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THEORY OF ACCELERATED ORBITS IN AVF CYCLOTRONS

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INTRODUCTION

A great amount of papers is dedicated to the orbit theory in AVF cyclotrons (see e.g. /1,2/). In these papers more attention is paid to the study of particle motion about a fixed radius, assuming that the radius gain per turn during acceleration is negligibly small. Acceleration is either not studied at all, or is treated separately/3/.

Recently several papers on the general theory of accelerated orbits have appeared. We shall especially mention the paper by Hagedoorn and Corsten /4/. In what follows an attempt will be made to develop a similar formalism for direct application to AVF cyclotrons.

The motion of charged particles in AVF cyclotrons appears to be one of the most complicated in comparison with the other cyclic accelerators. Its complete description can be achieved by numerical integration of Hamilton's equations of motion in the canonical variables. Fortunately for the accelerator theorists, the role of different factors influencing the particle trajectory can be usually put by order of magnitude to some hierarchy. Consequently, taking into account more and more factors one can get more and more detailed description of the trajectory order by order. The study of particle motion "step by step" is usually well suited with experimental data and is preferable for its convenient physical meaning.

1. HAMILTONIAN FORMULATION OF THE ORBIT THEORY IN CYCLIC ACCELERATORS

It is well known /5/ that the Lagrangian of a particle with rest mass m_o and charge q, moving in electromagnetic field, defined by a scalar potential φ and vector potential $\vec{A}$ is

$$L = -m_o c^2 (1-v^2/c^2)^{1/2} + q\vec{v}.\vec{A} - q\varphi , \qquad (1.1)$$

where $\vec{v}=d\vec{r}/dt$ is the particle's velocity and c is the velocity of light in vacuum. The first step is to define the design orbit of the particle $\vec{r}_o(s)$, where s is the curve length along the design orbit. If the external field has a symmetry plane (median plane), the design orbit is a plane curve lying on it. The design orbit is completely determined by the curvature K and the centre of curvature in every point.

Define now the natural coordinate system in which the position vector $\vec{r}(s)$ is

$$\vec{r}(x,z,s) = \vec{r}_o(s) + x\vec{n}(s) + z\vec{b}(s), \tag{1.2}$$

where x is the deviation from the design orbit in the direction of the unit normal vector $\vec{n}(s)$, z - the deviation in the direction of the unit binormal vector $\vec{b}(s)$. The triple $(\vec{n},\vec{b},\vec{\tau})$ ($\vec{\tau}$ being the unit tangent vector) defines a local coordinate system along the design orbit, and satisfies the equations

$$\vec{\tau}(s)=d\vec{r}_o/ds \quad ; \quad d\vec{\tau}/ds=-K\vec{n} \quad ; \quad d\vec{n}/ds=K\vec{\tau} + \kappa\vec{b} \quad ; \quad d\vec{b}/ds=-\kappa\vec{n}$$

called the Fresnet formulae, where $\kappa(s)$ is the torsion (κ=0 for plane curves). Using the expressions

$$\vec{v} = d\vec{r}/dt = v_x\vec{n} + v_z\vec{b} + v_s(1+Kx)\vec{\tau}$$

$$(1-v^2/c^2)^{1/2}=\{1-[v_x{}^2+v_z{}^2+v_s{}^2(1+Kx)^2]/c^2\}^{1/2}$$

$$\vec{v}.\vec{A}=v_xA_x+ v_zA_z + (1+Kx)v_sA_s$$

and the relation

$$H = v_xp_x+ v_zp_z+ v_sp_s- L,$$

where

$$p_u= \partial L/\partial v_u= m_ov_u(1-v^2/c^2)^{-1/2}+ qA_u \qquad ; \qquad u=(x,z)$$

$$p_s= \partial L/\partial v_s= m_ov_s(1+Kx)^2(1-v^2/c^2)^{-1/2}+ q(1+Kx)A_s$$

we can write down the Hamiltonian H as

$$H = m_o c^2 (1-v^2/c^2)^{-1/2} + q\varphi$$

or

$$H = c\{m_o^2 c^2 + (p_x - qA_x)^2 + (p_z - qA_z)^2 + (p_s/(1+Kx) - qA_s)^2\}^{1/2} + q\varphi . \quad (1.3)$$

It is well known[6] that a new Hamiltonian can be constructed from (1.3) in the new independant variable s, so that t, H become new canonical conjugated variables.

$$H = -p_s = -(1+Kx)\{(H-q\varphi)^2/c^2 - m_o^2 c^2 - (p_x - qA_x)^2 - (p_z - qA_z)^2\}^{1/2} - (1+Kx)qA_s . \quad (1.4)$$

Suppose now that the vector potential $\vec{A}$ consists of two parts $\vec{A}_o$ the guiding magnetic field of the accelerator and $\vec{A}_a$ - the accelerating field. The vector potential $\vec{A}_a$ and the scalar potential φ obey the Lorentz gauge condition

$$\mathrm{div}\,\vec{A}_a + c^{-2}\partial\varphi/\partial t = 0 . \quad (1.5)$$

Moreover we have

$$\vec{B} = \mathrm{rot}\vec{A}_o \quad ; \quad \vec{E}_a = -\mathrm{grad}\varphi - \partial\vec{A}_a/\partial t \quad ; \quad \vec{B}_a = \mathrm{rot}\vec{A}_a . \quad (1.6)$$

The next step is to compute $\vec{A}_o$. In doing so we review the method of Teng following the paper by T.Suzuki[7]. Consider a gauge condition of the form

$$xA_{ox} + zA_{oz} = 0 .$$

We get

$$A_{ox} = -zF(x,z,s) \quad ; \quad A_{oz} = xF(x,z,s) \quad ; \quad A_{os} = G(x,z,s) , \quad (1.7)$$

where $F(x,z,s)$ and $G(x,z,s)$ are yet unknown functions. The first eq (1.6) yields

$$2F + (x\partial/\partial x + z\partial/\partial z)F = B_s$$

$$GKx(1+Kx)^{-1} + (x\partial/\partial x + z\partial/\partial z)G = zB_x - xB_z . \quad (1.8)$$

