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**A QUALITATIVE THEORY  
OF DIOCOTRON INSTABILITY  
AND METHODS OF ITS SUPPRESSION**

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## INTRODUCTION

The diocotron instability of immersed electron beams <sup>1,2</sup> is a serious obstacle in a way of increasing beam density and correspondingly achieving high power level of generated short-wavelength radiation as well as high ion acceleration rate in such beams.

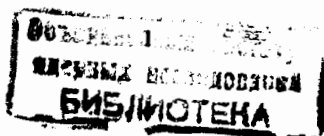
## QUALITATIVE ANALYSIS

As is well known <sup>1-4</sup>, the instability originates from a shear in electron drift velocity in cross fields: self-electric field and focusing longitudinal magnetic field. Let us first consider the problem in planar geometry approximation (Fig. 1) retaining however the condition  $F_r=0$  at the lower (inner) beam boundary which is characteristic of cylindrical geometry. The electron drift velocity with account of self-magnetic field  $B_0$  is equal to

$$v_{dr} = - \frac{cF_r}{eB_z} = - \frac{c(E_r - \beta_z B_0)}{B_z} = - \frac{cE_r}{\beta^2 B_z}, \quad (1)$$

where  $\beta = (1 - \beta_z^2)^{-1/2}$  is Lorentz-factor. In a beam of homogeneous density the drift velocity varies linearly over its cross-section (Fig. 1).

Let now an additional azimuthal (i.e. in the direction of the drift motion) force  $F_\theta$  affect a probe particle. The resultant transverse force  $F'$  is shown in Fig. 1 with dashes. Under its action the probe particle drifts downwards and meets the decreasing force  $F_r$ . The drift velocity is also decreasing so that the effective mass occurs to be negative:



$$m_{\text{eff}} = F_{\theta} / \dot{v}_{\text{dr}} = -(\frac{e}{c} B_z)^2 / \frac{dF_r}{dr} < 0 \quad (2)$$

This situation leads to instability if the interaction between electrons in the azimuthal direction has predominantly charge character (the case of capacitive impedance in the negative mass instability theory <sup>1</sup>).

On the basis of the conducted analysis one can come to the following conclusion: if the transverse force, responsible for the drift, increases in the direction of its action, then the beam can be unstable, provided the interaction has predominantly charge character, and is definitely stable in the case of current-wise interaction.

This rule can be easily generalized to the case of cylindrical geometry, in which the shear in angular velocity is essential, determined by the derivative  $d/dr(F_r/r)$ . If it is positive, the beam is liable to instability in the case of capacitive impedance and stable at inductive one.

#### METHODS OF STABILIZATION

The above formulated criterion suggests two ways of suppressing the diocotron instability.

- 1) to reverse the sign of effective mass;
- 2) to alter the sign of impedance (or at least diminish its value).

One of the proposed methods of the effective mass sign reversal is to transmit a current along the central rod <sup>3</sup> and/or place on it a negative charge <sup>4</sup>. To compensate for the self-field gradient of a thin-wall beam (of thickness  $\Delta = r_0 - r_1 \ll r_0$ ) the transmitted current should be but too high:

$$I \geq \frac{r_0}{(\beta_z J)^2 \Delta} I_b \quad (3)$$

where  $I_b$  and  $r_0$  are the beam current and outer radius.

Another method consists in partially neutralizing the beam space-charge with ions. However the ions under the action of the beam self-electric field move inwards and not only fail to cancel the velocity shear but even enhance it. Therefore this method is effective either for so short a pulse beam that the ions do not displace noticeably over its duration, or in the case of a strong

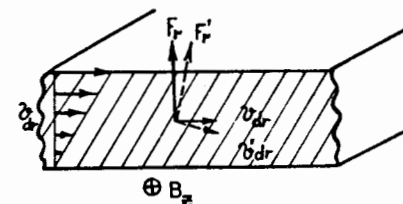


Fig. 1

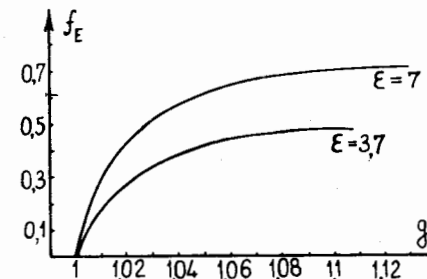


Fig. 2

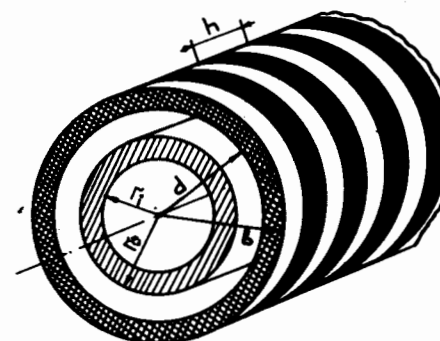


Fig. 3

magnetic field when ion Larmor radius is small compared to the beam thickness.

A classical method of reducing the beam impedance is screening<sup>1</sup>. The effect of solid conducting screens (inner and outer cylinders) has been studied in detail in Ref. [5]. The calculation showed that in order to suppress the diocotron instability the screen spacing to the beam surface should be small as compared with the beam thickness, which makes the screen alignment tolerance too close. Besides this, the solid screen can support axial current, thus reducing the modulated beam longitudinal self-electric field and making its use for collective ion acceleration inefficient.

In the relativistic case the impedance may be significantly reduced or even altered in sign by applying dielectric or anisotropically conducting screens. Since axial current in such screens is forbidden they attenuate the Coulomb repulsion between the beam electrons without noticeably changing their interaction as currents.

