ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E9 - 8315

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ON MECHANISMS OF ENERGY TRANSFORMATION AT THE FORMATION OF WHISTLER SOLITONS

1974

ЛАБОРАТОРИЯ ВЫЧИСЛИТЕЛЬНОЙ ТЕХНИКИ И АВТОМАТИЗАЦИИ

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ON MECHANISMS OF ENERGY TRANSFORMATION AT THE FORMATION OF WHISTLER SOLITONS

Submitted to ЖТФ

1. Introduction

Attempts are undertaken recently to create the theory of plasma turbulence which makes allowance for a strong interaction between waves; this interaction is ignored when waves are described by the kinetic equation of the weak turbulence theory. For waves having a weak dispersion - Langmuir, for example, - the method of averaging over rapid time, proposed in work /1/, is available, and the stationary solution of soliton type was obtained for Langmuir waves /2/.

Afterwards, this solution was shown to be valid only for very slow solitons and more general solutions which described subsonic solitary waves have been found. Besides, the processes of forming solitons and their interaction with each other and with ion-sound (s) waves /3-5/ were studied by means of computer.

An affirmation was also made that the principle process in forming and interaction of solitons was s -wave generation leading to H.F. energy dissipation.

One-dimensional approach is more significant value for the study of oscillation in magnetic field; a modulation instability development, for instance, leads to the formation of stable solitons (see, for example, /6/).

In present work we study via computer the dynamics of forming and interaction of whistler solitons and examine the dissipation process mentioned above.

2. Basic Relations

Whistlers (w), propagating along the magnetic field $\vec{H}_0$ parallel to Z-axis, have the spectrum $\omega^w = \omega_{He} k^2 c^2/\omega_{pe}^2$, i.e., the frequency ω strongly depends on wave number k_0, in this sense whistler differs qualitatively from Langmuir plasmon. It has no rest mass (see /4, 5/) making up the main part of the total energy of Langmuir solitons. The L.F. energy of whistler soliton consists of the kinetic energy of whistler-quasiparticles, of the potential energy of their interaction and of the energy of hydrodynamic perturbations moving together with a soliton as well.

We shall obtain dynamic equation for a complex envelope of field amplitude h of whistlers

$$h_x - i h_y = h \cdot \exp(-i\omega_0(k_0)t) ,$$

$(\omega_0(k_0) = \omega_{He} k_0^2 c^2/\omega_{pe}^2(n_0)$, n_0 is an unperturbed plasma density, δn is a perturbation) following /6/ and employing the fact that for whistlers $\omega \simeq \omega(n_0)(1 - \frac{\delta n}{n_0})$.

Then we obtain the equation describing packets narrow enough in k-space, i.e.,

$$i\frac{\partial h}{\partial t} + \frac{\omega_{He}}{\omega_{pe}^2} c^2 \frac{\partial^2 h}{\partial z^2} + \omega^w \frac{\delta n}{n_0} h = 0 . \quad (1)$$

We should note that in consequence of relation $k_0 \gg \Delta k$, the instability of w-turbulence is of the modulation type contrary to the Langmuir plasmon gas instability, which can be both modulation and quasidecay one.

Equation (1) is to be added with the wave equation for density perturbation with account of magnetic pressure

$$\frac{\partial^2 \delta n}{\partial t^2} - v_s^2 \frac{\partial^2 \delta n}{\partial z^2} = \frac{\partial^2}{\partial z^2} \frac{|h|^2}{8\pi m_i} \quad (2)$$

($v_s^2 = T_e/m_i$, m_i is an ion mass).

We should underline here a difference between Eq. (1) and corresponding equation describing Langmuir turbulence, namely, the change in the sign of the nonlinear term appears. This is connected with the fact that for whistlers $d\omega^w/dn = -\frac{\omega^w}{n} < 0$, while for Langmuir oscillations

$\frac{d\omega^\ell}{dn} = \frac{\omega^\ell}{2n} > 0$. As a result whistler solitons have supersonic velocities and a density hump is localized together with field $|h|^2$, while a density pit is formed in subsonic Langmuir solitons by H.F. pressure $|E|^2$.

It must be mentioned that if decay process $\ell \to \ell' + s$ is allowed for supersonic Langmuir packets $v_g^\ell > v_s$, then for whistler decay, $w \to w' + s$, on the contrary, is forbidden when $v_g^w > v_s$ and is allowed in subsonic region, i.e., when $k < \frac{v_s \omega_{pe}^2}{2\omega_{He} c^2}$.

