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THEORETICAL TREATMENT OF A CLASSICAL TRANSVERSE FEEDBACK SYSTEM USING Z-TRANSFORM

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INTRODUCTION

The coherent transverse beam oscillation damper systems are used in all charged particle accelerators to prevent particle losses due to transverse instabilities and to damp transverse injection oscillations. Theoretical investigation of the damper system for UNK-I has been carried out using Z-transform method of solving the problem of coherent transverse oscillation damping[1,2]. It has been found that such system including two pairs of pick-up electrodes (PU) and damping kickers (DK) connected by delayed negative feedback with digital electronics damps the transverse instability with the increment of about $0.1\omega_o$. But in many acting and being designed accelerators the growth rate of transverse oscillations is more slower and a classical transverse feedback system with one PU and DK connected by feedback with digital electronics is enough for damping. In this article the equations including the kick effect and the PU-signal transformation in feedback are obtained. The solving of these equations is carried out using Z-transform method. The results are analyzed for an ideal feedback and for a feedback with a notch filter. All numerical results are made for LHC[3].

DAMPER SYSTEM CONFIGURATION AND BASIC EQUATIONS

A classical transverse feedback system for each of two directions of beam transverse oscillations includes one PU and DK connected by delayed negative feedback where the signal from PU transfers to DK in opposite direction to the beam motion.

As in[2,4] the equation of beam center oscillations for complex value of deviation $\tilde{v}\sqrt{\beta}$ from the closed orbit can be written in the following form:

$$\frac{d^2\tilde{v}}{d\varphi^2} + \tilde{Q}^2\tilde{v} = \frac{Q^2\beta\sqrt{\beta}}{m\gamma_o v_o^2} F_{ext}, \qquad \tilde{Q} = Q + \Delta Q(\omega), \qquad (1)$$

where (v,φ) are generalized co-ordinates in normalized phase space to be applied for describing the transverse beam motion

in accelerator; $m\gamma_o v_o$ is the longitudinal momentum of a beam particle which has the angular frequency ω_o; β is the normalized betatron wave length; Q is the number of betatron oscillations per turn; $\Delta Q(\omega)$ is the complex tune shift for Q to be conditioned by an electromagnatic field of the displaced beam and to be proportional to transverse coupling impedance $Z_\perp$[4]. The sign (~) means that the complex value $\tilde{v}$ concernes the travelling wave of transverse displacement with the azimuthal wave number k and complex angular frequency ω. The physical value of displacement is the real value of $\tilde{v}$. For particle oscillating in transverse direction its deviation $v\sqrt{\beta}$ will be equal to displacement $\tilde{v}\sqrt{\beta}$ of the beam center for eigenwave:

$$\omega = \omega_o\left(k - Q - \Delta Q(\omega)\right). \qquad (2)$$

The damper system effect is characterizied by F_{ext} force. Out of the DK region, when $F_{ext}=0$, we have in (1) an ordinary betatron equation where the frequency, however, is the complex value and depends on ω. In the DK region we must solve the differential equation (1) with non-zero right-hand side. As in[1] we shall suppose that the kicker is short. Then we can describe the DK effect in a matrix form[1,5]:

$$\hat{V}(\varphi_K^+) = \hat{I}\hat{V}(\varphi_K^-) + Q\hat{T}\Delta\hat{V}(\varphi_K),$$

where $\hat{I}$ is the unit matrix and the matrix $\hat{T}$ is given by

$$\hat{T} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

In column matrix $\hat{V}(\varphi)$ the first element is $\tilde{v}(\varphi)$ and the second one is equal to $d\tilde{v}(\varphi)/d\varphi$. The column matrix $\Delta\hat{V}(\varphi_K)$ describes the kicker effect. Its second element is zero but the first one Δv is given by

$$\Delta v = \frac{l_K\sqrt{\beta_K}}{m\gamma_o v_o^2} F(\varphi_K), \qquad (3)$$

