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DYNAMICS OF LANGMUIR TURBULENCE. FORMATION AND INTERACTION OF SOLITONS

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Having used Eq. (10.27) cited in book /1/ it may be shown that the so-called quasistationary Langmuir turbulence spectrum derived in paper /2/ is unstable (see also /3, 4 /). Let k_0 correspond to the peak in a spectrum of turbulence (energy containing region); then if a wave number of a low frequency perturbation is significantly less than k_0, i.e., $k \ll k_0$, the instability of spectrum can be named as a modulation instability (apropos it is inherent in a monochromatic ℓ -wave), if $\kappa > k_0$, then instability may be classified as of quasidecay type.

The condition of the instability

$$\frac{W}{nT} > \kappa k^2 d_e^2 \quad (\kappa \text{ is a numerical coefficient}) \tag{1}$$

can be easily obtained from qualitative considerations proceeding from the dispersion relation /1/

$$\omega^{\ell} = \omega_{pe} + \frac{3}{2} k_0^2 d^2/\omega_{pe} - \frac{1}{8}\omega_{pe} \frac{W}{nT} \tag{2}$$

and taking it as a definition of energy of a nonrelativistic quasiparticle (plasmon). The first term in Eq. (2) corresponds to a rest mass, the second one does to a kinetic energy and the third term corresponds to a potential energy of plasmon interaction (a minus sign stands for an attraction). The inequality (1) means that the attraction energy of two plasmons exceeds their relative kinetic energy, that is why they couple. From this it follows, that both the modulation instability and the quasidecay one are the limit cases of an instability of ℓ -wave spectrum with respect to coupling of plasmons.

The growth rate of this instability versus k is plotted in Fig. 1, where

$$k_{tran} = \frac{1}{d_e}\left(\frac{2}{27}\mu\frac{W}{nT}\right)^{1/4}, \quad k_{max} = \frac{1}{d_e}\sqrt{\frac{W}{6nT}}, \quad \mu = \frac{m_e}{m_i}$$

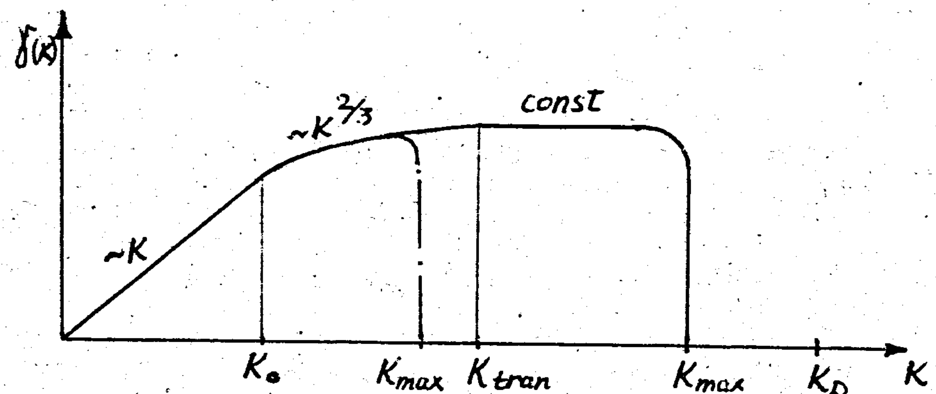

Fig. 1

Denoting $\frac{W}{nT} = \alpha\frac{m_e}{m_i}$ we find that $\left(\frac{k_{max}}{k_{tran}}\right) = \alpha^{1/4}$, so we have $k_{max} > k_{tran}$ if $\alpha > 1$ and $k_{max} < k_{tran}$ if $\alpha < 1$.