Let us introduce in set (1)-(2) dimensionless variables

$$z_d = \frac{\omega_{He}}{\omega_{pe}^2} \frac{c^2}{v_s} z, \quad t_d = \frac{\omega_{He}}{\omega_{pe}^2} \frac{c^2}{v_s^2} t, \quad \frac{\delta n_d}{n_0} = \frac{\omega_{pe}^4 v_s^2}{\omega_{He}^2 k_0^2 c^4} n ,$$

$$|h_d|^2 = 8\pi n_0 m_i \frac{v_s^4}{c^4} \frac{\omega_{pe}^4}{k_0^2 \omega_{He}^2} |h|^2 , \quad k_d = \frac{\omega_{pe}^2}{\omega_{He}} \frac{v_s}{c^2} k .$$

The equations (1), (2) take the form

$$i\frac{\partial h}{\partial t} + \frac{\partial^2 h}{\partial z^2} + nh = 0 , \quad (3)$$

$$\frac{\partial^2 n}{\partial t^2} - \frac{\partial^2 n}{\partial z^2} = \frac{\partial^2 |h|^2}{\partial z^2} . \quad (4)$$

This set has a soliton /6/ moving at a speed $v_g^w = \frac{2\omega_{He} k_0 c^2}{\omega_{pe}^2} > v_s$ as a solution. This solution may be written using our dimensionless variables as following

$$h = h_0(z - Mt) \cdot \exp(ikz - i\Omega t) , \quad h_0(\xi) = h_m \operatorname{sech}(\xi h_m \lambda/\sqrt{2}),$$

$$\lambda^2 = (M^2-1)^{-1},\ M = 2k_0 = \frac{v_g^w}{v_s},\ \Omega = \frac{M^2}{4} - \frac{h_m^2}{2(M^2-1)}, \tag{5}$$

$$n = \frac{|h|^2}{M^2-1}. \tag{6}$$

Besides, set (3), (4) has the following evolution invariants:

$$S_1 = \int |h|^2\, dz, \tag{7}$$

$$S_2 = \int \left(-n|h|^2 + |h'_z|^2 + \frac{n^2}{2} + \frac{V^2}{2}\right) dz, \tag{8}$$

where $\frac{\partial n}{\partial t} + \frac{\partial V}{\partial z} = 0$

These invariants may be easily calculated for soliton (5), (6)

$$S_1 = 2\sqrt{2(M^2-1)}. \tag{9}$$

$$S_2 = \frac{\sqrt{2}}{3}\,\frac{h_m^3(5-M^2)}{(M^2-1)^{3/2}} + \frac{h_m M^2}{2}\sqrt{2(M^2-1)}. \tag{10}$$

One can see that S_1 and S_2 are connected with a relation

$$S_2 = \frac{S_1^3(5-M^2)}{48(M^2-1)^3} + \frac{S_1 M^2}{4}. \tag{11}$$

The condition $\Delta k \ll k_0$ under which Eq. (1) of the basic set being valid restricts the soliton amplitude h_m essentially.

Really, the characteristic half-width of w-soliton in k-space is $\Delta k = h_m/\sqrt{2(M^2-1)}$, and $k_0 = M/2$, therefore the condition of validity of Eq. (1) is

$$h_m \ll M\sqrt{(M^2-1)/2}. \tag{12}$$

Having used dimensional variables one can rewrite this restriction in the form

$$h_{dm} \ll \sqrt{8\pi n_0 m_i}\; v_s\sqrt{2(M^2-1)}.$$

The soliton width being proportional to $\frac{\sqrt{(M^2-1)}}{h_m M}$ in dimensional variables does not practically depend on k_0. At a fixed soliton field amplitude h_m the density amplitude $\delta n_m \sim 1/(M^2-1)$, therefore, the nonlinear term in Eq. (1) (for soliton solution) is proportional to $M^2/(M^2-1)$ (because $\omega^w \sim M^2$).

Let us find out when $S_2 < 0$. First, it is necessary that $M > \sqrt{5}$ (see (11)). Beside that $S_2 < 0$ leads to

$$h_m^2 > \frac{3}{2}\,\frac{(M^2-1)^2 M^2}{(M^2-5)}$$

that can be written at $M \gg 1$ as

$$h_m > \sqrt{\frac{3}{2}}\, M^2. \tag{13}$$

Comparing (12) and (13) one can see that S_2 is positive in the region of validity of Eq. (1), in other words, the kinetic energy is bigger than the potential one.

