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EIGENPOTENTIALS OF SELF-CONSISTENT CYLINDRICAL BRILLOUIN RELATIVISTIC ELECTRON BEAM (REB)

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1 INTRODUCTION

The problem of quantitative determination of the drifting cylindrical Brillouin REB parameters and the main relations between them which be held, secure REB's self-consistency in an external homogeneous magnetic field, when thermal and radiation effects are negligible, has been solved by de Packh and Ulrich in 1960 [1].

They proved the forward valocity of all electrons in the Brillouin REB to be constant and found the securing its self-consistency radial dependence of the relativistic factor $\gamma(r)$ for $0 \leq r \leq a$, where a is the beam's external radius in the form:

$$\gamma(r) = \gamma_0 \frac{r_0^2 + r^2}{r_0^2 - r^2}, \tag{1}$$

where

$$r_0 = a\left(\frac{\gamma_a + \gamma_0}{\gamma_a - \gamma_0}\right)^{1/2}, \tag{2}$$

(γ_0 and γ_a are the relativistic factors of electrons on the axis and surface of the beam, respectively). There was also calculated the constant external magnetic field, H, necessary to provide self-consistency of the REB:

$$H = H_a = \frac{4 r_c^3}{\sigma (r_c^2 - a^2)^2}, \quad \left(\sigma = \frac{e}{mc^2} = 5{,}867 \times 10^{-4},\ e = |e|\right) \qquad (3)$$

and the total current, I, in the self-consistent beam (in units of $\frac{5mc^2}{e} \simeq 8500$ A):

$$I = 4(\gamma_0^2 - 1)^{1/2} \frac{a^2 r_c^2}{(r_0^2 - a^2)^2}, \qquad (4)$$

They have also given the calculated values of I for many fixed γ_a and γ_0 and the highest values of I_{max} at fixed γ_a's for the corresponding to I_{max} γ_0 values as well as formulae relating the REB parameters on the axis and surface of the beam.

Self-consistency realization including REB formation before its coming to the drift region by exact calculation of electron trajectories requires the use of not only radial dependences of $\gamma(r)$ and of the connected with it scalar eigenpotential $V_g(r)$, but also of radial dependences of the other parameters, such as diamagnetic field of REB, its corresponding vector eigenpotential.

Further, there are presented the results of the calculation of the φ-component $\mathcal{A}_\varphi(r)$ of the total vector potential in a self-consistent drifting cylindrical REB, of the diamagnetic field, $h(r)$, and the other characteristics in

addition to the data reported in[1] followed with their brief analysis.

2. DIAMAGNETISM OF SELF-CONSISTENT BRILLOUIN CYLINDRICAL REB

We will proceed from the exact relativistic equation for electron trajectories in an axisymmetrical laminar REB[2]. In the absence of initial magnetization (on the cathode) in the drift space in the cylindrical system of coordinates, r, φ, z, and Gaussian system of units it has the form:

$$\left.\begin{aligned} \frac{d^2 r}{dz^2} &= \frac{\sigma}{\beta_z^2 \gamma}\left\{\frac{\partial V_g}{\partial r} - \left[1 + \left(\frac{dr}{dz}\right)^2\right]\frac{2I(r)}{cr}\beta_z - \frac{\sigma}{\gamma}\mathcal{A}_\varphi \frac{\partial \mathcal{A}_\varphi}{\partial r}\right\}; \\ \beta_z &= \frac{1}{\gamma}\left(\frac{\gamma^2 - 1 - \sigma^2 \mathcal{A}_\varphi^2}{1 + \left(\frac{dr}{dz}\right)^2}\right)^{1/2}; \quad \beta_\varphi = \frac{v_\varphi}{c} = \frac{\sigma}{\gamma}\mathcal{A}_\varphi; \\ \gamma &= 1 + \sigma[V_c + V_g(r)]; \end{aligned}\right\} \qquad (5)$$

where eV_c = const > 0 is the kinetic energy of an electron emitted from the cathode to "infinity"; $eV_g(r) < 0$ is the electron kinetic energy loss in the electric field of the REB, $I(r) > 0$ is the REB current in a circle of radius r;

$$\mathcal{A}_\varphi = \mathcal{A}_\varphi(r)_{ext} + \mathcal{A}_{\varphi g}(r) = \frac{r}{2}[H + \bar{h}(r)], \qquad (6)$$

where the external magnetic field H = const > 0; $\bar{h}(r) < 0$ is the average over the circle area πr^2 value of $h(r)$.