The effect of a screen can be characterized with electric and magnetic field screening factors,  $f_E$  and  $f_M$ , which are zero in free space and turn into unity for a solid perfectly conducting screen of the same inner radius (if an external screen is considered). For a dielectric tube with dielectric constant  $\epsilon$ , inner and outer radii  $d$  and  $b$  correspondingly, one can easily find<sup>6</sup>

$$f_E = \frac{(\epsilon^2 - 1)(g^2 - 1)}{g^2(\epsilon + 1)^2 - (\epsilon - 1)^2}, \quad f_M = 0, \quad (4)$$

where  $g = (b/d)^{1/2}$ ,  $l$  is the number of electric field variations in azimuth. The dependence  $f_E(g)$  is shown in Fig. 2 for  $\epsilon = 3.7$  (quartz glass) and  $\epsilon = 7$  (window glass).

The screening effect of the tube may be enhanced by application of circular conducting strips with step in axial direction  $h$  not surpassing the beam-screen clearance (Fig. 3). For  $h \rightarrow 0$  the screening factors are  $f_E = 1$ ,  $f_M = 0$ . Let us investigate this limiting case in more detail. The dispersion equation of linear theory<sup>2</sup> being properly modified looks as

$$\left(\frac{\omega}{\omega_D}\right)^2 - B\left(\frac{\omega}{\omega_D}\right) + C = 0, \quad (5)$$

where

$$B = 1\left(1 - \frac{r_i^2}{r_o^2}\right) + j^2 \frac{r_o^{2l} - r_i^{2l}}{d^{2l}}, \quad (6)$$

$$C = 1\left(1 - \frac{r_i^2}{r_o^2}\right)\left(1 - j^2 \frac{r_i^{2l}}{d^{2l}}\right) - \left(1 - \frac{r_i^{2l}}{r_o^{2l}}\right)\left(1 - j^2 \frac{r_o^{2l}}{d^{2l}}\right), \quad (7)$$

$\omega_D = \omega_p^2/2j^2\Omega$  is the diocotron frequency,  $\omega_p^2 = 4\pi e^2 n/m$  is nonrelativistic beam plasma frequency squared,  $\Omega = eB_z/mc$ ,  $l$  is the azimuthal wave number.

Computed from eqs (5)-(7) values of  $d/r_o$ , which correspond to the complete suppression of all unstable diocotron modes, are shown in Fig. 4 as function of the ratio  $r_o/r_i$  with  $r_o$  and  $r_i$  being the beam outer and inner radii. The curve 1 is calculated for a solid conducting screen (in this case the effect does not depend on  $j$ ). The curves 2 and 3 are found for anisotropic screen ( $f_E = 1$ ,  $f_M = 0$ ) at the electron relativistic factor values  $\beta = 2$  and  $\beta = 3$  respectively.

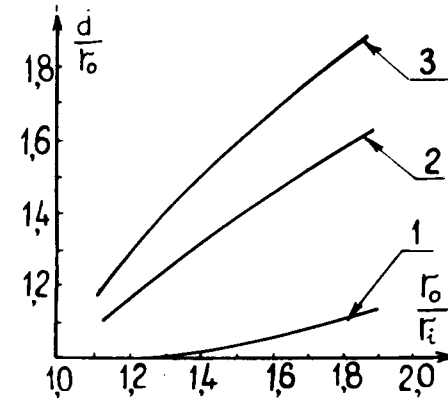


Fig. 4

Figure 4 shows that at a given value of  $r_o/r_i$  the anisotropic screen radius  $d$  which provides complete suppression of diocotron instability, by far exceeds the radius of solid screen necessary to produce the same effect. This essentially facilitates the

beam transport, rises the degree of instability suppression. It should be noted that such screens may be placed inside the acceleration section since they do not reduce the strength of both external accelerating and longitudinal self-electric fields.

#### REFERENCES

1. J.D.Lawson. The Physics of Charged-Particle Beams. Clarendon Press, Oxford, 1977.
2. R.B.Miller. An Introduction to the Physics of Intense Charged-Particle Beams. Plenum Press, N.Y., 1982.
3. E.Ott, J.-M.Wersinger. Phys.Fluids, 23, 324 (1980).
4. R.H.Levy. Phys.Fluids, 8, 1288 (1965).
5. Z.V.Kalandia et al. Zh.Tekhn.Fiz., 53, 1889 (1983).
6. Yu.I.Alexahin. JINR Report 9-87-196, Dubna, 1987.

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Алексахин Ю.И., Казаха В.И., Перельштейн Э.А. Е9-88-464  
Качественная теория диокотронной  
неустойчивости и методы ее подавления

Сформулировано простое необходимое условие развития диокотронной неустойчивости трубчатого электронного пучка в сильном продольном магнитном поле. С его помощью анализируются способы стабилизации пучка. Показано, что экранируя пучок диэлектрической или анизотропно-проводящей трубой, можно полностью подавить диокотронную неустойчивость.

Работа выполнена в Общественном научно-методическом отделе ОИЯИ.

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Alexahin Yu.I., Kazacha V.I., Perelstein E.A. E9-88-464  
A Qualitative Theory of Diocotron Instability  
and Methods of its Suppression

A simple necessary condition is formulated of the onset of the diocotron instability of a hollow electron beam in a strong longitudinal magnetic field. With its help methods of the beam stabilization are analyzed. It is shown that the diocotron instability can be completely suppressed by screening the beam with dielectric or anisotropically conducting tube.

The investigation has been performed at the Scientific-Methodical Division, JINR.

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