where l_K is the kicker length. Taking into account[1,2] it is not difficult to obtain the matrix equation where the transformation for a beam state matrix from the n-th turn to the (n+1)-th turn is determined by a transfer matrix. Let us suggest that the transfer matrix of any turn is $\hat{M}$ for unperturbed machine and the transfer matrix from DK to PU is $\hat{A}$ in which the phase advance between these two points is equal to $\alpha\pi$. It is necessary to note that all elements for these two matrices depend on ω. The DK effect at the azimuthal angle φ_K to be determined above as $\Delta\hat{V}(\varphi_K)$ will be defined further at the n-th turn as matrix $\Delta\hat{V}[n]$. Taking into account all these assumptions we can write

$$\hat{V}[n+1] = \hat{M}\hat{V}[n] + Q\hat{A}\hat{T}\Delta\hat{V}[n], \qquad (4)$$

where $\hat{V}[n] = \hat{V}(\varphi_P+2\pi n)$ is the column matrix determining the beam center state at the azimuthal angle φ_P at the n-th turn.

Before solving (4) it is necessary to determine the column matrix $\Delta\hat{V}[n]$. In accordance with feedback structure scheme the kick strength depends on the beam center displacement in PU position and on the PU-signal transformation in feedback electronics. Let us suggest that at the n-th turn the input voltage $U_{in}[n]$ at feedback is proportional to a beam center deviation in PU. Hence, put $U_{in}[n] = K_{in}\mathrm{Re}\tilde{v}[n]$. The kick strength will be proportional to the output voltage $U_{out}[n]$ at feedback. This means in (3) that $\Delta v[n] = K_{out}U_{out}[n]$. For an ideal feedback we should write that $U_{out}[n]=U_{in}[n]$. The time delay τ for such feedback is equal to the time of particle flight from PU to DK; hence $\tau=(\varphi_K-\varphi_P)/\omega_o$. Thus, for an ideal feedback the equation for $\Delta\hat{V}[n]$ can be written as

$$\Delta\hat{V}[n] = \tilde{K}\hat{V}[n], \qquad \tilde{K}=K_{in}K_{out}.$$

It is not difficult to obtain the coupling equations between U_{in} and U_{out} for other structure schemes of feedback with digital electronics as discrete time systems[6]. For example, such equation for a comb filter including p pure delays with time T_o and broadband amplifiers with gain b can be written as $U_{out}[n]= U_{in}[n]- b_1U_{in}[n-1]-\ldots- b_pU_{in}[n-p]$. Thus, the basic equation for such feedback is given by

$$\Delta\hat{V}[n] = \tilde{K}\Big[\hat{V}[n] - \sum_{m=1}^{p} b_m\hat{V}[n-m]\Big], \qquad (5)$$

It is essential for equation (5) that the time delay value in feedback corresponds to such one when the kick and the PU-sample belong to the same particles. Equation (5) for $p=0$

responds to the ideal feedback and these conditions can be realized in broadband amplifier with the linear phase-frequency characteristic. So the growth rate of transverse beam instability depends on ω then the $\tilde{K}$ value depends on ω, too. More detailed discussion about $\tilde{K}$ and gain-transfer characteristic in the radiotechnical sense has been performed in[2].

Substituting (5) into (4) we get the following equation:

$$\hat{V}[n+1] = \hat{M}\hat{V}[n] + Q\tilde{K}\hat{A}\hat{T}\Big[\hat{V}[n] - \sum_{m=1}^{p} b_m\hat{V}[n-m]\Big]. \qquad (6)$$

As in[1,2] the matrix equation (6) will be solved using Z-transform[6] for sequence $\hat{V}[n]$:

$$\hat{V}(z)= \sum_{n=0}^{\infty} \hat{V}[n]z^{-n}, \qquad \hat{V}[n] = \frac{1}{2\pi i}\oint_C \hat{V}(z)z^{n-1}dz. \qquad (7)$$

Here C is a closed curve surrounding all singular points of $\hat{V}(z)$ on complex z-plane. This method is convenient for analyzing different structure schemes of feedback systems with digital electronics and has wide applications for discrete time systems.