From the Gauss theorem in the symmetry of our problem and with β_z = const we have:

$$\frac{\partial V_g}{\partial r} = \frac{2I(r)}{c\beta_z r}. \qquad (7)$$

Let us now consider the condition of self-consistency, which requires $\frac{d^2 z}{dr^2} = \frac{dz}{dr} = 0$ for all r in the range $0 \leq r \leq a$ and has in correspondance with equations (5), (7) the form

$$\frac{2I(r)}{cr}\left(\frac{1}{\beta_z}-\beta_z\right)=\frac{\sigma}{\gamma}\mathcal{A}_\varphi\frac{\partial \mathcal{A}_\varphi}{\partial r}. \qquad (8)$$

Again we will use the integral of de Packh and Ulrich $\beta_z=\beta_c=(1-\frac{1}{\gamma_0^2})^{1/2}$; $(\gamma_c=1+\sigma[V_c+V_e(0)])$ thanks to which

$$\frac{1}{\beta_z}-\beta_z=\frac{1}{\gamma_0(\gamma_c^2-1)^{1/2}}. \qquad (9)$$

At the same time from the expression for β_z in eq.(5) we have:

$$\frac{1}{\beta_z}-\beta_z=\frac{1+\sigma^2\mathcal{A}_\varphi^2}{\gamma(\gamma^2-1-\sigma^2\mathcal{A}_\varphi^2)^{1/2}}, \qquad (10)$$

from where, taking into account exps.(6) and (9) we find that

$$\mathcal{A}_\varphi(r)=\frac{r}{2}[H+\bar{h}(r)]=\frac{1}{\sigma}\left(\frac{\gamma^2}{\gamma_0^2}-1\right)^{1/2}. \qquad (11)$$

Note that we would have come to the same result if had integrated eq.(8) taking into account the following from eqs.(7) and (5) expression:

$$\frac{2I(r)}{cr}=\frac{\beta_c}{\sigma}\frac{\partial\gamma}{\partial r} \qquad (12)$$

and expression (9).

By substituting into (11) the expression for $\gamma(r)$ (1) we find

$$\mathcal{A}_\varphi(r)=\frac{2r_0 r}{\sigma(r_0^2-r^2)}, \qquad (11')$$

from where it follows that

$$\bar{h}(r)=\frac{4r_c}{\sigma(r_c^2-r^2)}-H, \qquad (13)$$

which with account of exp.(3) takes the form

$$\bar{h}(r)=\frac{4r_c}{\sigma}\left[\frac{1}{r_0^2-r^2}-\frac{r_0^2}{(r_0^2-a^2)^2}\right], \qquad (13')$$

and the corresponding to it distribution of the diamagnetic field of the beam is

$$h(r)=\bar{h}+\frac{r}{2}\frac{\partial\bar{h}}{\partial r}=\frac{4r_0^3}{\sigma}\left[\frac{1}{(r_c^2-r^2)^2}-\frac{1}{(r_c^2-a^2)^2}\right]. \qquad (14)$$

For the complete account of the real magnetism of the REB we must also find the contribution of the beam current φ -component reflection from the conducting walls of the vacuum chamber. However, because of the fact that the total reflected current is, like REB, cylindrically symmetrical with its inner radius being larger than a, the magnetic field it creates $h_{z\,refl}$ = const throughout the REB.

So in calculation it is sufficient to have in mind that the value of H in the formulas contains this small addition.

Remember that the z-component of the vector eigenpotential, $A_{ze}(r)$ is exactly accounted for in eq.(5) where $\frac{2I(r)}{cr}=-h_{\varphi e}(r)$ /2/.

3. SCALAR EIGENPOTENTIAL AND OTHER REB PARAMETERS

The radial dependence of the eigenpotential, $V_e(r)$, follows from the expression of de Packh and Ulrich[1], which in combination with detalized $\gamma(r)$ structure given in eq.(5)

and the corresponding above-mentioned definition of $\gamma_0 = 1 + \sigma[V_c + \bar{V}_\ell(0)]$ allows one to write that

$$\bar{V}_\ell(r) = \bar{V}_\ell(0) + \frac{\gamma_0}{\sigma}\frac{2r^2}{r_0^2 - r^2}, \qquad (0 \le r \le a). \qquad (15)$$

The constant $\bar{V}_\ell(0) < 0$ in (15) multiplied by $-e$ is the work to be done to carry the electron from the inner surface of the earthed conducted wall of the vacuum chamber to the beam axis.

