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E9-90-520

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HIGH-FREQUENCY HEATING OF ELECTRONS IN ECR ION SOURCES

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1990

I. INTRODUCTION

ECR ion sources use a magnetic mirror geometry to trap charged particles. It is well known /1/, that the motion of a charged particle in a simple mirror can be decomposed in three parts: i) Larmor rotation around the guidung centre, placed on a field line; ii) azimuthal drift of the guiding centre, caused by a radial gradient of the field; iii) longitudinal oscillation along a field line, by reflection between the "mirror points". This fact is briefly reviewed in Sec. III.

Unfortunately, plasma confinement in a simple mirror geometry is unstable. To improve stability multipole fields are commonly used. The effect of sextupole field is considered in the present paper.

The purpose of this paper is to calculate the energy gain of an electron when it passes through a resonance zone. It was shown in our recent paper /2/, that the amplitude growth in the case of fast passing through resonance is inversely proportional to the square root of the passing rate at exact resonance. In Sec. IV we just apply a formula derived in /2/, and study the effect of sextupole field on plasma confinement.

II. THE MAGNETIC FIELD AND THE RF FIELD OF ECR ION SOURCE

The static magnetic field of an ECR ion source is rotationally symmetric, so that in cylindrical coordinates the magnetic vector potential $\vec{A}_o$ has only one nonzero component – $A_{o\varphi}$. Substituting the expression

$$A_{o\varphi}(r,z)=\sum_{n=o}^{\infty}a_n(z)r^{n+\lambda}$$

into the equation

$$\nabla^2A_{o\varphi}=\frac{\partial}{\partial r}\left[\frac{1}{r}\;\frac{\partial(rA_{o\varphi})}{\partial r}\right]+\frac{\partial^2A_{o\varphi}}{\partial z^2}=0 \qquad (2.1)$$

it is easy to obtain λ=1 and

$$a_{2n}(z)=\frac{(-1)^n B_{zo}^{(2n)}(z)}{2^{2n+1}n!(n+1)!} \quad ; \quad a_{2n+1}(z)=0$$

where $B_{zo}(z)$ is the axial component of magnetic field evaluated along the axis of the machine, and upper index denotes differentiation with respect to z. Therefore one has

$$A_{o\varphi}(r,z)=\sum_{n=o}^{\infty}\frac{(-1)^n B_{zo}^{(2n)}(z)}{2^{2n+1}n!(n+1)!}\, r^{2n+1}. \qquad (2.2)$$

Let us define now the magnetic field $B_{zo}(z)$ along the axis by the expression

$$B_{zo}(z)=B_o\left(1+\mu\,\frac{4z^2}{L^2}\right) \qquad (2.3)$$

where

$$\mu=\frac{B_{max}}{B_o}-1 \quad ; \quad B_{max}=B_{zo}\left(\pm\frac{L}{2}\right) \qquad (2.4)$$

and L is the length of the system (the distance between the "mirror points"). The quantity μ usually satisfies the inequality $0<\mu<1$.

Next we calculate the magnetic vector potential $\vec{A}_m$ of the multipole magnetic field, used for plasma confinement. Noting that the only nonzero component is $A_{mz}(r,\varphi)$ one immediately obtains the solution of the equation

$$\nabla^2 A_{mz}=r\frac{\partial}{\partial r}\left(r\frac{\partial A_{mz}}{\partial r}\right)+\frac{\partial^2 A_{mz}}{\partial\varphi^2}=0 \qquad (2.5)$$

in the form

$$A_{mz}(r,\varphi)=\sum_{m=1}^{\infty}G_m r^m\cos(m\varphi+\psi_m). \qquad (2.6)$$

For a pure multipole field of multiplicity m the coefficient G_m is given by

$$G_m=\frac{[B_{mr}^2(r_p)+B_{m\varphi}^2(r_p)]^{1/2}}{m r_p^{m-1}}=\frac{B_\perp(r_p)}{m r_p^{m-1}} \qquad (2.7)$$

where r_p is the pole radius of multipole magnet. Here we study the effect of sextupole magnetic field on plasma confinement.