Using the theorem about calculating the circulation integral by its singularities we can write:

$$\hat{V}[n]= \sum_k \mathrm{Rez}\,\hat{V}(z_k)z_k^{n-1}.$$

It is clear from this formula that the stable motion of particles is possible when all singular points z_k for $\hat{V}(z)$ on complex z-plane lie inside the circle with radius R=1:

$$|z_k|\leqslant 1. \qquad (8)$$

Using Z-transform (7) for equation (6) and taking into account its properties[6] we get that all singular points z_k are the roots of equation[1,2]

$$\det(z_k\hat{I} - \hat{M}_T) = 0, \qquad (9)$$

where

$$\hat{M}_T= \hat{M} + Q\tilde{K}\Big[1 - \sum_{m=1}^{p} b_m z_k^{-m}\Big]\hat{A}\hat{T}. \qquad (10)$$

The decrement for damped oscillations will be determined by the singular point with maximum absolute value of z_k:

$$D=-\ln(\max|z_k|).$$

Thus, determining the positions of all the singular points z_k on complex z-plane we can find parameters of a damper system where betatron oscillations are stable and calculate the decrement for damped transverse oscillations of particles in the accelerator.

CLASSICAL TRANSVERSE FEEDBACK SYSTEM

A classical transverse feedback system corresponds to the ideal feedback with $p=0$ in (10). In this case $\hat{M}_T=\hat{M}+Q\tilde{K}\hat{A}\hat{T}$ and singular points z_k when $|\Delta Q(\omega)|\ll Q$ are

$$z_k= \cos(2\pi\tilde{Q})+(\tilde{K}/2)\sin(\alpha\pi\tilde{Q}) \pm$$

$$\pm\ i\sqrt{\left[\sin(2\pi\tilde{Q})-(\tilde{K}/2)\cos(\alpha\pi\tilde{Q})\right]^2-\tilde{K}^2/4}. \qquad (11)$$

It is not difficult to see that for $\tilde{K}=0$ we get $z_k=\exp(\pm i2\pi\tilde{Q})$. Hence, from (8) we find that oscillations are stable if $\mathrm{Im}\tilde{Q}=0$. This coincides with the famous result for the stability criterion in the theory of betatron motion of particles in the accelerator. When $\mathrm{Im}\tilde{Q}=0$, the $\max|z_k|$ values versus $\tilde{K}$ for $0.62<\alpha\mathrm{Re}\tilde{Q}<1.62$ and $\mathrm{Re}\tilde{Q}=70.31$ [3] are shown in Fig.1. In this region of α the damping of transverse oscillations is obtained for negative values of $\tilde{K}$. It is easy to find that the sign of $\tilde{K}$ in stable region of parameters coincides with $\varkappa=\mathrm{sign}[\sin((2-\alpha)\pi\mathrm{Re}\tilde{Q})]$ and we always have $\varkappa\tilde{K}>0$. The damping is absent for such PU-DK distances when $\alpha\pi\mathrm{Re}\tilde{Q}=2\pi\mathrm{Re}\tilde{Q}-\pi m$, where m is integer that is less than $2\mathrm{Re}\tilde{Q}$. When betatron oscillation phase advance from PU to DK is in the neighbourhood of $\pi m+\pi/2$ the damping rate has maximum values (curves 5,6,7 and 8 on Fig.1). It is necessary to emphasize that the highest damping rate occurs for $(2-\alpha)\pi\mathrm{Re}\tilde{Q}=103.377\pi$ not for 103.5π. This result differs a little from statement [7] about distance from PU to DK for the highest damping rate.

When the instability occurs, the stable regions for $\tilde{K}$ are narrowing when $|\mathrm{Im}\tilde{Q}|$ is increased. Figure 2 for $(2-\alpha)\pi\mathrm{Re}\tilde{Q}=\pi m+\pi/2$ shows such dependences. It has been seen from Fig.2 that for $|\mathrm{Im}\tilde{Q}|>0.08$ a classical feedback cannot damp the instability. To prevent fast instability the damper

system including two pairs PU and DK connected by feedback must be used. Thus, for UNK-I such system damps the transverse instability with $|\mathrm{Im}\tilde{Q}|=0.1$ [2].