Equations (1.8) enable us to apply Euler's theorem for homogeneous terms, supposing that F and G are series of homogeneous polinomials in x and z, so that

$$F = 1/2B_s^{(0)} + 1/3B_s^{(1)} + 1/4B_s^{(2)} + \dots$$

$$G_x = (1+Kx/2)B_x^{(0)} + (1/2+Kx/3)B_x^{(1)} + (1/3+Kx/4)B_x^{(2)} + \dots$$

$$G_z = (1+Kx/2)B_z^{(0)} + (1/2+Kx/3)B_z^{(1)} + (1/3+Kx/4)B_z^{(2)} + \dots \qquad (1.8a)$$

$$G = (zG_x - xG_z)(1+Kx)^{-1},$$

where $B^{(0)}, B^{(1)}, B^{(2)}$ etc. denote homogeneous polinomials in x and z of orders 0,1,2 etc. Neglecting the beam current, Maxwell's equations for $\vec{B}$ are

$$\mathrm{rot}\,\vec{B} = 0 \quad ; \quad \mathrm{div}\,\vec{B} = 0\,. \qquad (1.9)$$

The first equation (1.9) gives the opportunity to express the field by a scalar potential ψ as

$$\vec{B} = \mathrm{grad}\,\psi \qquad (1.10)$$

and the second equation is simply the Laplace equation for ψ

$$\Delta\,\psi = 0. \qquad (1.11)$$

If we use the midplane symmetry the scalar potential is odd in z and

$$\psi = (a_o + a_1x + a_2x^2/2! + \dots)z - (b_o + b_1x + b_2x^2/2! + \dots)z^3/3! + (c_o + c_1x + \dots)z^5/5! + \\ + \dots \qquad (1.12)$$

where the coefficients are functions of s and b's, c's etc. are related to a's by equation (1.11) as

$$b_o = a_o'' + Ka_1 + a_2$$

$$b_1 = -2Ka_o'' - K'a_o' + a_1'' - K^2a_1 + Ka_2 + a_3 \qquad (1.12a)$$

$$b_2 = 6K^2a_o'' + 6KK'a_o' - 4Ka_1'' - 2K'a_1' + 2K^3a_1 + a_2'' - 2K^2a_2 + Ka_3 + a_4$$

....

$$c_o = b_o'' + Kb_1 + b_2$$

....

The prime denotes differentiation with respect to s. The coefficients a's have the following simple meaning

$$a_o = (B_z)_{x=z=o} \quad ; \quad a_1 = (\partial B_z/\partial x)_{x=z=o} \quad ; \quad a_2 = (\partial^2 B_z/\partial x^2)_{x=z=o} \dots \qquad (1.12b)$$

It is easy to see from eqs. (1.10) and (1.12) that the magnetic field can be computed if B_z evaluated in the median plane is known, so we can find the functions F and G by (1.8a) and therefore complete the calculation of $\vec{A}_o$ using (1.7).

The accelerating structure in AVF cyclotrons is usually a dee system where $\vec{A}_a=0$, and $\varphi\neq 0$. Taking the midplane symmetry into account we cast the potential φ in the form

$$\varphi(x,z,s)=A_o+A_1x+A_2x^2/2!+\ldots-(B_o+B_1x+B_2x^2/2!+\ldots)z^2/2!+(C_o+C_1x+\ldots)z^4/4. \quad (1.13)$$

If the series (1.13) is substituted in the Laplace equation (1.11) for φ, the relations between B's C's etc. and A's obtained, are exactly of the form (1.12a).

Let us introduce a canonical transformation by the generating function

$$S_2 = x\hat{p}_x + z\hat{p}_z + E\sigma + q\int\varphi(x,z,s,\sigma)d\sigma,$$

where $\sigma=-t$ is a canonical variable conjugated to H. Then

$$\hat{x} = \partial S_2/\partial\hat{p}_x = x \quad ; \quad \hat{z} = \partial S_2/\partial\hat{p}_z = z \quad ; \quad \hat{\sigma} = \partial S_2/\partial E = \sigma$$

$$p_u = \partial S_2/\partial u = \hat{p}_u - q\int E_u(x,z,s,\sigma)d\sigma = \hat{p}_u - q\tilde{E}_u \quad ; \quad u=(x,z) \quad (1.14)$$

$$H = \partial S_2/\partial\sigma = E + q\varphi(x,z,s,\sigma) = m_o\gamma c^2 + q\varphi(x,z,s,\varphi),$$

where $E_u=-\partial\varphi/\partial u$; $u=(x,z)$. The new Hamiltonian is

$$\hat{H}=-(1+Kx)\{E^2/c^2-m_o{}^2c^2-(\hat{p}_x-q\tilde{E}_x-qA_{ox})^2-(\hat{p}_z-q\tilde{E}_z-qA_{oz})^2\}^{1/2}-q(1+Kx)A_{os}-$$
$$-q(1+Kx)\tilde{E}_s \quad (1.15)$$

with the notation

$$\tilde{E}_s(x,z,s,\sigma)=\int E_s(x,z,s,\sigma)d\sigma=-(1+Kx)^{-1}\int\partial\varphi(x,z,s,\sigma)/\partial s d\sigma \quad (1.16)$$

In the new variables

$$\tilde{p}_u=\hat{p}_u/p_o=\hat{p}_u/(m_oc) \quad ; \quad u=(x,z) \quad ; \quad \tau=c\sigma \quad ; \quad \gamma=E/E_o=E/(m_oc^2) \quad (1.17)$$

the scaled Hamiltonian reads as

$$\tilde{H}=\hat{H}/p_o=-(1+Kx)\{\gamma^2-1-(\tilde{p}_x-\tilde{q}\tilde{E}_x-\tilde{q}A_{ox})^2-(\tilde{p}_z-\tilde{q}\tilde{E}_z-\tilde{q}A_{oz})^2\}^{1/2}-\tilde{q}(1+Kx)A_{os}-$$
$$-\tilde{q}(1+Kx)\tilde{E}_s \quad (1.18)$$

where $\tilde{q}=q/p_o$. The terms $\tilde{E}_x$ and $\tilde{E}_z$ are small compared with A_{ox} and A_{oz} and may be omitted. The part of $\tilde{H}$ in (1.18) depending on $\vec{A}_o$ denoted by $\tilde{H}_b$ is

$$\tilde{H}_b=\beta\gamma(1+Kx)\{-[1-(\bar{p}_x-\varepsilon A_{ox})^2-(\bar{p}_z-\varepsilon A_{oz})^2]^{1/2}-\varepsilon A_{os}\}, \quad (1.19)$$

where

$$\varepsilon=q/p=q/(p_o\beta\gamma) \quad ; \quad \bar{p}_u=\hat{p}_u/p=\hat{p}_u/(p_o\beta\gamma) \quad ; \quad u=(x,z). \qquad (1.20)$$