Further we assume that the RF field is a TE_o mode. The electric field has only an $E_\varphi^{rf}(r,z,t)$ component, which can be written in the form[3]

$$E_\varphi^{rf}(r,z,t)=\frac{\omega E_o}{k_c}J_1(k_c r)\cos kz\,\cos(\omega t+\psi_o) \qquad (2.8)$$

where

$$k_c^2=\frac{\omega^2}{c^2}-k^2=\frac{\nu_{1n}^2}{r_{res}^2} \qquad (2.9)$$

$J_n(w)$ is a Bessel function of the first kind, r_{res} is the resonator radius, E_o is the amplitude of electric field and ν_{1n} is the n-th root of $J_1(w)$ [i.e. $J_1(\nu_{1n})=0$]. This field can be derived from a vector potential

$$A_\varphi^{rf}(r,z,t)=-\frac{E_o}{k_c}J_1(k_c r)\cos kz\,\sin(\omega t+\psi_o). \qquad (2.10)$$

The general vector potential should be a sum of (2.2), (2.6) and (2.10) so that $\vec{A}=(0,A_\varphi,A_z)$ where

$$A_\varphi=A_{o\varphi}+A_\varphi^{rf}=\frac{B_o r}{2}+\frac{2\mu B_o z^2}{L^2}r-\frac{\mu B_o r^3}{2L^2}-\frac{E_o}{k_c}J_1(k_c r)\cos kz\,\sin(\omega t+\psi_o) \qquad (2.11a)$$

$$A_z=A_{3z}=\frac{B_\perp r^3}{3r_p^2}\cos(3\varphi+\psi_3). \qquad (2.11b)$$

III. HAMILTONIAN FORMULATION OF MULTIPERIODIC PARTICLE MOTION IN A MIRROR FIELD

The non-relativistic Hamiltonian function of a particle of charge q and mass m, moving in the field (2.11) is:

$$H=\frac{p_r^2}{2m}+\frac{1}{2m}\left(\frac{p_\varphi}{r}-qA_\varphi\right)^2+\frac{1}{2m}(p_z-qA_z)^2. \qquad (3.1)$$

Let us introduce a canonical transformation, given by the generating function[4]

$$F_1(x,y,X,Y)=\left[\mathrm{sgn}(q)\alpha yY-\frac{Y^2}{2}-\frac{\alpha^2y^2}{2}\right]\mathrm{tg}X-\alpha xY+\mathrm{sgn}(q)\frac{\alpha^2}{2}xy \qquad (3.2)$$

where

$$\mathrm{sgn}(q)=\frac{q}{|q|} \quad ; \quad \alpha=(m\omega_o)^{1/2} \quad ; \quad \omega_o=\frac{|q|B_o}{m}. \qquad (3.3)$$

This canonical transformation represents the particle motion in cartesian coordinates x,y with canonical momenta p_x, p_y in terms of guiding centre motion given by

$$x=\frac{1}{\alpha}[P_y+\mathrm{sgn}(q)(2P_x)^{1/2}\sin X] \quad ; \quad y=\frac{1}{\alpha}[\mathrm{sgn}(q)Y+(2P_x)^{1/2}\cos X] \qquad (3.4a)$$

$$p_x=\frac{\alpha}{2}[-Y+\mathrm{sgn}(q)(2P_x)^{1/2}\cos X] \quad ; \quad p_y=\frac{\alpha}{2}[\mathrm{sgn}(q)P_y-(2P_x)^{1/2}\sin X]. \qquad (3.4b)$$

Defining new canonical variables q_1, q_2, P_1, P_2 by

$$Y=(2p_2)^{1/2}\sin q_2 \quad ; \quad P_y=(2p_2)^{1/2}\cos q_2 \qquad (3.5a)$$

$$X=q_1 \quad ; \quad P_x=P_1 \qquad (3.5b)$$

and making use of the relations

$$r^2=x^2+y^2 \quad ; \quad \phi=\mathrm{arctg}\frac{y}{x} \quad ; \quad p_r=\frac{1}{r}(xp_x+yp_y) \quad ; \quad p_\varphi=xp_y-yp_x \qquad (3.6)$$

