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E9-88-445

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A SYNTHESIS METHOD FOR THE DESIGN OF RELATIVISTIC MAGNETICALLY-FOCUSED BEAM SOURCES

Submitted to VII International Conference
on High-Power Particle Beams
Karlsruhe, F.R.G., July 4-8, 1988

1988

INTRODUCTION

In a variety of applications, such as generation of high-power short-wavelength electromagnetic radiation, high-quality (desirably laminar) intense relativistic electron beams are needed. An effective approach to laminar beam source design is the synthesis method[1], which includes two main steps. The first one, called the internal problem, is to build a solution to cold-fluid equations possessing the specified characteristics,and the second step (the external problem) consists in subsequent determination of electrode shapes outside the beam required to realise this solution.

BASIC EQUATIONS

In the stationary case the laminar flow equations[2] may be cast into the form

$$\mathrm{rot}(\mathrm{rot}\vec{\eta} + \varkappa^2\mu\vec{\eta}) = -\varkappa^2\vec{\eta} \ , \ \varkappa^2\vec{\eta}\cdot\nabla\mu = 0 \ , \ \Delta\gamma = \varkappa^2\gamma \ , \ \gamma^2 = 1 + \vec{\eta}^2 , \qquad (1)$$

where $mc\vec{\eta}$ is the electron momentum, $\varkappa c = \sqrt{4\pi e\rho/m\gamma}$ is local value of beam plasma frequency and ρ is charge density. The variable μ ,conserved along the trajectories,at the cathode is proportional to the ratio of magnetic field normal component to the emitted current density. An equation for $\varkappa$, deduced from the set (1), explicitly depends on the choice of the coordinate system, which is convenient to make so that the surface $x_1=0$ coincides with cathode[3]. For the axially symmetric flow ($\partial/\partial x_3=0$) this equation has the form

$$(\vec{\eta}\cdot\nabla)^2\ln\varkappa^2 + \varkappa^2(1-\mu\vec{\eta}\cdot\mathrm{rot}\vec{\eta}) = -(\nabla\gamma)^2 + (\nabla\eta_3)^2 + \left(\frac{\eta_3}{h_3}\right)^2 + (\vec{\eta}\cdot\nabla\ln h_3)^2 +$$
$$+\frac{1}{h_1^2}\left(\frac{\partial\eta_1}{\partial x_1} + \frac{\eta_2}{h_2}\frac{\partial h_1}{\partial x_2}\right)^2 + \frac{1}{h_2^2}\left(\frac{\partial\eta_1}{\partial x_2} - \frac{\eta_2}{h_1}\frac{\partial h_2}{\partial x_1}\right)^2 + \frac{1}{h_1^2}\left(\frac{\partial\eta_2}{\partial x_1} - \frac{\eta_1}{h_2}\frac{\partial h_1}{\partial x_2}\right)^2 + \frac{1}{h_2^2}\left(\frac{\partial\eta_2}{\partial x_2} + \frac{\eta_1}{h_1}\frac{\partial h_2}{\partial x_1}\right)^2 , \qquad (2)$$

where h_i, $i=1,2,3$ are the Lamé coefficients. Eq.(2) may be regarded as the generalization of laminar beam envelope equation to the case of non-uniform charge density over the beam cross-section.

Another useful characteristic, the current function $\psi(x_1, x_2)$, is related to the azimuthal self-magnetic field as

$$\psi = \frac{eh_3 B_3}{2mc^2} = -\frac{h_3}{2}(\operatorname{rot}\vec{\eta} + \mu\varkappa^2\vec{\eta})_3 \qquad (3)$$

and equals to the current in terms of the unit $I_o = mc^3/e = 17$ kA, which flows through the circle enclosed by coordinate line x_1=const, x_2=const. The current density may be expressed with the help of function ψ as

$$\frac{1}{I_o} j_1 = \varkappa^2\eta_1 = \frac{2}{h_2 h_3}\frac{\partial\psi}{\partial x_2}, \quad \frac{1}{I_o} j_2 = \varkappa^2\eta_2 = -\frac{2}{h_1 h_3}\frac{\partial\psi}{\partial x_1}. \qquad (4)$$