The instability develops to appear in an effective flow of energy along the spectrum towards the bigger $k \sim k_{max}$ This is its initial linear stage. A saturation of the instability can not be studied within the frame-work of the weak turbulence theory. Having used the results of work /4/ it may be shown that the equations sought are

$$i\frac{\partial\psi}{\partial\tau} + \frac{\partial^2\psi}{\partial\xi^2} - \nu\psi = 0, \quad \left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\xi^2}\right)\nu = \frac{\partial^2}{\partial\xi^2}|\psi|^2, \qquad (3)$$

i.e. the Schrödinger equation with a selfconsistent potential. Here we denote

$$\tau = \frac{2}{3}\omega_{pe} q\mu t, \quad \xi = \frac{2}{3}\frac{\sqrt{q\mu}}{d_e}x, \quad \nu = \frac{3}{4}\frac{\delta n}{q\mu n_0},$$

$$\psi^2 = \frac{3}{64}\frac{E^2}{q\mu\pi n_0 T}, \quad T = T_e + T_i, \quad q = \frac{T}{T_e}, \quad v_s = \sqrt{\frac{T}{m_i}},$$

$$M_s = \frac{v_g}{v_s}, \qquad v_g = 3k_0 d_e^2/\omega_{pe}$$

is a group velocity of plasmon.

The set of equations (3) has stationary solutions of nonlinear wave and soliton types. When a soliton is subsound (i.e., $M_s < 1$) it is HF energy cluster in space, in the case $M_s > 1$ the solition is better to name antisoliton as it corresponds to a gap in an envelope of a plane Langmuir wave.

The plots for functions ψ and ν are shown in Fig. 2 and Fig. 3, respectively.

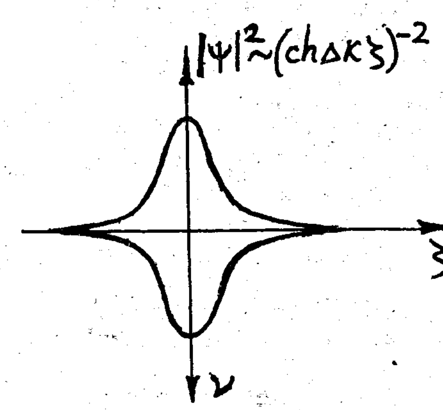

Fig. 2

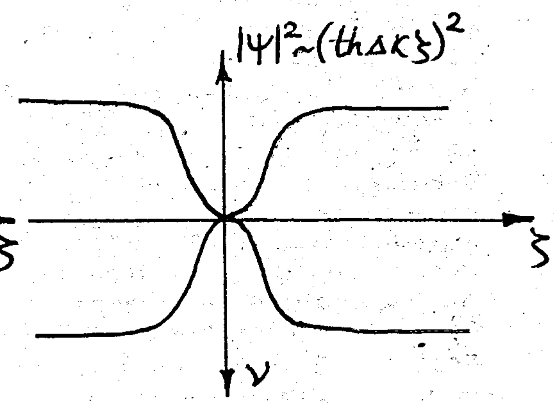

Fig. 3

Sometimes one investigates the so-called inertionless or quasistationary modifications of the set (3) which formally can be obtained by omitting the term proportional to

($\frac{\partial^2}{\partial t^2}$) in the equation for ν function. Then we have

$$i \frac{\partial \psi}{\partial \tau} + \frac{\partial^2 \psi}{\partial \xi^2} + |\psi|^2 \psi = 0 . \qquad (4)$$

Many works are now devoted to the study of this equation (see, for example, /5, 6, 7/). Equation (4) has two evident integrals of motion (others are possible /5/)

$$S_1 = \int |\psi|^2 d\xi , \qquad S_2 = \int (|\frac{\partial \psi}{d\xi}|^2 - \frac{1}{2} |\psi|^4) d\xi . \qquad (5)$$

The set (3) has the first integral, the second one is not concerved and must be modified.

In work /4/ the asymptotic theory of interaction of solitons based on Eq. (4) has been developed. It was shown there that, as a result of this interaction, solitons did not change their forms, they only changed their relative velocity.

We studied via computer the dynamics of formation and interaction of solitons with each other and with the Langmuir plane wave on the base of Eq. (4) and of the system (3). The results are essentially different.