one obtains

$$r^2=\frac{2}{\alpha^2}[p_1+p_2+2\,\mathrm{sgn}(q)(p_1p_2)^{1/2}\sin(q_1+q_2)] \qquad (3.7a)$$

$$rp_r=2\,\mathrm{sgn}(q)(p_1p_2)^{1/2}\cos(q_1+q_2) \qquad (3.7b)$$

$$\varphi=\mathrm{arctg}\,\frac{(2p_1)^{1/2}\cos q_1+\mathrm{sgn}(q)(2p_2)^{1/2}\sin q_2}{\mathrm{sgn}(q)(2p_1)^{1/2}\sin q_1+(2p_2)^{1/2}\cos q_2} \qquad (3.7c)$$

$$p_\varphi=\mathrm{sgn}(q)(p_2-p_1). \qquad (3.7d)$$

In order to proceed further we suggest two assumptions. First we presume that $(p_1/p_2)^{1/2}\ll 1$, which implies that the particle Larmor radius is much smaller than its guiding centre radius. This condition is satisfied for particles apart from a small region near the axis, and from (3.7c) one finds approximately $\varphi=\mathrm{sgn}(q)q_2$. The second assumption is that the effect of RF field and sextupole can be treated as perturbation and only terms of first order in E_o and $B_\perp$ are kept in the Hamiltonian (3.1).

Insert now expressions (3.7) into eq. (3.1) and average over fast oscillations arround the guiding centre. The new Hamiltonian is:

$$H=H_o+\varepsilon H_{rf}+\varepsilon H_s \qquad (3.8)$$

where

$$H_o=\frac{p_z^2}{2m}+\omega_o\left(1+\mu\frac{4z^2}{L^2}+\mu^2\frac{4z^4}{L^4}\right)p_1-\frac{2\mu}{mL^2}(p_1^2+2p_1p_2)+\mu^2\frac{4z^4}{L^4}\omega_o p_2+$$
$$+\frac{\mu^2}{mL^2}\left[\frac{p_1^3+p_2^3+9p_1p_2(p_1+p_2)}{m\omega_o L^2}-(p_1^2+p_2^2+4p_1p_2)\frac{4z^2}{L^2}\right] \qquad (3.9a)$$

$$H_{rf}=-2\omega_e\,\mathrm{sgn}(q)\left[\frac14+\mu\frac{z^2}{L^2}-\frac{\mu}{m\omega_o L^2}(p_1+p_2)\right](p_1p_2)^{1/2}\cos kz\,\cos(q_1+q_2-\omega t-\psi_o) \qquad (3.9b)$$

$$H_s=-p_z\,\frac{\omega_\perp r^3}{3r_p^2}\,\mathrm{sgn}(q)\cos[3\,\mathrm{sgn}(q)q_2+\psi_3] \qquad (3.9c)$$

$$\omega_e=\frac{|q|\mathcal{E}_o}{m} \quad ; \quad \omega_\perp=\frac{|q|B_\perp}{m} \quad ; \quad \mathcal{E}_o=E_o\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+1)!}\left(\frac{k_c r}{2}\right)^{2k}\approx \mathrm{const} \qquad (3.10)$$

and ε is a formal parameter, serving as account of perturbation.

In order to complete the multiperiodic representation of particle motion in the mirror field we perform one more canonical transformation given by /5/

$$F_2(q_1,q_2,z,P_1,P_2,P_3)=q_1P_1+q_2P_2+m\omega_z\int dz\left(\frac{2P_3}{m\omega_z}-z^2\right)^{1/2} \qquad (3.11)$$

$$Q_1=q_1+\frac{P_3}{4P_1}\sin 2Q_3 \quad ; \quad p_1=P_1 \quad ; \quad Q_2=q_2 \quad ; \quad p_2=P_2 \qquad (3.12a)$$

$$z=\left(\frac{2P_3}{m\omega_z}\right)^{1/2}\sin Q_3 \quad ; \quad p_z=(2m\omega_zP_3)^{1/2}\cos Q_3 \qquad (3.12b)$$

where

$$\omega_z=\frac{2}{L}\left(\frac{2\mu\omega_oP_1}{m}\right)^{1/2}. \qquad (3.13)$$

