^2$; $n''_+ = a''_2 q + a''_2 q^2$ ($a'{}''_{1,2}$ do not depend on $q$), and expanding the exponent in a series we get

$$S^+_\omega = \frac{\pi\tilde{L}_3}{2n'_+} \; Im\, f_+(1) \; (1+O(q^2))$$

and, since from (17) it follows that $Im\, f_+(1) = Im\,(n^2_+) = 2n'_+ n''_+$ ,



we have

$$S^+_\omega = \pi\tilde{L}_3 n''_+ \; (1+O(q^2)). \qquad (18)$$

Finally by substituting (18) and a similar expression for $S^-_\omega$ into (16a) we get

$$S_\omega = \frac{\omega V}{cR^2} I_{RJ} \; (n''_+ + n''_-);$$

thus the emission coefficient of an interacting plasma is

$$\eta_\omega = R^2 V^{-1} S_\omega = \frac{\omega}{c} I_{RJ} \; (n''_+ + n''_-). \qquad (19)$$

Here $n''_\pm = Im(n_\pm)$ are calculated by (17) and (13) keeping the second-order terms in $q$. One can easily obtain the spontaneous emission coefficient from here, by neglecting these terms in (13) and also taking $n'_\pm = 1$. The result is

$$\eta^0_\omega = \frac{\omega}{2c} I_{RJ} Im \left[\kappa_{11}(1)+\kappa_{22}(1)\right] = \qquad (20)$$
$$= \int d^3p \; f_0(\vec{p}) \; I^0_\omega(\vec{p}).$$

Here $f_0(\vec{p})$ is the momentum distribution function and $I^0_\omega(\vec{p})$ is the spectral density of the radiation of a single electron with momentum $\vec{p}$ in an external magnetic field, given by the Shott's formula.

The number of photons/sec radiated from unit volume, in unit solid angle and in unit frequency interval, calculated with and without interactions, are

$$N_\omega = \frac{\eta_\omega}{\hbar\omega} = \frac{T\omega^2}{8\pi^3 \hbar c^3} (n''_+ + n''_-) \; , \quad N^0_\omega = \frac{\eta^0_\omega}{\hbar\omega} = \frac{T\omega^2}{16\pi^3 \hbar c^3} \; Im\left[\kappa_{11}(1)+\kappa_{22}(1)\right]$$

respectively.

## III. Results obtained in the beam reference frame

We wish to calculate the correlation factor $\alpha$ defined as $\alpha \equiv 1-\eta_\omega/\eta^0_\omega = 1-N_\omega/N^0_\omega$ , where the emission coefficients $\eta^0_\omega(\theta)$ and

$\eta_\omega(\theta)$ are given by Equations (19) and (20) respectively. In these expressions the elements $\kappa_{ij}$ are taken for $n=1$, thus they are functions of the angle $\theta$, the harmonic number $m=\omega/\omega_0$, the temperature parameter $\mu=m_0c^2/T$ and the density parameter $q$. We used a variation of Trubnikov's method proposed by Drummond et al.[4] (see Appendix) to calculate numerically $\kappa_{ij}(\theta,m)$ for various values of $q$ and $\mu$ in the region $m\geq 2$, $0.1<\theta<\pi/2$. The results were verified by comparing the angle dependence of $\eta^0(\theta,m)$ with Fig. 4 in Ref. 4 and the dependence on m with Figures 1, 2, and 3 in Ref. 3. For low values of m and $\theta$, (when $m\sin(\theta)\sim 2$), the yield of more than one integration loops was taken into account[4].

We obtained:

1) the radiation is shielded by the correlations, i. e. $\alpha$ is always positive;

2) the condition $\lambda=2\pi/r_D$ may actually be considered as an approximate limit for the appearance of coherence effects, i. e. $\alpha\to 0$ when $\lambda<2\pi/r_D$. Since $r_D\omega/c=(m/\mu q)^{1/2}$ this corresponds to harmonics $m>m_{max}=\mu q$.

