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COMPUTER SIMULATION OF DYNAMICS OF STRONG-NONLINEAR PHENOMENA DESCRIBED BY THE SCHRÖDINGER EQUATION WITH THE SELFCONSISTENT POTENTIAL

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Langmuir turbulence is described by a set of the following equations

$$i\frac{\partial E}{\partial t}+\Delta_{xx}E-\frac{\alpha}{x^2}E=VE, \qquad (1)$$

$$\left(\frac{\partial^2}{\partial t^2}-\Delta_{xx}\right)V=\Delta_{xx}|E|^2. \qquad (2)$$

Here: $\alpha = 0,1,2$ respectively for the plane, cylindrical and spherical geometry, $\Delta_{xx}=\frac{1}{x^{\alpha}}\frac{\partial}{\partial x}x^{\alpha}\frac{\partial}{\partial x}$ and t , x , E , V stand for dimensionless time, coordinate, electrical field and density perturbation associated with corresponding dimensional variables by relations $t=\frac{2}{3}\omega_{pe}\,q\mu\,t_d$,

$X=\frac{2}{3}\frac{\sqrt{q\mu}}{d_e}X_d$, $V=\frac{3}{4}\frac{\delta n}{q\mu n_0}$,

$E^2=\frac{3}{64}\frac{E_d^2}{q\mu\pi n_0 T}$, $T=T_e+T_i$, $q=T/T_e$, $d_e=v_e/\omega_{pe}$,

$v_e=\sqrt{T_e/m_e}$,

k_0 is characteristic wave number of ℓ -wave packet, $v_g=3k_0 v_e^2/\omega_{pe}$, $v_s=\sqrt{T/m_i}$, $M=v_g/v_s$ is Mach number.

Ignoring the term $\partial^2 V/\partial t^2$ in Eq. (2) we shall get the so-called quasistatic approximation in which $V=-|E|^2$.

Let us, first, examine the plane picture. Stationary solutions of system (1) - (2) are the following solutions [1]

$$E = E_m \exp\left[i\left(\frac{M}{2}x - \Omega t\right)\right] \operatorname{sech}\left[\frac{E_m}{\sqrt{2(1-M^2)}}(x - Mt)\right],$$
$$\Omega = \frac{M^2}{4} - \frac{E_m^2}{2(1-M^2)}, \quad V = -\frac{|E|^2}{1-M^2}. \tag{3}$$

The evolution described by system (1) - (2) of arbitrary initial packets results in forming some solutions (3) and generating ion-sound waves. Quasistatic description does not lead to any stationary state; solutions oscillate for infinitely long time (see [2]).

System (1) - (2) has L.F. energy integral

$$S_2 = \int \left(V|E|^2 + |E_x'|^2 + \frac{V^2}{2} + \frac{u^2}{2}\right) dx = \text{const},$$
$$\frac{\partial V}{\partial t} + \frac{\partial u}{\partial x} = 0. \tag{4}$$

This energy consists of the potential energy of plasmons in the selfconsistent field V, the kinetic energy of soliton and relative plasmon motions inside soliton, and also kinetic and potential energy of hydrodynamic perturbation associated with H.F. field motion and s-waves generation. Neglecting $\partial^2 V/\partial t^2$ forbids generation and propagation of s-waves, whereas these waves play the main role in the dynamics of the strong ℓ-turbulence, their momentum and energy must be taken into account in conservation laws.

Soliton (3) has $S_2 = \frac{(5M^2-1)}{(1-M^2)^3}\frac{S_1^3}{48} + \frac{M^2 S_1}{4}$

($S_1 = \int |E|^2 dx = \text{const}$), so the soliton of such a type having velocity M can be formed from an initial packet for which we have $S_2(0) \geq \frac{S_1^3}{48}\frac{(5M^2-1)}{(1-M^2)^3} + \frac{M^2 S_1}{4}$.

Supersonic ($M_0 > 1$) initial packets, loosing their kinetic energy to generate s-waves, transform to subsonic ones then solitons with $S_2 > 0$, i.e., $M > 1/\sqrt{5}$ can arise. An asymmetrical packet with $k_0 = 0$ radiating s-waves in one direction mainly becomes symmetrical and gets a velocity $M \neq 0$.

The solitons (3) can coalesce at definite conditions generating an excess of energy via s-waves (the coalescence region of two encountering similar solitons has been found in [1]). These phenomena cannot, in principle, take place for the quasistatic approximation that, as it was shown in [1], practically has no field of application at interaction and is invalid, in general, when forming the solitons.