The transformed Hamiltonian averaged over fast longitudinal oscillations reads as

$$\mathcal{H}_o=\omega_oP_1+\omega_zP_3+\frac{2\mu}{mL^2}\left(1+\frac{P_2}{P_1}\right)P_3^2\sin^4Q_3-\frac{2\mu}{mL^2}(P_1^2+2P_1P_2)-\frac{\mu\omega_z}{m\omega_oL^2}\left(P_1+4P_2+\frac{P_2^2}{P_1}\right)P_3\sin^2Q_3+$$
$$+\frac{\mu^2}{m^2\omega_oL^4}[P_1^3+P_2^3+9P_1P_2(P_1+P_2)] \qquad (3.14a)$$

$$\mathcal{H}_{rf}=-2\omega_e\left[\frac14-\frac{\mu(P_1+P_2)}{m\omega_oL^2}+\frac{2\mu P_3}{m\omega_zL^2}\sin^2Q_3\right]\mathrm{sgn}(q)\sqrt{P_1P_2}\,f(P_1,P_3)*$$
$$*\cos(Q_1+Q_2-\omega t-\psi_o) \qquad (3.14b)$$

$$\mathcal{H}_s=-\frac{2\omega_\perp}{3m\omega_or_p^2}\left(\frac{\omega_z}{\omega_o}\right)^{1/2}P_2\sqrt{P_2P_3}\,\mathrm{sgn}(q)*$$
$$*\{\cos[3\,\mathrm{sgn}(q)Q_2+Q_3+\psi_3]+\cos[3\,\mathrm{sgn}(q)Q_2-Q_3+\psi_3]\} \qquad (3.14c)$$

where

$$f(P_1,P_3)=\sum_{s=-\infty}^{\infty}J_{4s}\left(k\frac{\sqrt{2P_3}}{\sqrt{m\omega_z}}\right)J_{2s}\left(\frac{P_3}{4P_1}\right). \qquad (3.15)$$

The next step is to introduce resonant canonical variables $a_i,\tilde{P}_i$ by a canonical transformation with the following generating function:

$$\tilde{F}_2(Q_i,\tilde{P}_i;t)=(Q_1+Q_2-\omega t-\psi_o)\tilde{P}_1+Q_2\tilde{P}_2+Q_3\tilde{P}_3. \qquad (3.16)$$

The new canonical variables are:

$$a_1=Q_1+Q_2-\omega t-\psi_o \; ; \; P_1=\tilde{P}_1 \; ; \; a_2=Q_2 \; ; \; P_2=\tilde{P}_1+\tilde{P}_2 \; ; \; a_3=Q_3 \; ; \; P_3=\tilde{P}_3. \qquad (3.17)$$

For the resonant Hamiltonian one finds

$$\tilde{\mathcal{H}}_o=\Delta\omega\tilde{P}_1+\omega_z\tilde{P}_3+\frac{2\mu}{mL^2}\left(2+\frac{\tilde{P}_2}{\tilde{P}_1}\right)\tilde{P}_3^2\sin^4a_3-\frac{2\mu}{mL^2}(3\tilde{P}_1^2+2\tilde{P}_1\tilde{P}_2)-$$
$$-\frac{\mu\omega_z}{m\omega_oL^2}\left(6\tilde{P}_1+6\tilde{P}_2+\frac{\tilde{P}_2^2}{\tilde{P}_1}\right)\tilde{P}_3\sin^2a_3+\dots \qquad (3.18a)$$

$$\tilde{\mathcal{H}}_{rf}=-2\omega_e\left[\frac{1}{4}-\frac{\mu(2\tilde{P}_1+\tilde{P}_2)}{m\omega_o L^2}+\frac{2\mu\tilde{P}_3}{m\omega_z L^2}\sin^2 a_3\right]sgn(q)(\tilde{P}_1^2+\tilde{P}_1\tilde{P}_2)^{1/2}f(\tilde{P}_1,\tilde{P}_3)\cos a_1 \quad (3.18b)$$

$$\tilde{\mathcal{H}}_s=-\frac{2\omega_1}{3m\omega_o r_p^2}\left(\frac{\omega_z}{\omega_o}\right)^{1/2}\tilde{P}_2(\tilde{P}_2\tilde{P}_3)^{1/2}sgn(q)*$$

$$*(\cos[3sgn(q)a_2+a_3+\psi_3]+\cos[3sgn(q)a_2-a_3+\psi_3]) \quad (3.18c)$$

and $\Delta\omega=\omega_o-\omega$ is the actual frequency depart (resonance detuning).