The relative momenta $\bar{p}_x$ and $\bar{p}_z$ and the deviations x and z are usually small, so that the square root in (1.19) may be expanded in power series in $x, \bar{p}_x, z, \bar{p}_z$. After quite tedious calculations one finds[7]

$$\tilde{H}_b=\tilde{H}_{bo}+\tilde{H}_1+\tilde{H}_2+\tilde{H}_3+\tilde{H}_4+\dots \qquad (1.21)$$

with

$$\tilde{H}_{bo}=-\beta\gamma$$

$$\tilde{H}_1=\beta\gamma(\varepsilon a_o-K)x$$

$$\tilde{H}_2=\beta\gamma(\bar{p}_x^{\,2}+\bar{p}_z^{\,2})/2 + \varepsilon\beta\gamma[(Ka_o+a_1)x^2-a_1z^2]/2$$

$$\tilde{H}_3=\beta\gamma Kx(\bar{p}_x^{\,2}+\bar{p}_z^{\,2})/2 + \varepsilon\beta\gamma a_o' z(z\bar{p}_x-x\bar{p}_z)/3 +$$

$$+ \varepsilon\beta\gamma[(Ka_1+a_2/2)x^3- (Ka_1+a_2+b_o/2)xz^2]/3 \qquad (1.22)$$

$$\tilde{H}_4=\beta\gamma(\bar{p}_x^{\,2}+\bar{p}_z^{\,2})^2/8+\varepsilon\beta\gamma(Ka_o'+3a_1')xz(z\bar{p}_x-x\bar{p}_z)/12+\varepsilon^2\beta\gamma a_o'^2z^2(x^2+z^2)/18+$$

$$+\varepsilon\beta\gamma[(Ka_2/2+a_3/6)x^4-(Ka_2+a_3/3+Kb_o/2+b_1/2)x^2z^2+b_1z^4/6]/4$$

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Let us now briefly review some basic relations from the theory of AVF cyclotrons. The isochronous field is given by[8]

$$B_{is}(R) = B(0)(1-R^2/R_\infty^{\,2})^{-1/2}, \qquad (1.23)$$

where

$$B(0)=(m_o\omega_o)/q=(A/Z)(m_p\omega_o)/e \quad ; \quad R=1/K \quad ; \quad R_\infty=c/\omega_o \qquad (1.24)$$

and ω_o is the angular frequency of motion on the design orbit, A - the mass number, Z - the charge state, m_p - the proton mass, e - the elementary charge. The curvature K of the design orbit is found from the relation

$$K=[qB_{is}(R)]/p_e \quad , \text{or} \quad K=(\beta_eR_\infty)^{-1} \qquad (1.25)$$

the p_e and β_e being the design momentum and the design relative velocity of the particle respectively. Furthermore[1]

$$(B_z)_{z=o}=B_z(R,\vartheta)=\bar{B}(R)[1+F(R,\vartheta)], \qquad (1.26)$$

where

$$\bar{B}(R) = \langle B_z(R,\vartheta)\rangle = (2\pi)^{-1}\int_0^{2\pi} B_z(R,\vartheta)d\vartheta \qquad (1.27)$$

$$F(R,\vartheta)=[B_z(R,\vartheta)-\langle B_z(R,\vartheta)\rangle]/\langle B_z(R,\vartheta)\rangle=[B_z(R,\vartheta)-\bar{B}(R)]/\bar{B}(R) \qquad (1.28)$$

and ϑ is the azimuthal angle depending on the curve length, so that

$$d\vartheta = K ds\,. \qquad (1.29)$$

The function $F(R,\vartheta)$ is periodic in ϑ and may be Fourier analized to give

$$F(R,\vartheta) = \sum_n [A_n(R)\cos n\vartheta + B_n(R)\sin n\vartheta]\,. \qquad (1.30)$$

Define now the relative change in guiding field for a deviation from a fixed radius R

$$\mu(x,\vartheta)=B_z(R+x,\ \vartheta)/\bar{B}(R) \qquad ,\text{or} \qquad B_z(R+x,\ \vartheta)=\bar{B}(R)\mu(x,\vartheta) \qquad (1.31)$$

whose expansion in Taylor series in x/R reads

$$\mu(x,\vartheta)=1+\mu' x/R+\mu''(x/R)^2/2!+\ldots+\sum_n [(A_n+A_n' x/R+A_n''(x/R)^2/2!+\ldots)\cos n\vartheta+$$

$$+(B_n+B_n' x/R+B_n''(x/R)^2/2!+\ldots)\sin n\vartheta]\,, \qquad (1.32)$$

where

$$\mu'=(R/\bar{B})d\bar{B}/dR \qquad ; \qquad \mu''=(R^2/\bar{B})d^2\bar{B}/dR^2 \qquad \text{etc.} \qquad (1.33)$$

$$G_n'=RdG_n/dR \qquad ; \qquad G_n''=R^2d^2G_n/dR^2 \qquad \text{etc.} \qquad (1.34)$$

and $G_n=(A_n,B_n)$. Note that

$$G_n'=(R/\bar{B})d(\bar{B}G_n)/dR=\mu' G_n+RdG_n/dR \qquad \text{etc.} \qquad (1.34a)$$

Usually $\mu' G_n$ is much smaller than RdG_n/dR and therefore eq. (1.34a) is transformed to eq. (1.34). In the case of N - fold symmetry the flutter profile $F(R,\vartheta)$ consists of two parts: the structural part including harmonics of the form $n=kN$ and a nonstructural part (the remainder of the sum (1.30)) which we shall treat as a perturbation.