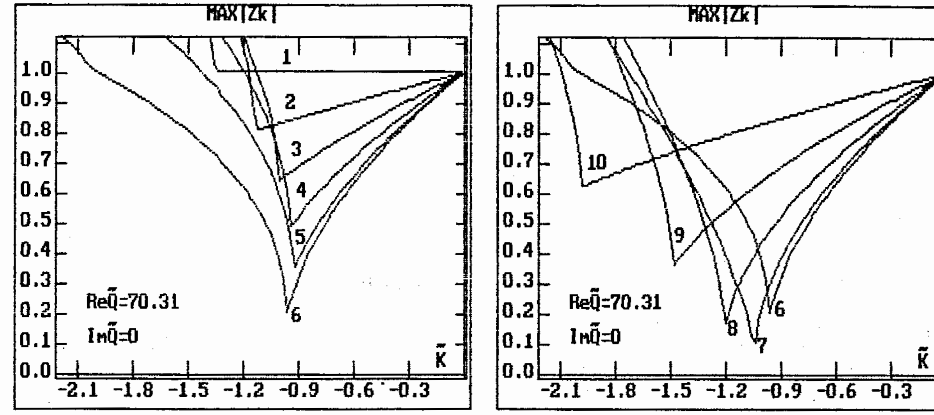

Fig.1. The $\max|z_k|$ values versus $\tilde{K}$ for $\alpha\mathrm{Re}\tilde{Q}=0.62+0.1(n-1)$. The digits near curves are n-values.

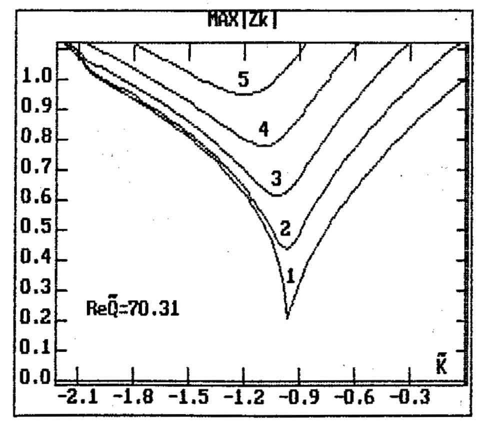

Fig.2. The $\max|z_k|$ values versus $\tilde{K}$. $|\mathrm{Im}\tilde{Q}|=0.1(n-1)$. $\alpha\pi\mathrm{Re}\tilde{Q}=1.12\pi$.

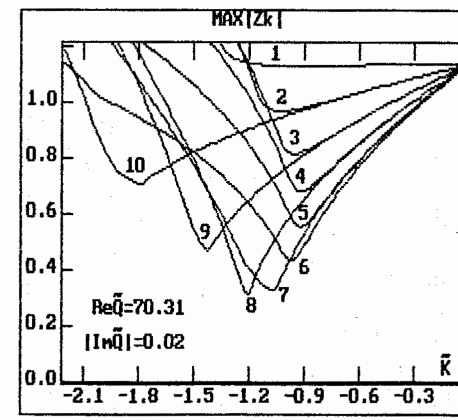

Fig.3. The $\max|z_k|$ values versus $\tilde{K}$. $\alpha\mathrm{Re}\tilde{Q}=0.62+0.1(n-1)$.

The stable regions for $\tilde{K}$ with different α, when $|\mathrm{Im}\tilde{Q}|=0.02$, are shown in Fig.3. As can be seen from Fig.3 the instability is not damped for small values of $\tilde{K}$ that differs from dependences in Fig.1. Thus, when the distance from PU to DK is $(2-\alpha)\pi\mathrm{Re}\tilde{Q}=\pi m+\pi/2$ (curve 6 in Fig.3), the damping occurs for $|\tilde{K}|>0.23$. The highest damping rate for $|Im\tilde{Q}|=0.02$ corresponds to $(2-\alpha)\pi Re\tilde{Q}=103.344\pi$ which differs from PU-DK distance in the same rate for $Im\tilde{Q}=0$.