IV. MAXIMUM ENERGY GAIN OF AN ELECTRON, PASSING THROUGH RESONANCE

Here we apply our previous results /2/ about fast passing through resonance in order to calculate the maximum energy gain of an electron, passing through resonance zones in ECR ion source. The unperturbed resonant frequency ω_1 is found from eq. (3.18a) to be

$$\omega_1=\frac{\partial\tilde{\mathcal{H}}_o}{\partial\tilde{P}_1}=\Delta\omega+\frac{\omega_z\tilde{P}_3}{2\tilde{P}_1}-\frac{2\mu\tilde{P}_2\tilde{P}_3^2}{mL^2\tilde{P}_1^2}\sin^4 a_3-\frac{4\mu}{mL^2}(3\tilde{P}_1+\tilde{P}_2)+\frac{\mu\omega_z\tilde{P}_2\tilde{P}_3}{2m\omega_o L^2\tilde{P}_1^2}\left(\frac{\tilde{P}_2}{\tilde{P}_1}-6\right)\sin^2 a_3 \quad (4.1)$$

The resonant frequency ω_1 exhibits fast oscillations for given $\tilde{P}_1,\tilde{P}_2$ and $\tilde{P}_3$. Solving equation $\omega_1=0$ one can find the resonant values of a_1 ($a_1\simeq\omega_z t+\phi_o$). This equation has in general four roots, which define four resonance zones placed symmetrically in couples on both sides of the mirror plane.

Following /2/ we define a canonical transformation $(a_i,\tilde{P}_i)\to(\alpha_i,\mathcal{J}_i)$ by

$$\alpha_i=a_i+\sum_{n=1}^{\infty}\varepsilon^n\frac{\partial G_n(a_k,\mathcal{J}_k;t)}{\partial\mathcal{J}_i} \quad ; \quad \tilde{P}_i=\mathcal{J}_i+\sum_{n=1}^{\infty}\varepsilon^n\frac{\partial G_n(a_k,\mathcal{J}_k;t)}{\partial a_i}. \quad (4.2)$$

The functions $G_n(a_k,\mathcal{J}_k;t)$ should be determined so that $\mathcal{J}_i$ become exact invariants of motion. For the maximum energy gain of an electron passing through m-th resonance zone we obtain [see formula (3.19) in /2/]

$$\Delta E_{1max}^{(m)}=\omega_o\Delta P_{1max}^{(m)}=2\omega_o\omega_e\left[\frac{2\pi}{|\dot{\omega}_1(a_{3m})|}\right]^{1/2}\left|\frac{1}{4}-\frac{\mu(2\mathcal{J}_1+\mathcal{J}_2)}{m\omega_o L^2}+\frac{2\mu\mathcal{J}_3}{m\omega_z L^2}\sin^2 a_{3m}\right|*$$

$$*(\mathcal{J}_1^2+\mathcal{J}_1\mathcal{J}_2)^{1/2}f(\mathcal{J}_1,\mathcal{J}_3) \quad (4.3)$$

where a_{3m} is the m-th root of equation $\omega_1(a_3)=0$ and

$$\dot{\omega}_1(a_{3m})=\omega_z(\mathcal{J}_1)\left(\frac{\partial\omega_1}{\partial a_3}\right)_{a_3=a_{3m}}. \quad (4.4)$$