If the radius R of the vacuum chamber is larger than a, then using eq.(15) and the Gauss theorem it is easy to find that for $a < r \le R$

$$\bar{V}_\ell(r) = \bar{V}_\ell(0) + \frac{\gamma_0}{\sigma}\frac{2a^2}{r_0^2 - a^2} + \frac{2I}{c\beta_0}\ln\frac{r}{a}. \qquad (16)$$

In the interval $0 \le r \le a$ from eq.(15) it follows that

$$\frac{\partial \bar{V}_\ell}{\partial r} = \frac{4\gamma_0 r_0^2 r}{\sigma(r_0^2 - r^2)^2}, \qquad (7')$$

and in the interval $a < r \le R$, if exp.(16) and (4) are taken into account

$$\frac{\partial \bar{V}_\ell}{\partial r} = \frac{4\gamma_0 r_0^2 a^2}{(r_0^2 - a^2)^2}\frac{1}{r}. \qquad (17)$$

And

$$\bar{V}_\ell(0) = -\int_0^R \frac{\partial \bar{V}_\ell(r)}{\partial r}dr = -\frac{2\gamma_0 a^2}{\sigma(r_0^2 - a^2)}\left(1 + \frac{2r_0^2}{r_0^2 - a^2}\ln\frac{R}{a}\right). \qquad (18)$$

It is evident, that the slowing down of an electron in REB due to its charge having a maximum value of

$$-\frac{2\gamma_0 a^2}{\sigma(r_0^2 - a^2)}\left(1 + \frac{2r_0^2}{r_0^2 - a^2}\ln\frac{R}{a}\right)$$

on the axis is not followed by any energy dispersion of the particles in the beam, when they bombard an earthed conducting target placed in the beam, because in this case the effect is automatically totally compensated before the electrons reach the target.

By comparing dependences (7) and (7′) in the interval $0 \le r \le a$ we find

$$I(r) = \frac{2c(\gamma_0^2 - 1)^{1/2} r_0^2 r^2}{\sigma(r_0^2 - r^2)^2}, \qquad (19)$$

with the total current

$$I = \frac{c(\gamma_0^2 - 1)^{1/2}}{2\sigma\gamma_0^2}(\gamma_a^2 - \gamma_0^2), \qquad (20)$$

and the radial dependence of particles density $n(r) = \frac{\partial I(r)/\partial r}{2\pi e c\beta_0 r}$ being

$$n(r) = \frac{2\gamma_0 r_0^2}{\pi e\sigma}\frac{r_0^2 + r^2}{(r_0^2 - r^2)^3}. \qquad (21)$$

It is convenient to take a dimensionless radius $\rho = \frac{r}{a}$ as an argument in all the dependences under consideration. Then, using the denotion $\frac{\gamma_a + \gamma_0}{\gamma_a - \gamma_0} = \Gamma$, they can be presented in the form:

$$\gamma(\rho) = \gamma_0\frac{\Gamma + \rho^2}{\Gamma - \rho^2}, \qquad (1')$$

$$\bar{V}_\ell(\rho) = -\frac{2\gamma_0}{\sigma(\Gamma - 1)}\left(1 + \frac{2\Gamma}{\Gamma - 1}\ln\frac{R}{a}\right) + \frac{2\gamma_0}{\sigma}\frac{\rho^2}{\Gamma - \rho^2}, \qquad (15')$$

$$\bar{h}(\rho) = \frac{4\Gamma^{1/2}}{\sigma a}\left[\frac{1}{\Gamma - \rho^2} - \frac{\Gamma}{(\Gamma - 1)^2}\right], \qquad (13'')$$

$$A_\varphi(\rho) = \frac{2\Gamma^{1/2}}{\sigma}\frac{\rho}{\Gamma - \rho^2} \qquad (11'')$$

$$h(\rho) = \frac{4\Gamma^{3/2}}{\sigma a}\left[\frac{1}{(\Gamma-\rho^2)^2} - \frac{1}{(\Gamma-1)^2}\right], \qquad (14')$$

$$I(\rho) = \frac{2c(\gamma_0^2-1)^{1/2}\Gamma}{\sigma}\,\frac{\rho^2}{(\Gamma-\rho^2)^2}, \qquad (19')$$

$$n(\rho) = \frac{2\gamma_0\Gamma}{\pi e\sigma a^2}\,\frac{\Gamma+\rho^2}{(\Gamma-\rho^2)^3}. \qquad (21')$$

4. DISCUSSION OF RESULTS

Now even most powerful modern sources of REB allow transportation of beam currents much lower their predicted maximum values, I_{max}, for γ_a used in this sources. The main reason is the absence of any solution of the problem of theoretical prediction of a full set of electron-optical parameters of electron guns (EG) necessary to provide not only zero beam emittance at its outlet but also realization of all the above-mentioned radial dependences and constant parameters of self-consistent REB before its coming into drift region.