The linear equation for V is not longer correct for solitons with $M \to 1$ as in (3) $V \to \infty$ and the wave equation (2) must be replaced with an equation of the Boussinesq type with the right part:

$$\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial x^2} - \beta\frac{\partial^2 V^2}{\partial x^2} - \gamma\frac{\partial^4 V}{\partial x^4} = \frac{\partial^2}{\partial x^2}|E|^2 \tag{5}$$

(β and γ are constants).

The energy integral for the set (1). (5) is now

$$S_2 = \int \left[V|E|^2 + |E_x'|^2 + \frac{V^2}{2} + \frac{u^2}{2} + \frac{\beta V^3}{3} - \frac{\gamma (V_x)^2}{2}\right] dx. \tag{6}$$

The importance of correct description of L.F. processes in the region $M \to 1$ is determined by, the fact, that the main part of L.F. energy is contained, in this case, in hydrodynamic perturbations.

The option of boundary conditions plays an extreme role when computer studying the dynamics of strong ℓ-turbulence. The so-called "wave" boundary conditions were proposed in [1,2] taking into account that s-waves and clusters of H.F. field energy can go out from the area of solution. The periodical boundary condition and that with fixed ends were shown in [1,3] to lead to a physical reversion of the process after s-waves have reflected from ends, or come from the neighbour periods.

An attempt to investigate an evolution of spherically symmetrical packets of ℓ-waves was made in [4]. It was shown there that the set (1) - (2) could not have a self-similarity solution in this case. The process of forming of "spheritons" was observed via computer. An initial stage of "spheriton" accelerated motion towards the center was studied analytically. This stage and the regime(following it)of uniform motion were investigated by means of computer. The correctness of computer calculations may be guaranteed up to a certain moment because of a strong narrowing of "spheriton" moving towards the center, and enlarging the field amplitude. We have found the possibility of (computational) stopping of a spheriton when a few points of meshwork cover the energy containing area.

The width of a selfconsistent packet ΔR decreases in cylindrically symmetrical case proportionally to a distance R_m from the center; and the energy density $|E|^2$ increases proportionally R_m^{-2} but not R_m^{-4} , as it was in spherical case. We can afford to see via computer the process of moving of packets practically up to the center $R=0$.

It should be underlined here that one-dimensional model does not allow us to conclude of efficiency of absorption of ℓ -waves because the singularity of $|E|^2$ may appear, for this two- or three-dimensional calculations must be done in hydrodynamic approximation using sufficiently frequent meshwork.

The one-dimensional simulation has more practical sense for oscillations in the presence of a magnetic field. We have the following set of equations for the formation and interaction of whistler (w) solitons[5] ($\omega^w = \omega_{He} k_0^2 c^2/\omega_{pe}^2$)

$$i\frac{\partial h}{\partial t} + \frac{\omega_{He}\, c^2}{\omega_{pe}^2}\frac{\partial^2 h}{\partial z^2} + \omega^w \frac{\delta n}{n} h = 0\,, \qquad (7)$$

$$\frac{\partial^2 \delta n}{\partial t^2} - v_s^2 \frac{\partial^2 \delta n}{\partial z^2} = \frac{\partial^2}{\partial z^2}\frac{|h|^2}{8\pi m_i}\,. \qquad (8)$$

The sign of the term describing nonlinear interactions in Eq. (7) is opposite to that in Eq. (1). This leads to the fact that stationary solutions of the set (7) - (8) are supersonic solitons (unlike subsonic Langmuir ones) in which clusters of H.F. field energy and density move together.

Eq. (7) is valid only for packets with $\Delta k \ll k_0$,

so it describes solitons at $h_m \ll \sqrt{8\pi n_0 m_i}\; v_s \sqrt{2(M^2-1)}$.

In a hot plasma, where $v_i \ll \omega^w/k \ll v_e$, , weak-damped whistlers and their solitons exsist at $\mu \ll v_A/$. $/v_s \ll 1$, moreover there may occur nearly sonic solitons $M \gtrsim 1$. In a cold plasma $\omega^w/k \gg v_e$ the condition of exsistence of weak-damped whistlers is $1 \ll v_A/v_s$ therefore the velocities of solitons are now

$$M = v_g/v_s = \frac{2v_A}{v_s}\frac{kc}{\omega_{pi}} \gg 1\,.$$

We have examined the formation and collisions of w -solitons. Their velocities M decrease due to collisions. The solitons can go to a frequency region $\omega \sim \omega_{Hi}$ at $\omega^w/k \gg v_e$ by means of decrease of k_0 as a result of collisions of solitons with each other and with s -waves. There is a strong linear absorption in this region. Moreover the value of ω^w decreases together with M or, in other words, H.F. energy transforms to a L.F. sound one that is a new channel of turbulent dissipation of H.F. energy for decayless spectra.

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