The mean field B(R) can be expressed as

$$\bar{B}(R) = B_{is}(R)[1+\Delta B(R)/B_{is}(R)]\,, \qquad (1.35)$$

where $\Delta B(R)$ is a small perturbation to the isochronous field due to imperfections of the cyclotron magnet. Let us for convenience write down some coefficients a's from (1.12b)

$$a_o=\bar{B}(R)\mu(0,\vartheta)=B_{is}(1+\Delta B/B_{is})(1+F)$$

$$a_1=\bar{B}(R)(\partial\mu/\partial x)_{x=o}=KB_{is}(1+\Delta B/B_{is})(\mu'+F') \qquad (1.36)$$

$$a_2=\bar{B}(R)(\partial^2\mu/\partial x^2)_{x=o}=K^2B_{is}(1+\Delta B/B_{is})(\mu''+F'')\,,$$

where $F^{(n)}=R^n\partial^n F/\partial R^n$.

The entire Hamiltonian is obtained if the term $-\tilde{q}[(1+Kx)\tilde{E}_s]_{x=z=o}$ is added to $\tilde{H}_b$ in (1.21) and (1.22). We are not interested here in the focusing effect of the accelerating gap so we have dropped terms of higher order in x,z that arise from the last term in (1.18). Involving thin lense approximation for the accelerating gap we get

$$-\tilde{q}R[(1+Kx)\tilde{E}_s]_{x=z=o}=-(Z/A)[eU_d/(m_p c)](kN_o\omega_o)^{-1}f(\vartheta)\cos(kN_o\omega_o\tau/c-\varphi_o), \tag{1.37}$$

where U_d is the dee voltage, k is the harmonic acceleration mode, N_o is the number of dees, φ_o is the initial phase of RF-field, and $f(\vartheta)$ is a function determining the location of accelerating gaps, so that

$$f(\vartheta)=\sum_p \{\delta(\vartheta-\vartheta_o-2\pi p)-\delta(\vartheta-\vartheta_o-\vartheta_u-2\pi p)+\delta(\vartheta-\vartheta_o-2\pi/N_o-2\pi p)\}. \tag{1.38}$$

In equation (1.38) $\delta(x)$ is the Dirac δ-function, ϑ_o is the angle between the reference point of flutter profile and the nearest accelerating gap and ϑ_u is the dee angle.

Making profit of eqs. (1.12a), (1.17), (1.20), (1.25) and (1.36) from (1.22) we obtain

$$\tilde{H}_o=-R\beta\gamma-(Z/A)[eU_d/(m_p c)](kN_o\omega_o)^{-1}f(\vartheta)\cos(kN_o\omega_o\tau/c-\varphi_o)$$
$$\tilde{H}_1=\beta_e\gamma_e x(1+h_x)-\beta\gamma x$$
$$\tilde{H}_2=R(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})/(2\beta\gamma)+\beta_e\gamma_e(g_x x^2-g_z z^2)/(2R)$$
$$\tilde{H}_3=x(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})/(2\beta\gamma)+[\beta_e\gamma_e/(\beta\gamma)]zh_z(z\tilde{p}_x-x\tilde{p}_z)/(3R)+ \tag{1.39}$$
$$+\beta_e\gamma_e(g_{3x}x^3-g_{3z}xz^2/2)/(3R^2)$$
$$\tilde{H}_4=(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})^2/(8K\beta^3\gamma^3)+\dots$$

where

$$h_x = F + \Delta B/B_{is}$$
$$g_x = 1+\mu'+F+F'+\Delta B/B_{is}$$
$$g_z = \mu'+F'$$
$$h_z = \partial F/\partial\vartheta \tag{1.40}$$
$$g_{3x} = \mu'+F'+(\mu''+F'')/2$$
$$g_{3z} = \partial^2F/\partial\vartheta^2+K^{-1}(\partial K/\partial\vartheta)(\partial F/\partial\vartheta)+3(\mu'+\mu''+F'+F'').$$

The Hamiltonian expansion (1.39) describes the motion now in terms of a new independant variable ϑ instead of s.

2. CALCULATION OF THE DESIGN ORBIT

Let us now perform a canonical transformation with a generating function

$$F_2 = x\tilde{\tilde{p}}_x + z\tilde{\tilde{p}}_z + (\tau-\tau_e)(\tilde{\gamma}+\gamma_e) \tag{2.1}$$

so that

$$\tilde{u}=\partial F_2/\partial\tilde{\tilde{p}}_u=u \quad ; \quad \tilde{p}_u=\partial F_2/\partial u=\tilde{\tilde{p}}_u \quad ; \quad u=(x,z) \tag{2.1a}$$
$$\tilde{\tau} = \partial F_2/\partial\tilde{\gamma} = \tau-\tau_e \quad ; \quad \gamma = \partial F_2/\partial\tau = \tilde{\gamma}+\gamma_e .$$

Assuming that the quantities $\tilde{\tau}$, $\tilde{\gamma}$ are small compared with τ_e and γ_e respectively we obtain

$$d\tau_e/d\vartheta=-R/\beta_e \quad ; \quad d\gamma_e/d\vartheta=-(Z/A)[eU_d/(m_pc^2)]f(\vartheta)\sin(kN_o\omega_o\tau_e/c-\varphi_o) \tag{2.2}$$

and the transformed Hamiltonian reads as

$$\tilde{H}_o=R\tilde{\gamma}^2/(2\beta_e^3\gamma_e^3)+(Z/A)[eU_d/(m_pc^2)]kN_o(2R_\infty)^{-1}f(\vartheta)\tilde{\tau}^2\cos(kN_o\omega_o\tau_e/c-\varphi_o)$$
$$\tilde{H}_1 = \beta_e\gamma_e h_x\tilde{x} - \tilde{\gamma}\tilde{x}/\beta_e$$

$$\tilde{H}_2=R(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})/(2\beta_e\gamma_e)+\beta_e\gamma_e(g_x\tilde{x}^2-g_z\tilde{z}^2)/(2R)-R\tilde{\gamma}(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})/(2\beta_e^3\gamma_e^2)$$

$$\tilde{H}_3=\tilde{x}(\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})/(2\beta_e\gamma_e)+h_z\tilde{z}(\tilde{z}\tilde{p}_x-\tilde{x}\tilde{p}_z)/(3R)+\beta_e\gamma_e(g_{3x}\tilde{x}^3-g_{3z}\tilde{x}\tilde{z}^2/2)/(3R^2)$$

$$\tilde{H}_4 = (\tilde{p}_x^{\,2}+\tilde{p}_z^{\,2})^2/(8K\beta_e^3\gamma_e^3) + \dots \tag{2.3}$$