FEEDBACK SYSTEM WITH DIGITAL NOTCH FILTER

The feedback with a notch filter for *p=1* in (5) will be considered further. In this case for (10) and for $|\Delta Q(\omega)|\ll Q$ we obtain from (9) the following cube equation for z_k:

$$z_k^3 - 2[\cos(2\pi\tilde{Q})+ (\tilde{K}/2)\sin(\alpha\pi\tilde{Q})]z_k^2 + [1- \tilde{K}\sin((2-\alpha)\pi\tilde{Q})]z_k+$$

$$+ b\tilde{K}[\sin((2-\alpha)\pi\tilde{Q})+ \sin(\alpha\pi\tilde{Q})]= 0, \qquad (12)$$

where $b=b_1$. It is easy to see from (12) that the roots of the reduced equation with *b=0* are given by (11). For non-zero values of b we have three roots z_k which can be found by Cardano formula. It is necessary to emphasize once more that the stable region and the damping rate are determined by the maximum absolute value of z_k among all three values.

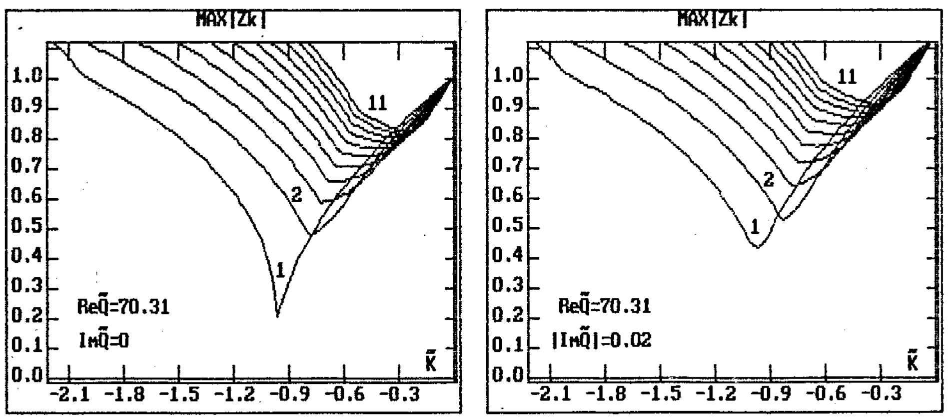

Fig.4. The $\max|z_k|$ values versus $\tilde{K}$. $\alpha\pi Re\tilde{Q}=1.12\pi$. $b=0.1(n-1)$.

Figure 4 shows the $\max|z_k|$ values versus $\tilde{K}$ for $(2-\alpha)\pi Re\tilde{Q}=\pi m+\pi/2$ when the instability is absent ($Im\tilde{Q}=0$) and occurs ($|Im\tilde{Q}|=0.02$). So the notch filter is used to weaken the influence of signals with frequencies multiple to revolution frequency. And that is why all curves in Fig.4 are shown for *b>0.* It is necessary to remind that the maximum effect of such a filter will be for $b=1$ /1/. Figure 4 shows

that the stable regions for $\tilde{K}$ are narrowing when b increases. At the same time it is necessary to note that for small $\tilde{K}$ the damping rate is larger than its values for *b=0.* Figure 5 shows the $\max|z_k|$ values versus $\tilde{K}$ for different PU-DK distances. It is necessary to emphasize that the highest damping rate (curve 2) will be for $\alpha\pi Re\tilde{Q}=0.826\pi$ (when $Im\tilde{Q}=0$) and for $\alpha\pi Re\tilde{Q}=0.849\pi$ (when $Im\tilde{Q}=0.02$). It can be seen also that for the smaller α (curve 1) the damping rate quickly decreases. Due to this reason it is difficult to realize the regime with the highest damping rate. The PU-DK distances as $(2-\alpha)\pi Re\tilde{Q}=\pi m+\pi/2$ or near this values are preferable (curves 4, 5, 6) because usually the damping regime corresponds to small values of $\tilde{K}$ for acting accelerators.