In order to illustrate 1) and also to compare our results with those obtained in Ref. 3, we calculate approximately the spectral density of the radiation from unit area of a transparent plasma slab of thickness L:

$$I(\omega)=\sum_{m=1}^{\infty} I_m(\omega) \simeq 4\pi L\eta_\omega(\theta=45^0).$$

Here $I_m(\omega)$ is the yield[6] of the harmonic m, and we assumed that the integral of $\eta_\omega(\theta)$ with respect to the solid angle is roughly equal to $4\pi\eta_\omega(\theta=45^0)$. Fig. 1 shows the reduction of



the normalized spectrum

$$\frac{I(\omega)}{4\pi I_{RJ}\Lambda} = \frac{m}{q}\left(n''_+ + n''_-\right)$$

with increasing the density; here $\Lambda\equiv L\omega_p^2/(\omega_0 c)$. The spontaneous spectrum ($q\to 0$) in Fig. 1 should be compared with Fig. 2 in Ref. 3.

In Figures 2(a) and 2(b) the correlation factor $\alpha$ and the values of $2\pi r_D/\lambda=(m/\mu q)^{1/2}$ are plotted versus $q$ for $\theta=45^0$, m=3 and two electronic temperatures: $T=50m_0c^2$ and $T=5m_0c^2$. It is seen, that $\alpha$ is close to 1 percent when $2\pi r_D/\lambda$ is of order of 1. Note, that the series expansions used in Sec.II are valid even though $q>1$ if $\kappa_{ij}\ll 1$.

Since the considerations in Sec. II concerned the case of an uniform plasma of macroscopic dimensions, it must be shown that the correlation effects may be comparable with the selfabsorption even for big values of $L/\lambda$. Fig. 3 shows two curves of equal values of the optical depth $(n''_+ + n''_-)L\omega/c$ and the factor $\alpha$, calculated with $q=0.1$, $\theta=45^0$, $L\omega/c=50$ and $100$, and drawn in the $(m,\mu)$-plane. An increase of $q$ from $0.1$ to $1$ leads only to insignificant translation of these curves towards lower frequencies. It can be seen from Fig.3 that the decrease of the spectrum caused by correlations dominates at high harmonics, and in the region of low harmonics becomes comparable with the absorption only for cold and dense plasmas. In this case, however, the total emission is small. In particular, at frequencies close to the first harmonic ($m\sim 1$) the correlation effects can be neglected for all temperatures of interest. One should also take into account the restriction (2) on the maximal absorption coefficient and also the fact that the radiated spectrum is a sharply decreasing function of the harmonic m, thus high values of m ($m \gg 2$) are not so interesting.

## IV. Transformation to the laboratory reference frame

The transformations of the frequency, angle and density are

$$\omega = \gamma\left(1-\beta\cos(\theta^{l})\right)\omega^{l}; \qquad \cos(\theta) = \frac{\cos(\theta^{l})-\beta}{1-\beta\cos(\theta^{l})}; \qquad (22)$$

$$N_e^{l} = \gamma N_e \qquad (23)$$

where $N_e^l, \omega^l, \theta^l$ are the electronic density, the radiation frequency and the propagation angle in the laboratory frame; $\beta = u/c$; $u$ is the beam velocity along the magnetic field; $\gamma = (1-\beta^2)^{1/2}$. For $q$ we get from (23)

$$q = 10^{-11} N_e^{l} \;/\; (B_0^2[kGs]\;\gamma), \qquad (24)$$

Now we use (22) and the fact, that the total number of photons at all angles and frequencies remains the same after the transformation, to obtain the spectral density of the photons in the laboratory frame

$$N_\omega^{0\,l} = \frac{N_\omega^0}{\gamma\left(1-\beta\cos(\theta^l)\right)} = \frac{r_0 B_0^2[kGs]\; m^2\left(\kappa''_{11}(1)+\kappa''_{22}(1)\right)}{16\pi^3 \hbar c\;\mu\;\gamma\left(1-\beta\cos(\theta^l)\right)}, \qquad (25)$$

where $r_0 = 2.81\ 10^{-13}$cm.

In the relativistic case ($\gamma \gg 1$) the relation between the harmonics is

$$m = M\left(1-\beta\cos(\theta^l)\right) = \begin{cases} M/(2\gamma^2), & \theta^l = 0 \\ M/\gamma^2, & \theta^l = 1/\gamma\ . \end{cases}$$

Here $M = \omega^l/\Omega$ is the harmonic number in the laboratory frame with $\Omega \equiv \omega_0/\gamma$.