PARAXIAL EXPANSION

At distances from the axis, that correspond to the current values small in comparison with electrostatic limit, solution to the system of equations (1)-(2) may be sought in the form of power series in the transverse coordinate, x_2 (paraxial expansion). In the case of solid beam substitution of series

$$\eta_1 = \sum_{n=0}^{\infty}\eta_{1n}x^{2n},\ \eta_{2,3} = \sum_{n=0}^{\infty}\eta_{2,3n}x^{2n+1},\ \varkappa = \sum_{n=0}^{\infty}\varkappa_n x^{2n},\ \mu = \sum_{n=0}^{\infty}\mu_n x^{2n},\ \psi = \sum_{n=0}^{\infty}\psi_n x^{2n+2} \ldots$$

into eq.(2) leads in the lowest order to the equation (h_{10}= 1 is assumed)

$$\frac{3+\eta_{10}^2}{1+\eta_{10}^2}\eta_{10}'^2 = 2\varkappa_o^2 - \beta^2 + f^2 - 4\eta_{10}^2\varkappa_o\left(\frac{1}{\varkappa_o}\right)'' \qquad (5)$$

where $f=\mu_o\varkappa_o^2\eta_{10}=\beta-2\eta_{30}/\lambda$, $\lambda=h_{20}$, $\beta=eB_{10}/mc^2$, B_{10} is the longitudinal component of magnetic field, prime denotes differentiation by x_1. An approximate substitution $\varkappa_o^2 \simeq 4I/(I_o \cdot \eta_{10} \cdot R^2)$ with I and R being the beam current and radius converts eq. (5) into the well-known laminar beam envelope equation[2,3]. However, in the contradistinction to the latter the former is an exact equation. In the next order (x_2^2) eq.(2) yields a linear differential equation for the variable $\tau = \varkappa_1/\varkappa_0$, which characterises the flow non-paraxiality:

$$\eta_{10}^2\psi_o\varkappa_o^2\left[\frac{1}{\varkappa_o^2}\left(\frac{\tau}{\psi_o}\right)'\right]' + (\varkappa_o^2+f^2)\tau = -\frac{1}{2}\left(\frac{\mu_1}{\mu_o}+\frac{\eta_{11}}{\eta_{10}}\right)f^2 + \frac{1}{4}\lambda'(\lambda'-\lambda\chi)f^2 +$$
$$+\frac{1}{8}\lambda^2\varkappa_o^2(\beta-f)^2(1-\mu_o^2\varkappa_o^2) + \frac{1}{8}[\lambda^2\beta'(\beta'-f') - \beta(\lambda^2\beta')'] - \gamma_o'(\gamma_1'-\gamma_o'h_{11}) - \frac{2\gamma_1^2}{\lambda^2} +$$
$$+\eta_{10}(\eta_{11}-\eta_{10}h_{11})\left[2\varkappa_o\left(\frac{1}{\varkappa_o}\right)'' + \sigma\chi\right] + \eta_{10}^2\sigma(h_{11}'-\chi h_{11}) + \frac{2}{\lambda^2}\left(\eta_{11}+\frac{1}{4}\lambda\lambda'\eta_{10}\chi\right)^2 + \qquad (6)$$
$$+\frac{3}{2}\eta_{10}'(\eta_{11}'-\eta_{10}'h_{11}-\eta_{10}h_{11}\chi) + \frac{2}{\lambda^2}\left[\eta_{11}+\frac{1}{4}\lambda\lambda'\eta_{10}\chi - \psi_o - \frac{1}{4}\mu_o\varkappa_o^2\lambda^2(\beta-f)\right]^2,$$