3. Formation and Interaction of w-Solitons

The process of whistler soliton formation has been followed by means of computer. We investigated system (3), (4) under the following initial data

$$h = h_0\cdot \exp[-(z-z_0)^2/2d^2]\cdot \exp\left(i\frac{M}{2}z\right),$$
$$h_0 = 1,\ M = 3,\ d = \sqrt{M^2-1}/h_0,\quad n = 0, \tag{14}$$

Using relations (5), (9) it is easy to see that such a packet is narrower than the soliton with the same S_1 and M, that is why the initial packet should broaden. The evolution of such a packet is shown in Fig. 1. One can see that density hump forming moves together with the widening whistler packet.

The radiation of ion-sound waves being necessary for the formation of w-soliton occurs in both directions; moving with the sound speed these waves lag behind the supersonic soliton, while the excess energy of relative motions in coordinate system associated with the soliton radiates in form of s-wave energy. The value of this energy can be found by using relation (8).

For the initial packet being twice wider than (14), i.e., $d = 2\sqrt{M^2-1}/h_0$, we have, when forming soliton, on the contrary, narrowing of the packet and increasing of the field amplitude up to the value $h_m = S_1/2\sqrt{2(M^2-1)}$ would occur.

Let us underline that an initial packet having certain values S_1^{in} and S_2^{in} may transform only to a soliton with a smaller value of total L.F. energy $S_2^{sol} < S_2^{in}$.

As the decay $w \to w' + s$ is forbidden at $v_g^w > v_s$ the Čerenkov braking of the formed w-soliton does not occur.

Whistler solitons at certain relations between h_{mi} and M_i $(i = 1, 2)$ can coalesce like Langmuir ones, irradiating ion-sound trains. If, for example, one w-soliton overtakes another when $|M_1 - M_2| \ll M_1$ one can obtain their coalescence by studying system (3)-(4). This system is generally speaking invalid for the description of the interaction of w-solitons accompanied by velocity changing ΔM being of order of M_i. It is clear qualitatively, however, that their interaction will lead to a decrease of their relative velocity.

Let us look, e.g., at the interaction of two identical w-solitons moving towards each other with $|M_i| = 3$, $h_{mi} = 1$ (Fig. 2). During the process of interaction of solitons one can see the characteristic interference picture, while at this stage the local minima of $|h|^2$ correspond to the density humps. Velocity shift ΔM_i in one act of such an interaction is small; s-wave generated as a result of the interaction are naturally localized between the two supersonic solitons running away each other

The decrease of w-soliton velocity as a result of many such collisions can lead to an essential decrease of

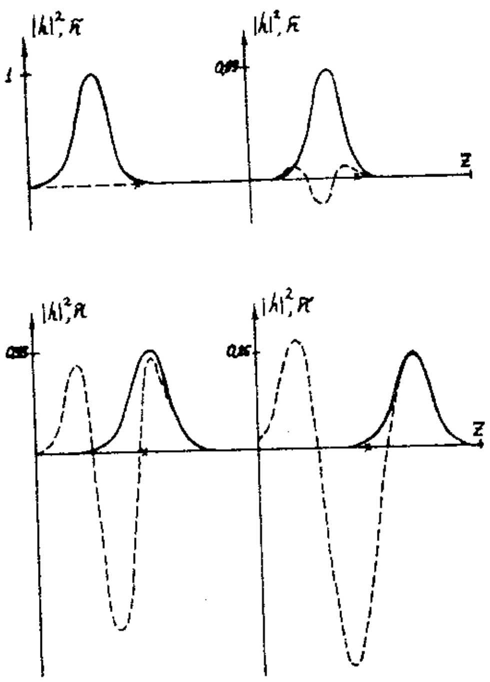

Fig. 1. The formation of whistler soliton. Solid line corresponds to $|h|^2$, dashed one is for density $\tilde{n} = n(M^2-1)$, $M = 3$.