Consider radiation of millimeter and submillimeter waves $\lambda^b$ in the beam reference frame. By excluding the magnetic field from $\lambda^b[cm] = 11/(mB_0[kGs])$ and Eq. (24) one can obtain that from $\lambda^b < 1$cm and $\lambda^b < 1$mm it follows that $q\gamma < 10^{-11} N_e^l m^2/121$ and $q\gamma < 10^{-13} N_e^l m^2/121$ respectively. As it was pointed out in Sec. III we are interested in harmonics m not much larger than 2. In order to obtain considerable values of $\alpha$ one should take $q > 0.1$ and so $\gamma$ can not be much more than $10$ for real laboratory densities. One can also see that for laboratory densities $N_e^l \ll 10^{13}$ the submillimeter waves are practically not affected by the correlations ($q \ll 0.1$ for all $\gamma$).

Bellow we give an example with $\lambda^b \sim 1$mm and a high magnitude of the correlation effects. We take $N_e^l = 10^{13}$, $\gamma = 3$, $\mu = 100$ and magnetic field $B_0 = 5.5kGs$ ($q = 1.1$). It is convenient in the laboratory frame to present the spectrum on an $(x,y)$ plane, where $x = M/(2\gamma^2)$ and $y = \theta^l\gamma$. Fig. 4 shows the noninteraction photon spectrum (25) and the values of $\alpha$. We do not show on this picture the maximum, which is at $x \sim 1$ for $y \sim 0$ and at $x \sim 1/2$ for $y \sim 1$, because it corresponds to emission close to the first harmonic in the intrinsic frame. If for instance L is taken to be of order $5cm$ then from $1 < x < 4$ it follows that $30 < L\omega/c < 120$ and, according to the results of Sec. III, in this region $\alpha$ is bigger or comparable with the radiation decrease.

## V. Summary and conclusions

We have studied the effects of two-particle correlations on the radiation spectrum of a moving magnetized plasma both in the intrinsic and laboratory reference frames. It is shown that these effects can be simply described by terms of the kind $\kappa_{ij}\kappa_{mn}$ in the absorption coefficients ($\kappa_{ij}$ is the plasma susceptibility tensor) in agreement with Kirchoff's formula. It is further shown that the correlations diminish the radiation and that they are subject to the well known criterion - they appear at wavelengths of order of the Debye shielding length. We compared two ideal situations: (1) radiation of noninteracting electrons including absorption of the waves in the plasma and (2) radiation of interacting electrons without wave-absorption. We conclude that the correlation effects may become important at high densities (of order $10^{13} cm^{-3}$) in the case of radiation of electron beams having small relativistic factors $\gamma$, and thickness of the order of several centimeters.

## VI. Acknowledgments

D.I. Kaltchev is grateful for the hospitality of Storage Ring Sector of the Laboratory for Nuclear Problems where this work was done.

## Appendix: Calculation of the plasma susceptibility tensor

For the considered case of an equilibrium plasma the momentum distribution of the electrons is relativistic Maxwellian:

$$f_0(p) = \frac{N_e \mu}{4\pi c^3 K_2(\mu)} \exp\left(-\mu(1+p^2/m_0^2c^2)^{1/2}\right),$$



where $\mu=m_0c^2/T$, $p$ is the momentum of the electron and $K_2$ is the usual Bessel function. After substituting $\omega=kc$ into the elements $\kappa_{ij}$ they are given by[6]

$$\kappa_{ij}(\theta,m)=-iq\ \mu^2\int_0^\infty d\xi\ q_{ij}\ , \qquad \text{(A1)}$$

where

$$q_{11}=(K_2/\rho^2)(\cos\xi\cos^2\theta+\sin^2\theta)-(K_3/\rho^3)m^2\sin^2\theta\cos^2\theta(\sin\xi-\xi)^2 \quad ;$$

$$q_{22}=(K_2/\rho^2)\cos\xi-(K_3/\rho^3)m^2\sin^2\theta\ (1-\cos\xi)^2\ ;$$

$$q_{33}=(K_2/\rho^2)(\cos\xi\sin^2\theta+\cos^2\theta)-(K_3/\rho^3)m^2(\xi\cos^2\theta+\sin^2\theta\sin\xi)^2\ ;$$

$$q_{12}=-(K_2/\rho^2)\sin\xi\cos\theta+(K_3/\rho^3)m^2\sin^2\theta\cos\theta(\sin\xi-\xi)(1-\cos\xi)\ ;$$

$$q_{23}=(K_2/\rho^2)\sin\theta\sin\xi-(K_3/\rho^3)m^2\sin\theta(1-\cos\xi)(\xi\cos^2\theta+\sin^2\theta\sin\xi);$$