where notations $\chi=\psi_o'/\psi_o$, $\sigma=\varkappa_o'/\varkappa_o$ are introduced. For other flow characteristics interconnection between coefficients of different orders may be found from eqs.(1),(4). In particular

$$\eta_{11} = -\eta_{10}h_{11} + \psi_o - \left(\frac{\psi_o'}{\varkappa_o^2}\right)' + \frac{1}{2}\mu_o\varkappa_o^2\lambda\eta_{30}, \quad \mu_1 = \text{const}\cdot\psi_o. \qquad (7)$$

The rest of relations have the same form as in the case of a non-vortical flow[3,4]. Correctness of eqs. (5)-(7) has been verified with the help of algebraic programming system REDUCE[5].

CATHODE ASYMPTOTICS

In the considered order of paraxial theory coordinate surfaces x_1=const, including that of the cathode (x_1=0), in the r-z plane of cylindrical coordinates are represented by fourth-order parabolas

$$z = a_1 r^2 + a_2 r^4. \qquad (8)$$

To simplify the formulae let us assume as a unit the characteristic length

$$x_o = \left(\frac{I_o}{8\pi j_{eo}}\right)^{1/2}, \qquad (9)$$

where j_{eo} is the emitted current density at the cathode center. In the following the values of x_1, h_2, h_3 are meant to be expressed in the terms of x_o. A subsidiary condition $\lambda(0)=1$ may be imposed without lack of generality.

As the initial conditions let us consider those of a space-charge limited flow: $\vec{\eta}(0)=0$, $\gamma'(0)=0$. Together with the requirement of ψ function being finite they lead to an equality $\psi'(0)=0$, which implies that tangential component of current density vanishes at the cathode[3]. In the vicinity of the cathode the characteristics of such a flow may be presented as power series in agregate $u=(3/2\ x_1)^{1/3}$. The above-mentioned conditions and eqs.(5)-(7) impose interrelationship between the series coefficients. Analytical computation with REDUCE program[5] gives the result:

$$\frac{1}{\sqrt{2}\cdot x_o\cdot\varkappa_o} = u + \sum_{k=1}^{\infty} u^{6k+1} t_{6k} - \frac{44}{45}a_1 u^4 - \frac{1}{126}u^5 + u^6 b_o\left(-\frac{27}{280}b_3 + \frac{9}{35}b_o a_1\right) +$$
$$+ u^8\left(\frac{44}{4725}b_3 b_o^3 - \frac{352}{14175}b_o^4 a_1 + \frac{22}{1575}a_1\right) + u^9\left(-\frac{27}{770}b_6 b_o - \frac{257}{15400}b_3^2 +\right.$$
$$+ \frac{514}{5775}b_3 b_o a_1 + \frac{9}{15400}b_o^4 - \frac{243}{1925}b_o^2 t_6 + \frac{19718}{433125}b_0^2 a_1^2 - \left.\frac{4511}{1455300}\right) + u^{10}\left(-\frac{13}{24948}b_3 b_o^5 +\right.$$
$$+ \frac{9881}{1746360}b_3 b_o + \frac{26}{18711}b_o^6 a_1 - \frac{3931}{654885}b_o^2 a_1 + \frac{392}{495}t_6 a_1 - \frac{286904}{3007125}a_1^3 + \left.\frac{1856}{13365}a_2\right) +$$
$$+ u^{11}\left(\frac{11}{7020}b_6 b_o^3 + \frac{87589}{27518400}b_3^2 b_o^2 - \frac{33941}{77395500}b_3 b_o^3 a_1 - \frac{11}{421200}b_0^6 + \frac{11}{1950}b_o^4 t_6 -\right.$$

$$- \frac{8359354}{290233125} b_0^4 a_1^2 + \frac{53}{442260} b_0^2 - \frac{49}{1170} t_6 + \frac{28694}{2764125} a_1^2) + u^{12} (- \frac{747}{40040} b_9 b_0 - \frac{1}{55} b_6 b_3 +$$