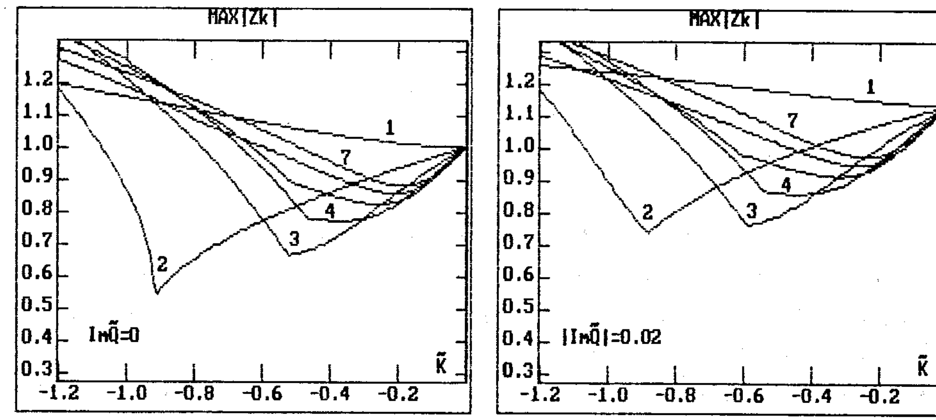

Fig.5. The $\max|z_k|$ values versus $\tilde{K}$ for $\alpha Re\tilde{Q}=0.75+0.1(n-1)$. $Re\tilde{Q}=70.31$. *b=1.*

CONCLUSION

Having been obtained for the classical transverse feedback system in the accelerator, equation (4) including the kick effect on particle motion and equation (5) corresponding to the PU-signal transformation with digital electronics in feedback have been efficiently solved using Z-transform method. This method may be easily used for more complicated structure schemes in feedback systems. The simplicity of obtained solutions allows one to provide optimization of damper systems for different parameters without intricate calculations. At the same time, it is necessary to note that final choice for PU and DK positions and for filter parameters may be done after theoretical investigation of the damper system capacity for operation in the presence of different external perturbations as it has been analysed in[1].

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Жабицкий В.М. E9-91-156
Исследование классической системы подавления поперечных колебаний пучка в ускорителе с использованием Z-преобразования

Для классической системы подавления поперечных колебаний пучка в ускорителе получены уравнения, учитывающие действие корректирующего дипольного магнита и преобразование сигнала с датчика положения центра тяжести пучка в цепи обратной связи при цифровой обработке сигнала. Решения этих матричных уравнений найдены с использованием одностороннего Z-преобразования, которое широко применяется для анализа систем дискретного времени. Полученные решения проанализированы для идеальной цепи обратной связи и для цепи с фильтром типа гребенчатого. Найдены условия стабильного движения частиц пучка для таких систем подавления. Получены величины декрементов затухания поперечных колебаний для различных расстояний между датчиком положения и толкателем. Приводятся области устойчивости и графики различных зависимостей для декрементов затухания колебаний пучка в проектируемом ускорителе LHC (ЦЕРН).

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

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Zhabitsky V.M. E9-91-156
Theoretical Treatment of a Classical Transverse Feedback System Using Z-Transform

The equation including the kick effect and the equation corresponding to the pick-up signal transformation with digital electronics in feedback are obtained for a classical transverse feedback system in an accelerator. The solution of these matrix equations has been found using Z-transform to be applied widely for the analysis of discrete time systems. These solutions are analyzed for an ideal feedback and for a feedback with a notch filter. The stability criterion for such damper systems is obtained. The damping rates for different distances from pick-up to kicker are found. The stability criterion and the dependences on graphics for damping rates are shown for LHC (CERN).

The investigation has been performed at the Particle Physics Laboratory, JINR.

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