The solution of the first equation (2.2) is trivial and we immediately write it down

$$\tau_e = \tau_{eo}-c(\vartheta-\vartheta_o)/\omega_o . \tag{2.4}$$

Substitute now (2.4) in the second equation (2.2) and note that the expression $f(\vartheta)\sin(kN_o\omega_o\tau_e/c-\varphi_o)$ is periodic in ϑ with a period $2\pi/N_o$. Therefore

$$f(\vartheta)\sin[kN_o(\vartheta-\vartheta_o)+\tilde{\varphi}_o]=(N_o/\pi)\{\alpha_o+\sum_p[\sin\tilde{\varphi}_o\cos pN_o(\vartheta-\vartheta_o)-$$
$$-\sin(\tilde{\varphi}_o+kN_o\vartheta_u)\cos pN_o(\vartheta-\vartheta_o-\vartheta_u)]\} \tag{2.5}$$

$$\alpha_o=-\sin(kN_o\vartheta_u/2)\cos(\tilde{\varphi}_o+kN_o\vartheta_u/2) \quad ; \quad \tilde{\varphi}_o=\varphi_o-kN_o\tau_eR_\infty^{-1} . \tag{2.5a}$$

It is evident from the above equations that the maximal energy gain is achieved if $kN_o\vartheta_u=(2s+1)\pi$ for $s=0,1,2,\dots$, that is

$$k = (2s+1)\pi(N_o\vartheta_u)^{-1} . \tag{2.6}$$

In this case equation (2.5) becomes

$$f(\vartheta)\sin[kN_o(\vartheta-\vartheta_o)+\tilde{\varphi}_o]=N_o\pi^{-1}\sin\tilde{\varphi}_o\{1+2\sum_p \cos(pN_o\vartheta_u/2)\cos pN_o(\vartheta-\vartheta_o-\vartheta_u/2)\} \tag{2.7}$$

and the second equation (2.2) is written in the form

$$d\gamma_e/d\vartheta=\lambda_o[1+2\sum_p \cos(pN_o\vartheta_u/2)\cos pN_o(\vartheta-\vartheta_o-\vartheta_u/2)] , \tag{2.8}$$

where

$$\lambda_o = (N_o/\pi)(Z/A)[eU_d/(m_p c^2)]\sin\tilde{\varphi}_o \ . \qquad (2.9)$$

Integration of equation (2.8) gives the design energy

$$\gamma_e=\gamma_{eo}+\lambda_o(\vartheta-\vartheta_o)+2\lambda_o\sum_p\cos(pN_o\vartheta_u/2)/(pN_o)[\sin pN_o(\vartheta-\vartheta_o-\vartheta_u/2)+\sin(pN_o\vartheta_u/2)] \ . \qquad (2.10)$$

In the case of $\vartheta_u=\pi/N_o$ eqs. (2.8) and (2.10) are further simplified to give

$$d\gamma_e/d\vartheta = \lambda_o[1+2\sum_p \cos 2pN_o(\vartheta-\vartheta_o)] \qquad (2.8a)$$

$$\gamma_e=\gamma_{eo}+\lambda_o(\vartheta-\vartheta_o)+\lambda_o\sum_p (pN_o)^{-1}\sin 2pN_o(\vartheta-\vartheta_o) \ . \qquad (2.10a)$$

3. INTRODUCTION OF "QUASIEQUILIBRIUM ORBIT" AND DISPERSION

Firstly we must note that the term "quasiequilibrium orbit" may not be the most suited and one can argue about it.

Let us now perform a second canonical transformation with a generating function

$$E_2 = (\tilde{x}-\tilde{x}_o)(\tilde{\tilde{p}}_x+\tilde{p}_o)+\tilde{z}\tilde{\tilde{p}}_z+\tilde{\tau}\tilde{\tilde{\gamma}} \qquad (3.1)$$

aiming to cancel the term $\beta_e\gamma_e h_x\tilde{x}$ in $\tilde{H}_1$. The quantity $\tilde{x}_o$ we call the "quasiequilibrium orbit" and $\tilde{p}_o$ the "quasiequilibrium momentum". Using the relations

$$\tilde{x} = \tilde{\tilde{x}} + \tilde{x}_o \quad ; \quad \tilde{p}_x= \tilde{\tilde{p}}_x+ \tilde{p}_o \qquad (3.2a)$$

$$\tilde{z} = \tilde{\tilde{z}} \quad ; \quad \tilde{p}_z= \tilde{\tilde{p}}_z \quad ; \quad \tilde{\tau} = \tilde{\tilde{\tau}} \quad ; \quad \tilde{\gamma} = \tilde{\tilde{\gamma}} \qquad (3.2b)$$

it is easy to get

$$d^2\tilde{x}_o/d\vartheta^2+\gamma_e^{-1}(d\gamma_e/d\vartheta)(d\tilde{x}_o/d\vartheta)+g_x\tilde{x}_o=-Rh_x \quad ; \quad \tilde{p}_o=(\beta_e\gamma_e/R)(d\tilde{x}_o/d\vartheta) \qquad (3.3)$$

and

$$\tilde{\tilde{H}}_o=\tilde{H}_o-\tilde{x}_o\tilde{\gamma}/\beta_e$$

$$\tilde{\tilde{H}}_1=-\tilde{\gamma}\tilde{\tilde{x}}/\beta_e \qquad (3.4)$$

$$\tilde{\tilde{H}}_2=R(\tilde{\tilde{p}}_x^{\,2}+\tilde{\tilde{p}}_z^{\,2})/(2\beta_e\gamma_e)+\beta_e\gamma_e(g_x\tilde{\tilde{x}}^2-g_z\tilde{\tilde{z}}^2)/(2R)+\ldots$$

....