$$q_{13}=-(K_2/\rho^2)\sin\theta\cos\theta(1-\cos\xi)+(K_3/\rho^3)m^2\sin\theta\cos\theta(\xi-\sin\xi) \times (\xi\cos^2\theta+\sin^2\theta\sin\xi)\ .$$

with $K_\nu(\rho)$, $\nu=2,3$ - Bessel functions and

$$\rho\ =\ m\left(-\xi^2\sin^2\theta+2\sin^2\theta(1-\cos\xi)-2i\xi\mu/m+(\mu/m)^2\right)^{1/2}.$$

The functions $K_\nu(\rho)$ have an infinite number of saddle points in the first quadrant of the complex $\xi$-plane: $\xi_n=x_n+iy_n$ ; $x_n>0$, $y_n>0$ $(n=0,1,2...)$. We follow the method described in Ref. 4. and replace the path of integration over the real axes in (A1) with a contour, that consist of the part of the imaginary axis $(0,iy_0)$ and successive loops passing all saddle points $\xi_n$. Each loop is given by the equation $Im\ \rho= const_n$. For $|\rho|\gg 1$ the functions $K_\nu(\rho)$ are not oscillating along this contour and the integrals in (A1) can be solved numerically.

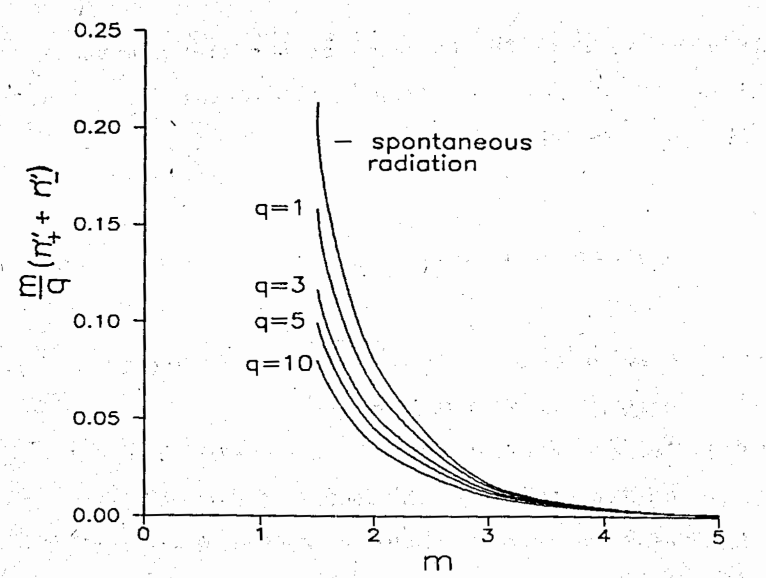


Fig.1 Decrease of the cyclotron radiation spectrum of a transparent plasma slab, due to correlation effects; $m_0c^2/T=10$.

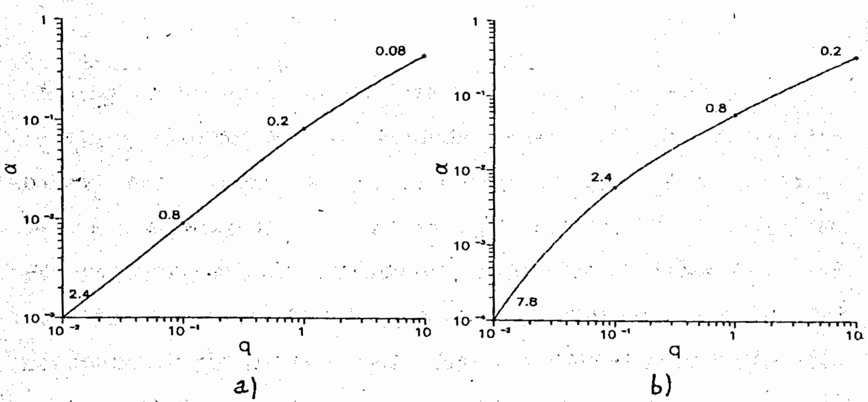


Fig.2 The correlation factor $\alpha$ versus $q$ for $\theta=45^0$, m=3 and two temperatures: a) $T=50m_0c^2$, b) $T=5m_0c^2$. The ratio $2\pi r_D/\lambda$ is also shown. Coherence effects appear ($\alpha>0.01$) when $2\pi r_D/\lambda$ is of order of 1.