$$+ \frac{25174}{525525} b_6 b_0 a_1 + \frac{126667}{5255250} b_3^2 a_1 + \frac{2}{105105} b_3 b_0^7 + \frac{60353}{113513400} b_3 b_0^3 - \frac{5625}{56056} b_3 b_0 t_6 -$$

$$- \frac{375812}{7882875} b_3 b_0 a_1^2 - \frac{16}{315315} b_0^8 a_1 - \frac{18679}{38697750} b_0^4 a_1 - \frac{16941}{875875} b_0^2 t_6 a_1 + \frac{8223464}{84459375} b_0^2 a_1^3 -$$

$$- \frac{608}{45045} b_0^2 a_2 - \frac{1610311}{212837625} a_1) + \ldots$$

$$\beta = \sum_{K=0}^{\infty} b_{3k} u^{3k} + u^7 (\frac{1}{14} b_3 - \frac{4}{21} b_0 a_1) - \frac{1}{120} b_0 u^8 + u^{10} (\frac{1}{35} b_6 + \frac{268}{1575} b_3 a_1 -$$

$$- \frac{1}{2100} b_0^3 + \frac{18}{175} b_0 t_6 - \frac{69404}{118125} b_0 a_1^2) + \ldots$$

$$\mu_0 = 2 b_0 , \quad \mu_1(0) = - \frac{9}{4} b_6 + 6 b_3 a_1 + \frac{7}{16} b_0^3 - \frac{81}{2} b_0 t_6 - \frac{283}{25} b_0 a_1^2 ,$$

$$\gamma = 1 + \frac{1}{2} u^4 + \frac{32}{45} a_1 u^7 + \frac{13}{504} u^8 + u^9 b_0 (\frac{1}{140} b_3 - \frac{2}{105} b_0 a_1) + u^{10} (\frac{1}{60} b_0^2 - \frac{8}{5} t_6 +$$

$$+ \frac{2284}{3375} a_1^2) + u^{11} (- \frac{2}{4725} b_3 b_0^3 + \frac{16}{14175} b_0^4 a_1 + \frac{272}{4725} a_1) + u^{12} (\frac{1}{770} b_6 b_0 - \frac{19}{15400} b_3^2 +$$

$$+ \frac{6}{275} b_3 b_0 a_1 - \frac{1}{46200} b_0^4 + \frac{9}{1925} b_0^2 t_6 - \frac{24034}{433125} b_0^2 a_1^2 + \frac{3127}{5821200}) + u^{13} (\frac{1}{62370} b_3 b_0^5$$

$$+ \frac{1301}{873180} b_3 b_0 - \frac{4}{93555} b_0^6 a_1 + \frac{139814}{3274425} b_0^2 a_1 - \frac{11128}{2475} t_6 a_1 + \frac{12634096}{15035625} a_1^3 - \frac{4736}{13365} a_2) +$$

$$+ u^{14} (- \frac{1}{24570} b_6 b_0^3 + \frac{10247}{27518400} b_3^2 b_0^2 - \frac{260581}{77395500} b_3 b_0^3 a_1 + \frac{1}{1474200} b_0^6 - \frac{1}{6825} b_0^4 t_6 +$$

$$+ \frac{1892599}{290233125} b_0^4 a_1^2 + \frac{953}{884520} b_0^2 - \frac{92}{819} t_6 + \frac{72074}{921375} a_1^2) + u^{15} (\frac{64}{25025} b_9 b_0 + \frac{2}{1925} b_6 b_3 -$$

$$- \frac{1296}{175175} b_6 b_0 a_1 - \frac{18296}{2627625} b_3^2 a_1 - \frac{2}{4729725} b_3 b_0^7 - \frac{88741}{1135134000} b_3 b_0^3 - \frac{1236}{175175} b_3 b_0 t_6 +$$