The last canonical transformation is given by the generating function

$$G_2=\hat{p}_x(\tilde{\tilde{x}}-\hat{\gamma}D)+(\beta_e\gamma_e/R)(dD/d\vartheta)\tilde{\tilde{x}}\hat{\gamma}-\beta_e\gamma_e(2R)^{-1}D(dD/d\vartheta)\hat{\gamma}^2+\tilde{\tilde{z}}\hat{p}_z+\tilde{\tilde{\tau}}\hat{\gamma} \ . \qquad (3.5)$$

Noting that

$$\tilde{\tilde{x}}=\hat{x}+\hat{\gamma}D \quad ; \quad \tilde{\tilde{p}}_x=\hat{p}_x+(\beta_e\gamma_e/R)(dD/d\vartheta)\hat{\gamma} \quad ; \quad \tilde{\tilde{z}}=\hat{z} \quad ; \quad \tilde{\tilde{p}}_z=\hat{p}_z \qquad (3.5a)$$

$$\tilde{\tilde{\tau}}=\hat{\tau}+\hat{p}_xD-(\beta_e\gamma_e/R)(dD/d\vartheta)\hat{x} \quad ; \quad \tilde{\tilde{\gamma}}=\hat{\gamma} \qquad (3.5b)$$

we have

$$d^2D/d\vartheta^2+\gamma_e^{-1}(d\gamma_e/d\vartheta)(dD/d\vartheta)+g_xD=R(\beta_e^2\gamma_e)^{-1} \qquad (3.6)$$

$$\hat{H}_o=\tilde{\tilde{H}}_o-D(2\beta_e)^{-1}\hat{\gamma}^2 \qquad (3.7)$$

$$\hat{H}_2=R(\hat{p}_x^2+\hat{p}_z^2)/(2\beta_e\gamma_e)+\beta_e\gamma_e(g_x\hat{x}^2-g_z\hat{z}^2)/(2R)+\ldots$$

....

The quantity D is called the dispersion function of the cyclotron.

4. CALCULATION OF THE "QUASIEQUILIBRIUM ORBIT" AND DISPERSION

The quantity λ_o in (2.9) is quite small so that $(d\gamma_e/d\vartheta)/\gamma_e \ll 1$ for a wide range of energies and we can explore the quasistatic approximation. This means that instead of R in eqs. (3.3) and (3.6) we take the mean value of R:

$$\langle R\rangle=R_o[1+\pi\lambda_o(\beta_{eo}^2\gamma_{eo}^3)^{-1}]. \qquad (4.1)$$

The condition for the gap-crossing resonance/3/ is

$$2pN_o - mN = \pm 1, \qquad (4.2)$$

where p and m are some arbitrary integers. It is clear that if the number of sectors N is even the eq. (4.2) is no longer valid. In such a way we exclude the gap-crossing resonance and do not consider its effects here.

Introduce a new independant variable ϕ and a new coordinate y_o, defined by:

$$\tilde{x}_o(\vartheta)=w_o(\vartheta)y_o(\phi) \quad ; \quad \phi = \psi_o(\vartheta)/\nu_o , \qquad (4.3)$$

where

$$d^2w_o/d\vartheta^2+g_xw_o=w_o^{-3} \quad ; \quad d\psi_o/d\vartheta=w_o^{-2} \quad ; \quad \nu_o=(2\pi)^{-1}\int^{2\pi} w_o^{-2}(\vartheta)d\vartheta. \qquad (4.4)$$

With these relations in hand eq. (3.3) is transformed as

$$d^2y_o/d\phi^2+\nu_o^2y_o=-\langle R\rangle\nu_o^2h_xw_o^3- -(d\gamma_e/d\vartheta)[\nu_o^2w_o^3(dw_o/d\vartheta)y_o+\nu_ow_o^3(dy_o/d\phi)]/\gamma_e. \qquad (4.5)$$

First of all we must find the solution of the first of eqs. (4.4). It is easy to obtain:

$$w_o(\vartheta)=1+w_{o1}(\vartheta)+w_{o2}(\vartheta)+\ldots \qquad (4.6)$$

$$w_{o1}(\vartheta)=\sum_k (k^2N^2-4)^{-1}(\bar{A}_{kN}\cos kN\vartheta+\bar{B}_{kN}\sin kN\vartheta) \quad ; \quad \bar{G}_{kN}=G_{kN}+G_{kN}' \qquad (4.6a)$$

$$w_{o2}=-\{\mu'+\Delta B/B_{is}-3\sum_k(k^2N^2-4)^{-2}(\bar{A}_{kN}^2+\bar{B}_{kN}^2)+(1/2)\sum_k(k^2N^2-4)^{-1}(\bar{A}_{kN}^2+\bar{B}_{kN}^2)\}/4 \qquad (4.6b)$$

$$\nu_o=1+(\mu'+\Delta B/B_{is})/2+(1/4)\sum_k(k^2N^2-4)^{-1}(\bar{A}_{kN}{}^2+\bar{B}_{kN}{}^2), \tag{4.6c}$$

$$\psi_o \simeq \nu_o\vartheta \quad ; \quad \text{or} \quad \phi \simeq \vartheta \tag{4.6d}$$

Inserting the formulae (4.6) into equation (4.5) one can find that

$$y_o/\langle R\rangle=\nu_o{}^{1/2}\sum_k(k^2N^2-\nu_o{}^2)^{-1}[A_{kN}\cos kN\vartheta+B_{kN}\sin kN\vartheta]-$$

$$-[\nu_o{}^{-3/2}\Delta B/B_{is}+(3/2)\sum_k(k^2N^2-4)^{-1}(A_{kN}\bar{A}_{kN}+B_{kN}\bar{B}_{kN})].$$

Turning back to the equation (4.3) we have

$$\tilde{x}_o/\langle R\rangle=\nu_o{}^{1/2}\sum_k(k^2N^2-\nu_o{}^2)^{-1}[A_{kN}\cos kN\vartheta+B_{kN}\sin kN\vartheta] - \tilde{x}_{o2}/\langle R\rangle, \tag{4.7}$$

where

$$\tilde{x}_{o2}/\langle R\rangle=\nu_o{}^{-3/2}\Delta B/B_{is}+(3/2)\sum_k(k^2N^2-4)^{-1}(A_{kN}\bar{A}_{kN}+B_{kN}\bar{B}_{kN})-$$

$$-(\nu_o{}^{1/2}/2)\sum_k(k^2N^2-\nu_o{}^2)^{-1}(k^2N^2-4)^{-1}(A_{kN}\bar{A}_{kN}+B_{kN}\bar{B}_{kN}). \tag{4.8}$$

The dispersion D is calculated exactly in the same manner, repeating the above considerations and therefore

$$D/\langle R\rangle=\nu_o{}^{-3/2}[\langle\beta_e\rangle^2\langle\gamma_e\rangle]^{-1}(1+w_{o1}+\dots). \tag{4.9}$$