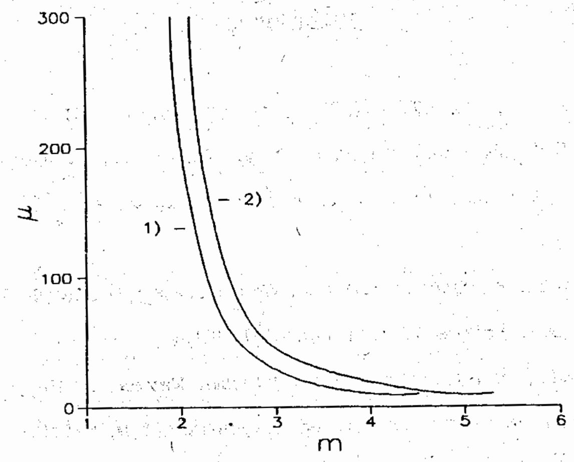


Fig.3 Curves obtained by solving the equation $(n''_+ + n''_-)L\omega/c=\alpha$ for $q=0.1$; $\theta=45^0$: 1) $L\omega/c=50$; 2) $L\omega/c=100$. On the left of each curve $(n''_+ + n''_-)L\omega/c>\alpha$ .

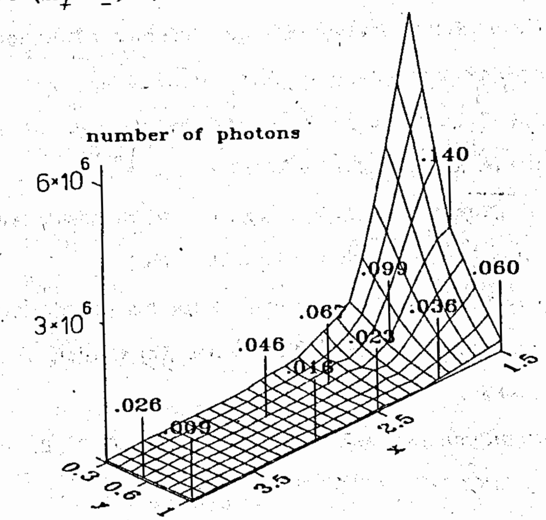


Fig.4 Spectral density of the photons radiated / second/ unit solid angle/ unit frequency interval in the laboratory frame, calculated without correlations, versus $x=M/(2\gamma^2)$, $y=\theta'\gamma$ and the values of $\alpha$. Here $q=1.1$, $\mu=100$; $\gamma=3$.

## References

[1] Pomeranchuck, I.Y., JETP, 16, 379 (1946); J.Phys.USSR, 9, 276 (1945).

[2] Drummond, W.E., Rosenbluth, M.N., Phys.Fluids, 3, 45 (1960).

[3] Hirshfield, J.I., Baldwin, D.E., Brown, S.C., Phys.Fluids, 4, 198 (1961).

[4] Drummond, W.E., Rosenbluth, M.N., Phys.Fluids, 6, 276 (1963).

[5] Trubnikov, B.A., Phys.Fluids, 4, 195 (1961).

[6] Baldwin, D.E., *Cinetic Theory of Plasma Waves in Magnetic Field* in *Achievements of Plasma Physics* (Mir, Moscow, 1974).

[7] Trubnikov, B.A., *Electromagnetic Waves in Relativistic Plasmas* in *Plasma Physics and Controlled Nuclear Fusion Research* (Acad.Sciences, USSR, 1958).

[8] Levin, M.L., Ritov, S.M., *Theory of Equilibrium Fluctuations in the Electrodynamics* (Science, Moscow, 1967).

[9] Parchomchuck, V.V., Pestrikov, D.V., *Thermal Noises of Intense Beam in a Storage Ring*, JTP, 50, 7 (1980).

[10] Dikanski, N.S., Pestrikov, D.V., *Physics of the intense beams in storage rings* (Science, Novosibirsk, 1989).

[11] Chattopadhyay, S., *Some fundamental aspects of fluctuations and coherence in charged-particle beams in storage rings* (CERN, Geneva, 1984).

[12] *Plasma Electrodynamics*, edited by Achieser, A.I. (Science, Moscow, 1974), p.518.

Received by Publishing Department
on January 21, 1992.