$$+ \frac{7632983}{118243125} b_3 b_0 a_1^2 + \frac{16}{14189175} b_0^8 a_1 + \frac{1679}{12162150} b_0^4 a_1 + \frac{31456}{875875} b_0^2 t_6 a_1 - \frac{10656824}{84459375} b_0^2 a_1^3 +$$

$$+ \frac{2048}{289575} b_0^2 a_2 + \frac{321956}{212837625} a_1) + u^{16} (- \frac{1}{5985} b_6 b_0^5 + \frac{4471}{307230} b_6 b_0 + \frac{2027}{4189500} b_3^2 b_0^4 +$$

$$+ \frac{2039}{512050} b_3^2 - \frac{2533372}{311070375} b_3 b_0^5 a_1 - \frac{4345907}{622140750} b_3 b_0 a_1 + \frac{1}{359100} b_0^8 - \frac{2}{3325} b_0^6 t_6 +$$

$$+ \frac{88931764}{4666055625} b_0^6 a_1^2 + \frac{46867}{663616800} b_0^4 - \frac{5603}{307230} b_0^2 t_6 + \frac{61364911}{933211125} b_0^2 a_1^2 - \frac{67}{19} t_{12} +$$

$$+ \frac{1497}{950} t_6^2 - \frac{20993576}{3526875} t_6 a_1^2 + \frac{6181585192}{7141921875} a_1^4 - \frac{1962496}{3809025} a_1 a_2 + \frac{234813757}{167231433600}) + \ldots$$

$$\tau = (- \frac{7}{96} b_0^2 + \frac{27}{4} t_6 + \frac{1}{6} \lambda'' + \frac{61}{50} a_1^2) + u^2 (\frac{9}{160} b_3^2 - \frac{3}{10} b_3 b_0 a_1 + \frac{2}{5} b_0^2 a_1^2 + \frac{1}{160}) + u^3 (\frac{1}{18} \lambda''' -$$

$$- \frac{3}{5} \lambda'' a_1 - \frac{176}{45} a_1^3 + \frac{176}{45} a_2) + u^4 (- \frac{1}{112} b_3^2 b_0^2 + \frac{1}{21} b_3 b_0^3 a_1 - \frac{4}{63} b_0^4 a_1^2 - \frac{55}{6048} b_0^2 + \frac{3}{4} t_6 -$$

$$- \frac{1}{189} \lambda'' + \frac{61}{450} a_1^2) + u^5 (- \frac{729}{2240} b_9 b_0 + \frac{27}{280} b_6 b_3 + \frac{99}{70} b_6 b_0 a_1 + \frac{58}{175} b_3^2 a_1 + \frac{39}{5600} b_3 b_0^3 -$$

$$- \frac{243}{280} b_3 b_0 t_6 - \frac{9}{112} b_3 b_0 \lambda'' - \frac{14491}{5250} b_3 b_0 a_1^2 - \frac{13}{700} b_0^4 a_1 + \frac{81}{35} b_0^2 t_6 a_1 + \frac{3}{14} b_0^2 \lambda'' a_1 +$$

$$+ \frac{23204}{7875} b_0^2 a_1^3 - \frac{36}{35} b_0^2 a_2 + \frac{13}{1575} a_1) + u^6 (\frac{13}{9576} b_6 b_0^5 - \frac{7841}{102144} b_6 b_0 - \frac{403}{109440} b_3^2 b_0^4 -$$

$$- \frac{2759}{102144} b_3^2 + \frac{8021}{123120} b_3 b_0^5 a_1 + \frac{120979}{3447360} b_3 b_0 a_1 - \frac{13}{574560} b_0^8 + \frac{13}{2660} b_0^6 t_6 -$$

$$- \frac{1243463}{8079750} b_0^6 a_1^2 - \frac{16267}{110315520} b_0^4 + \frac{267}{10640} b_0^2 t_6 - \frac{7}{216} b_0^2 \lambda'' + \frac{3472601}{18468000} b_0^2 a_1^2 + \frac{17325}{608} t_{12} -$$

$$- \frac{22041}{6080} t_6^2 - 4 t_6 \lambda'' - \frac{1210957}{57000} t_6 a_1^2 + \frac{1}{81} \lambda^{(IV)} - \frac{86}{405} \lambda''' a_1 + \frac{1}{27} \lambda''^2 +$$