5. CALCULATION OF THE BETATRON FREQUENCES

The Hamiltonian describing the betatron oscillations may be obtained in the following way. The terms quadratic in $\hat{x}$, $\hat{p}_x$, $\hat{z}$, $\hat{p}_z$ encountered in $\tilde{H}_3$ from (2.3), after the substitution $\tilde{x}=\hat{x}+\tilde{x}_o$, $\tilde{p}_x=\hat{p}_x+\tilde{p}_o$ are added to $\hat{H}_2$ from (3.7). The result is:

$$\hat{H}_b=\sum_u\{(2\beta_e\gamma_e)^{-1}RF_u\hat{p}_u{}^2+R_u\hat{u}\hat{p}_u+(2R)^{-1}\beta_e\gamma_eG_u\hat{u}^2\} \quad ; \quad u=(x,z) \tag{5.1}$$

where

$$F_x=1+\tilde{x}_o/R+3(d\tilde{x}_o/d\vartheta)^2(2R^2)^{-1} \quad ; \quad R_x=(d\tilde{x}_o/d\vartheta)/R \quad ; \quad G_x=g_x+2R^{-1}g_{3x}\tilde{x}_o$$

$$F_z=1+\tilde{x}_o/R+(d\tilde{x}_o/d\vartheta)^2(2R^2)^{-1} \quad ; \quad R_z=h_z\tilde{x}_o(3R)^{-1} \tag{5.2}$$

$$G_z=2(3R)^{-1}h_z(d\tilde{x}_o/d\vartheta)-g_z-(3R)^{-1}g_{3z}\tilde{x}_o.$$

In order to cast the Hamiltonian (5.1) in a canonical form we introduce a canonical transformation [1] whose generating function is:

$$F_2=\sum_u\{F_u{}^{-1/2}\hat{u}\bar{p}_u+(\beta_e\gamma_e/R)[(4F_u{}^2)^{-1}(dF_u/d\vartheta)-(2F_u)^{-1}R_u]\hat{u}^2\} \quad ; \quad u=(x,z)$$

The above canonical transformation cancels the cross term $R_u\hat{u}\hat{p}_u$ and the result is:

$$\hat{u}=F_u{}^{-1/2}\bar{\bar{u}} \quad ; \quad \hat{p}_u=F_u{}^{-1/2}\bar{\bar{p}}_u+(\beta_e\gamma_e/R)[(2F_u{}^2)^{-1}(dF_u/d\vartheta)-R_u/F_u]F_u{}^{-1/2}\bar{\bar{u}} \tag{5.3}$$

$$\overline{\overline{H}}_b=\sum_u [(2\beta_e\gamma_e)^{-1}R\overline{\overline{p}}_u^{\,2}+(2R)^{-1}\beta_e\gamma_e\hat{g}_u(\vartheta)\overline{\overline{u}}^2] + \Delta\overline{\overline{H}}_b \quad ; \quad u=(x,z) \qquad (5.4)$$

where

$$\hat{g}_u(\vartheta)=(2F_u)^{-1}(d^2F_u/d\vartheta^2)-3(4F_u^{\,2})^{-1}(dF_u d\vartheta)^2-R_u^{\,2}+F_uG_u+$$
$$+[R_u(dF_u/d\vartheta)-F_u(dR_u/d\vartheta)]/F_u \qquad (5.5)$$

$$\Delta\overline{\overline{H}}_b=\sum (2R)^{-1}\beta_e(d\gamma_e/d\vartheta)[(2F_u)^{-1}(dF_u/d\vartheta)-R_u]\overline{\overline{u}}^2. \qquad (5.6)$$

Introduce now the action-angle variables $(J_u,\ \alpha_u)$ by

$$S_1=\sum_u (2R)^{-1}\beta_e\gamma_e\overline{\overline{u}}^2\{w_u(dw_u/d\vartheta)-w_u^{\,-2}tg(\alpha_u+\psi_u-\nu_u\vartheta)\} \qquad (5.7)$$

$$\overline{\overline{u}}=(2RJ_u)^{1/2}(\beta_e\gamma_e)^{-1/2}w_u\cos(\alpha_u+\psi_u-\nu_u\vartheta)$$

$$(5.7a)$$

$$\overline{\overline{p}}_u=(2J_u\beta_e\gamma_e/R)^{1/2}[(dw_u/d\vartheta)\cos(\alpha_u+\psi_u-\nu_u\vartheta)-w_u^{\,-1}\sin(\alpha_u+\psi_u-\nu_u\vartheta)]$$

where [see eqs. (4.4)]

$$d^2w_u/d\vartheta^2+g_uw_u=w_u^{\,-3} \quad ; \quad d\psi_u/d\vartheta=w_u^{\,-2} \quad ; \quad \nu_u=(2\pi)^{-1}\int^{2\pi} w_u^{\,-2}(\vartheta)d\vartheta. \qquad (5.8)$$

The new Hamiltonian H_b is written as

$$H_b = H_{bo} + \Delta H_b \qquad (5.9)$$

$$H_{bo} = \sum_u \nu_uJ_u \qquad (5.9a)$$

$$\Delta H_b=\gamma_e^{\,-1}(d\gamma_e/d\vartheta)\sum_u J_u\{[w_u(dw_u/d\vartheta)+w_u^{\,2}((2F_u)^{-1}(dF_u/d\vartheta)-R_u)]*$$
$$*\cos^2(\alpha_u+\psi_u-\nu_u\vartheta)-(1/2)\sin2(\alpha_u+\psi_u-\nu_u\vartheta)\}. \qquad (5.9b)$$