The Table gives two concrete sets of providing REB self-consistency calculated dependences that should be realized at its inlet to the drift region of a maximum possible current, I_{max}, from the presently most powerful REB sources, the so-called ironless linear induction electron accelerator LIA-10 [3] and RADLAC-II [4]. In the first the EG yields $\gamma_a = 6$, in the second it reaches $\gamma_a = 11$. Having solved the equation $\gamma_0^6 - (2\gamma_a^2+\gamma_a^4)\gamma_0^2 + 2\gamma_a^4 = 0$ [1] we find the corresponding to I_{max} $\gamma_0 = 1.378$ at the outlet of the EG of LIA-10 and $\gamma_0 = 1.402$ of RADLAC-II. We take the external radius of the beam $a = 9$ cm for the LIA-10 (as in the original) and $a = 4$ cm for the RADLAC-II (but not a = 1.1 cm as in the original) for the emission density from the cathode not exceeding that in RADLAC-II. We assume also that $\ln\frac{R}{a} = 0$.

Table

	LIA-10, $I_{max} = 145$ kA, $\gamma_a = 6$; $\gamma_0 = 1.378$; $\Gamma = 1.6$; $H = 4.3$ kOe; $a = 9$ cm						RADLAC-II. $I_{max} = 507.2$ kA; $\gamma_a = 11$; $\gamma_0 = 1.402$; $\Gamma = 1.29$; $H = 29.3$ kOe; $a = 4$ cm					
ρ	0	0.4	0.6	0.8	0.9	1	0	0.4	0.6	0.8	0.9	1
$\gamma(\rho)$	1.38	1.69	2.2	2.3	4.2	6	1.4	1.8	2.5	4.15	6.1	11
$\sigma V_\varphi(\rho)$	-4.6	-4.3	-3.8	-2.8	-1.8	0	-9.6	-9.2	-8.5	-6.85	-4.9	0
$\bar{h}(\rho)$ (kOe)	-3.7	-3.6	-3.5	-3.3	-3.1	-2.7	-28	-28	-27	-26	-25	-23
$H+\bar{h}$ (kOe)	0.6	0.7	0.8	1	1.2	1.6	1.5	1.7	2.1	3	4	6.6
$A_\varphi(\rho)$ (kOe.cm)	0	1.2	2.1	3.6	4.9	7.2	0	1.4	2.5	4.75	7.2	13.25
$I(\rho)$ (kA)	0	4	12.2	36.1	67.6	145	0	5.4	17.9	65.2	151	507.2
$n(\rho)/n(0)$	1	1.5	2.6	6.5	12.6	31.2	1	1.7	3.4	11.6	31.3	153.4

There attracts attention the large value of the diamagnetic field of the beam, which in a self-consistent REB with I_{max} reaches 85% of the consistening field strength, H, for the LIA-10 and 95% for the RADLAC-II. No less surprising is the fact that **the largest portion of I_{max} is concentrated** in a thin near-surface layer of REB. Particularly, in the LIA-10, in this layer thickness $0.8 \leq \rho \leq 1$ this portion makes over 0.75 I_{max} and in the RADLAC-II over 0.87 I_{max}. So a high-current self-consistent cylindrical Brillouin REB with I_{max} has a tube-like cross-section distribution of current.

This and the other specific characteristics of such REB's make the problem of their realization a delicate and extremely difficult one both from the theoretical point of view and especially from the point of view of its technical implementation.

To approach the problem one naturally should start with the mastering of the method of exact calculation of electron trajectories in the transmission region from the nonmagnetized cathode of the EG to the exit point of REB into the drift region, then of formation by repeated variation of the cathode shape and other electron-optical parameters of EG of such an electron flow with ever zero emittance, that the resultant set of dependences between the parameters of the self-consistent REB was in exact correspondance with a concrete set of the above deduced formulas relating the relativistic factor and the total current in the electron gun under calculation.

5. CONCLUSIONS

The data being contained in this work are given in addition to the results obtained by de Packh and Ulrich/1/. These in combination with exact relativistic equation for electron trajectories in an axisymmetrical laminar REB/2/ may appear useful for exact calculation of real sources of self-consistent high-current cylindrical Brillouin REB's.

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