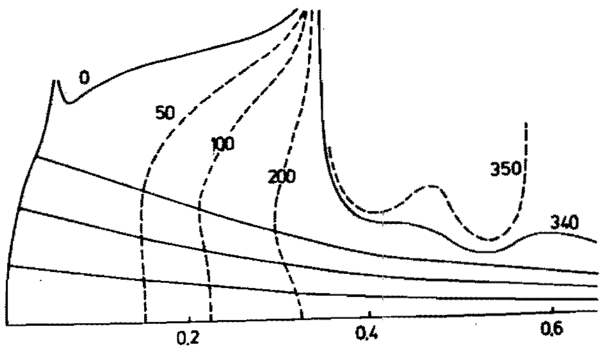
$$+ \frac{158}{405} \lambda'' a_1^2 + \frac{9817171}{1038825000} a_1^4 - \frac{186676}{115425} a_1 a_2 - \frac{1230209}{115831296}) + \ldots$$

ALGORITHM FOR SOLVING THE INTERNAL PROBLEM

As eq. (5) is the only relation between three functions η_{10}, β and $\varkappa_0$, there is a certain freedom in specifying two of them, which are chosen as generic functions. This freedom may be used in order to obtain the optimum source characteristics. The form of generic functions is not quite arbitrary: they should provide the required beam parameters at the source output and satisfy the initial conditions at the cathode.

In the case of a non-vortical flow ($\mu = 0$) it is convenient to choose $\beta(x_1)$ and $\varkappa_0(x_1)$ as the generic functions and consider eq. (5) as an equation for momentum of electrons at the axis $\eta_{10}(x_1)$. Owing to this equation, being the first-order differential equation, its solution (if it exists) tends to the final value at $x_1 \to \infty$ without oscillations, thus automatically ensuring the beam matching in the lowest order of paraxial theory. Subsequently solving eq. (6) gives information on the aberrations in the designed system which can be minimized by a proper choice of the cathode configuration (fixed by parameters a_1, a_2).

A computation example is shown in the Figure of the beam source with current 100A and energy 330kV. The beam compression factor in radius on entering the focusing magnetic field (not shown) is about 30 at a moderate level of aberrations. Equipotentials outside the beam (indicated in kV) are found by approximate method of Ref. [6]



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Received by Publishing Department
on June 22, 1988.

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Метод синтеза источников релятивистских магнитофокусируемых пучков

Описан метод расчета источников релятивистских пучков, который позволяет находить геометрию электродов, необходимую для формирования пучков с заданными характеристиками, в частности, согласованных пучков в продольном фокусирующем магнитном поле. Метод основан на параксиальном разложении негеометризованных уравнений ламинарных потоков пространственного заряда и включает анализ и устранение абераций надлежащим выбором формы эмиттирующей поверхности.

Работа выполнена в Общеинститутском научно-методическом отделе ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

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A Synthesis Method for the Design of Relativistic Magnetically-Focused Beam Sources

A system of equations governing stationary laminar space-charge flow is presented. In the case of solid axisymmetric flow it is reduced to a set of ordinary differential equations in the longitudinal coordinate. In the vicinity of the cathode the solution is given in the form of power series. A synthesis method of relativistic Pierce-type gun design is proposed, employing the obtained set of equations.

The investigation has been performed at the Scientifical-Methodical Division, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988