The solution of the first of eqs. (5.8) with regard of

$$\hat{g}_x=1+\sum_k (\widetilde{A}_{kN}\cos kN\vartheta+\widetilde{B}_{kN}\sin kN\vartheta)+\hat{g}_{x2} \qquad (5.10)$$

$$\hat{g}_{x2}=\mu'+(5/8)\sum_k k^2N^2(k^2N^2-1)^{-2}(A_{kN}^{\,2}+B_{kN}^{\,2})+$$

$$+(1/2)\sum_k(k^2N^2-1)^{-1}(A_{kN}\overline{\overline{A}}_{kN}+B_{kN}\overline{\overline{B}}_{kN})-\sum k^2N^2[(k^2N^2-1)(k^2N^2-4)]^{-1}*$$
$$(A_{kN}\overline{A}_{kN}+B_{kN}\overline{B}_{kN}) \qquad (5.10a)$$

$$\widetilde{A}_{kN}=\overline{A}_{kN}+(k^2N^2+2)(k^2N^2-1)^{-1}A_{kN}/2 \quad ; \quad \overline{A}_{kN}=A_{kN}+A_{kN}' \qquad (5.10b)$$

$$\overline{\overline{A}}_{kN} = 2A_{kN}'+ A_{kN}'' \qquad (5.10c)$$

is found to be

$$w_x = 1 + w_{x1}+ w_{x2}+ \dots \qquad (5.11)$$

where

$$w_{x1}=\sum_k (k^2N^2-4)^{-1}(\widetilde{A}_{kN}\cos kN\vartheta+\widetilde{B}_{kN}\sin kN\vartheta) \qquad (5.12)$$

$$w_{x2}=-(1/8)\sum_k(k^2N^2-10)(k^2N^2-4)^{-2}(\tilde{A}_{kN}{}^2+\tilde{B}_{kN}{}^2)-g_{x2}/4 . \qquad (5.13)$$

For the betatron frequency ν_x given by the third of eqs. (5.8) we find

$$\nu_x=1+(1/4)\sum_k(k^2N^2-4)^{-1}(\tilde{A}_{kN}{}^2+\tilde{B}_{kN}{}^2)+g_{x2}/2. \qquad (5.14)$$

Repeating the above considerations for the z-coordinate it is easy to obtain

$$\hat{g}_z=-\sum_k(\tilde{\tilde{A}}_{kN}\cos kN\vartheta+\tilde{\tilde{B}}_{kN}\sin kN\vartheta)+\hat{g}_{z2} , \qquad (5.15)$$

where

$$\tilde{\tilde{A}}_{kN} = A_{kN}'+k^2N^2(k^2N^2-1)^{-1}A_{kN}/2 \qquad (5.15a)$$

$$\hat{g}_{z2}=-\mu'-(1/2)\sum_k(k^2N^2-1)^{-1}(A_{kN}\bar{\bar{A}}_{kN}+B_{kN}\bar{\bar{B}}_{kN})-(1/8)\sum_k k^2N^2(k^2N^2-1)^{-2}*$$

$$(A_{kN}{}^2+B_{kN}{}^2)+(1/2)\sum_k k^2N^2(k^2N^2-1)^{-1}(A_{kN}{}^2+B_{kN}{}^2) \qquad (5.15b)$$

and

$$w_z = A_o[1-\sum_k k^{-2}N^{-2}(\tilde{\tilde{A}}_{kN}\cos kN\vartheta+\tilde{\tilde{B}}_{kN}\sin kN\vartheta)] \qquad (5.16)$$

$$A_o = [\hat{g}_{z2}+(1/2)\sum_k k^{-2}N^{-2}(\tilde{\tilde{A}}_{kN}{}^2+\tilde{\tilde{B}}_{kN}{}^2)]^{-1/4} . \qquad (5.17)$$

For the betatron frequency ν_z one finds

$$\nu_z = A_o{}^{-2} . \qquad (5.18)$$

6. THE PHASE MOTION

The phase motion (i.e. the motion in $\hat{\tau}$, $\hat{\gamma}$ variables) is governed by $\hat{H}_o$ from equations (3.7). Noting that

$$D = (\beta_e{}^2\gamma_e)^{-1}R\alpha_M , \qquad (6.1)$$

where α_M is the momentum compaction factor, and

$$\eta_M = (p_e/\omega_o)(d\omega_o/dp_e) = \gamma_e{}^{-2}-\alpha_M \qquad (6.2)$$

which is equal to zero ($\eta_M=0$) for isochronous cyclotrons, we obtain

$$\hat{H}_o=-\beta_e{}^{-1}\tilde{x}_o\hat{\gamma}+(Z/A)[eU_d/(m_pc^2)](2R_\infty)^{-1}kN_o\hat{\tau}^2f(\vartheta)\cos(kN_o\omega_o\tau_e/c-\varphi_o) \qquad (6.3)$$

With equations (2.5) and (2.6) in hand the Hamiltonian H_o given by the last equation is transformed to give

$$\hat{H}_o=-\beta_e{}^{-1}\tilde{x}_o\hat{\gamma}+\lambda_o\text{ctg}\tilde{\varphi}_o(2R_\infty)^{-1}kN_o\{1+2\sum_p\cos(pN_o\vartheta_u/2)\cos pN_o(\vartheta-\vartheta_o-\vartheta_u/2)\}\hat{\tau}^2. \qquad (6.4)$$

The Hamilton equations following from (6.4) are

$$d\hat{\tau}/d\vartheta = -\beta_e{}^{-1}\tilde{x}_o \qquad (6.5a)$$

$$d\hat{\gamma}/d\vartheta=-\lambda_o ctg\tilde{\varphi}_o(kN_o/R_\infty)\{1+2\sum_p \cos(pN_o\vartheta_u/2)\cos pN_o(\vartheta-\vartheta_o-\vartheta_u/2)\}\hat{\tau} \qquad (6.5b)$$

The phase motion is given by the solution of equation (6.5a) in the form

$$\hat{\tau}=\hat{\tau}_o+(R_\infty\tilde{x}_{o2}/\langle R\rangle)(\vartheta-\vartheta_o)-R_\infty\nu_o^{1/2}\sum_k [kN(k^2N^2-\nu_o^2)]^{-1} * \\ *\{A_{kN}[\sin kN\vartheta-\sin kN\vartheta_o]+B_{kN}[\cos kN\vartheta_o-\cos kN\vartheta]\}, \qquad (6.6)$$

CONCLUDING REMARKS

An attempt is made to apply a fully six-dimensional Hamiltonian formalism to the analysis of accelerated orbits in AVF cyclotrons. It should be mentioned (although it is quite obvious) that slight modifications are needed for the present theory to be applicable to spiral ridged cyclotrons. In the central region, however, the quantity Kx is not small, so that equations (1.21) and (1.22) are not valid. This disadvantage is overcome by a reasonable compromise of simplicity.

We do not discuss here the effects of adiabatic damping and adiabatic change of betatron frequences [see eq. (5.9)]. All the nonlinear effects are not studied too. We intend to treat them in a future publication.

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