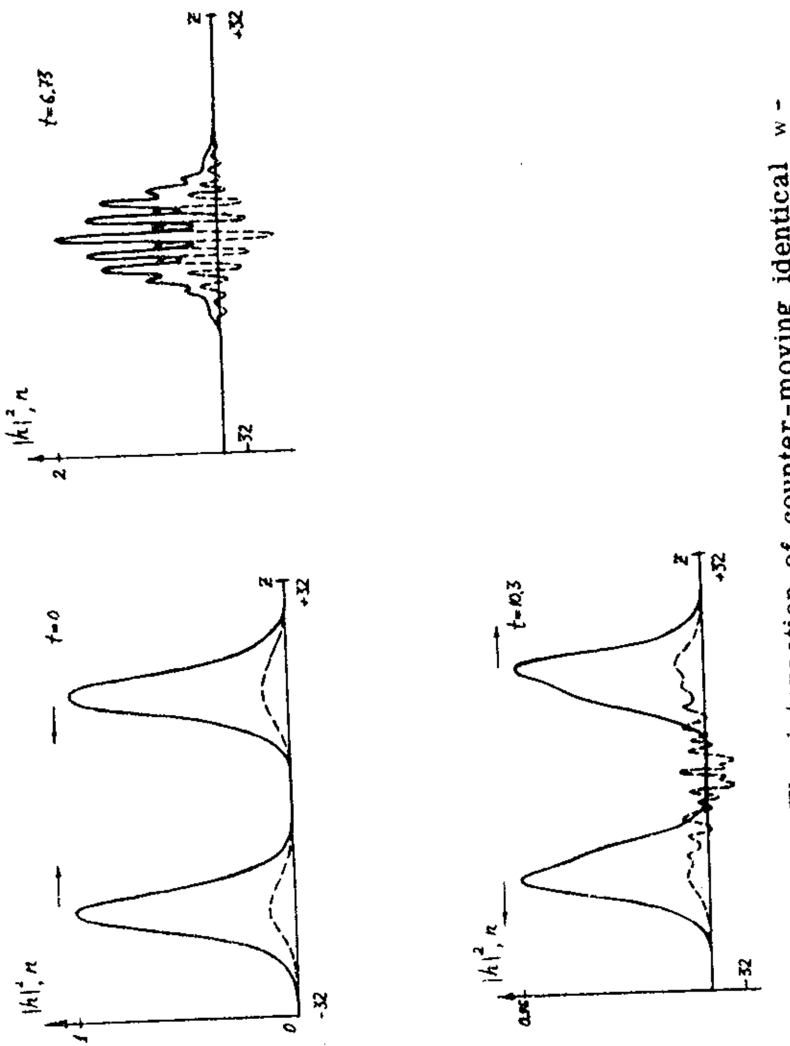

Fig. 2. The interaction of counter-moving identical w-solitons, $h_m = 1$, $M = 3$. The solid line corresponds to $|h|^2$, the dashed one does to plasma density perturbation n.

frequency $\omega^w \sim k_0^2 \sim M^2$. So due to counter collisions of w-solitons there exists the mechanism of H.F. energy transformation of decayless, in weak turbulence theory, spectrum of supersonic whistlers to L.F. sound energy.

Weak-damped whistlers and their solitons exist in a hot plasma $\mu \ll v_A/v_s \ll 1$ when $v_i \ll \omega^w/k \ll v_e$; w-solitons can have transonic velocities, $M \geq 1$. under these conditions. In a cold plasma $1 \ll v_A/v_s$, the condition of weak-damped whistlers to occur is $\omega^w/k \gg v_e$ while soliton velocity is hypersonic, $M = \frac{2 v_A}{v_s} \frac{k c}{\omega_{pe}} \gg 1$. Therefore in cold plasma the transition of solitons to heavy-damping region $\omega \sim \omega_{Hi}$ is possible as a result of many acts of deceleration due to their pair collisions.

We should stress a qualitative difference between the processes of interaction of s-waves with Langmuir and whistler solitons. A subsonic Langmuir soliton can both decelerate and accelerate by sonic impulses (the latter takes place when s-waves overtake soliton). For supersonic whistler solitons only deceleration is possible due to their interaction with ion-sound wave-trains. That is why the dissipation of whistler energy occured is irreversible.

The second term in Eqs. (2), (4) is small at $M \gg 1$ in comparison with the first one and can be neglected. It means physically that ion-sound waves generated, for example, during the formation of a soliton, move at a velocity ignorable in comparison with the soliton velocity. The correctness of such a neglect was verified by computation.

The truncated system describes correctly the formation of self-consistent solution (soliton), but leads to an infinite growth of density perturbation left by soliton.

4. Short Conclusions

1. The above analysis confirms that solitons may be formed as the result of the modulation instability in a gas of whistlers.

2. s -waves radiate in the process of whistler-soliton formation and interaction with each other (like that for Langmuir solitons).

3. As the results of counter collisions of whistler solitons their energy decreases, that is the new channel of turbulent dissipation of H.F. energy for decayless spectra.

We express our gratitude to Prof. E.P.Zhidkov for useful discussion of the results of this work and Prof. M.G.Mescheryakov for the support.

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Received by Publishing Department
on October 9, 1974.