Note that given the invariants $\mathcal{J}_1$ the equation $\left[\omega_1(a_3)\right]_{\tilde{P}_i=\mathcal{J}_i}=0$ may possess only two real roots for some values of $\mathcal{J}_i$. By the canonical perturbation theory it can be easily seen that $\tilde{\mathcal{H}}_s$ in eq. (3.18c) provides slight modulation of $\tilde{P}_2$ and $\tilde{P}_3$ according to

$$\tilde{P}_2=\mathcal{J}_2+3[\varkappa_-\cos(3a_2-a_3-\psi_3)+\varkappa_+\cos(3a_2+a_3-\psi_3)] \quad (4.5a)$$

$$\tilde{P}_3=\mathcal{J}_3-\varkappa_-\cos(3a_2-a_3-\psi_3)+\varkappa_+\cos(3a_2+a_3-\psi_3) \quad (4.5b)$$

where

$$\varkappa_-=sgn(q)\frac{2\omega_1}{3m\omega_o r_p^2(3\omega_2-\omega_3)}\left(\frac{\omega_z}{\omega_o}\right)^{1/2}\mathcal{J}_2\sqrt{\mathcal{J}_2\mathcal{J}_3} \quad (4.6a)$$

$$\varkappa_+=sgn(q)\frac{2\omega_1}{3m\omega_o r_p^2(3\omega_2+\omega_3)}\left(\frac{\omega_z}{\omega_o}\right)^{1/2}\mathcal{J}_2\sqrt{\mathcal{J}_2\mathcal{J}_3} \quad (4.6b)$$

$$\omega_2=\frac{3\mu}{4mL^2}\frac{\mathcal{J}_3^2}{\mathcal{J}_1}-\frac{4\mu}{mL^2}\mathcal{J}_1+\dots \quad ; \quad \omega_3=\omega_z+\frac{3\mu}{2mL^2}\left(2+\frac{\mathcal{J}_2}{\mathcal{J}_1}\right)\mathcal{J}_3+\dots \quad (4.6c)$$

The equation $\left[\omega_1(a_3)\right]_{\tilde{P}_i=\mathcal{J}_i}=0$ has four real roots with account of $\sin^2 a_3\le 1$ if the inequalities

$$0\le-\frac{\Delta\omega mL^2\mathcal{J}_1^2}{2\mu\mathcal{J}_2\mathcal{J}_3^2}-\frac{m\omega_z L^2\mathcal{J}_1}{4\mu\mathcal{J}_2\mathcal{J}_3}+\frac{2\mathcal{J}_1^2}{\mathcal{J}_2\mathcal{J}_3^2}(3\mathcal{J}_1+\mathcal{J}_2)\le 1 \quad (4.7a)$$

$$0\le\frac{\omega_z\mathcal{J}_1}{\omega_o\mathcal{J}_3}\left(\frac{\mathcal{J}_2}{\mathcal{J}_1}-6\right)\le 8 \quad (4.7b)$$

hold. The ineq. (4.7b) is fulfilled for a wide range of $\Delta\omega$, μ and $\mathcal{J}_i$. If one assumes that $\mathcal{J}_1\simeq\frac{m\omega_o r_L^2}{2}$, $\mathcal{J}_2\simeq\frac{m\omega_o r_{pl}^2}{2}$ and $\mathcal{J}_3\simeq\frac{m\omega_z z_m^2}{2}$, where r_L is the Larmor radius of electron, r_{pl} is the radius of plasma border and z_m is the maximum deviation from the mirror plane, a simple criterion for scaling the ECR ion source can be obtained.

Let us consider the following example: energy of electrons - 1 keV, cyclotron frequency at minimum B - $\frac{\omega_o}{2\pi}=9$ GHz, distance between "mirror points" - L=35 cm, radius of plasma border - $r_{pl}=1$ cm and $\mu=1.1$. For the frequency of RF field one finds $\frac{\omega}{2\pi}\ge 13.933784$ GHz.

V. CONCLUDING REMARKS

As a result of the investigation performed a formula for the maximum energy gain [formula (4.3)] of an electron passing through Electron Cyclotron Resonance has been derived. From eq. (4.1) it is clear that the resonance zone position depends on the resonance detuning, as well as on μ and on the amplitude of longitudinal oscillations.

The energy gain for one period of longitudinal oscillation will be the sum of energy gains when passing the resonance zones on one side of the mirror multiplied by a factor of four.

The author wishes to thank A.A.Efremov and especially Dr. V.B.Kutner for many illuminating discussions and interesting suggestions.

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Received by Publishing Department
on November 15, 1990.