Equation (4), if relation (6) (see below) is not performed, has no stationary solutions at all, we have functions oscillating in time (see Fig. 4a). The system (3), at the same initial ℓ -wave packets, goes, changing S_2, to the stationary solution of $|\psi| = |\psi|_m / \mathrm{ch}(\Delta k \xi)$ (where $\Delta k = |\psi|_m / \sqrt{2(1-M_s^2)}$) type, that accompanied by generation of ion-sound waves taking the energy and momentum out of area of HF packet localization (see Fig. 4b).

If S_2 for the soliton of $|\psi| = |\psi|_m / \mathrm{ch}(\Delta k \xi)$ type has been calculated, one obtains easily (6), that means the solutions of Eq. (4) can approach the soliton-type ones if only initial packets satisfy the condition

$$(S_2 / S_1^2) = -(1 - M_s^2)^{-2} (1 + 2M_s^2) / 96 . \qquad (6)$$

The investigation of dynamics of two-soliton interaction based on Eq. (4) confirms qualitatively the results of

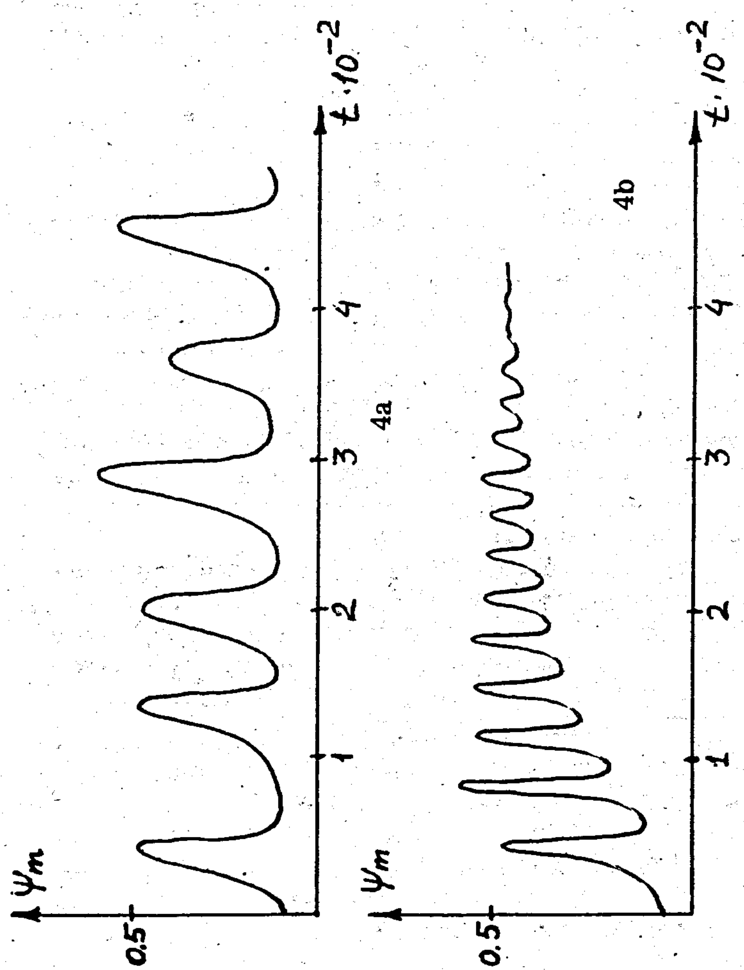

Fig. 4. The plots of peak value of an envelope of packet $|\psi(\xi, t)|$ as a function of time; $|\psi|_m(s\,t) = S_1 / 2\sqrt{2(1-M_s^2)}$ an amplitude of the stationary soliton which corresponds to a given value S_1, i.e., the total energy of HF field.

work /4/ : solitons either go through each other or reflect from each other keeping their forms.

The dynamical system (3) gives absolutely different result. Let us look at two (identical) solitons moving each other with equal group velocities. The binding condition (1) now is $S_I > M_s(1 - M_s^2)$, if this condition is performed (i.e., a potential energy of soliton interaction is greater than their relative kinetic energy) it is possible to form one soliton from two, i.e., their coupling. This result was proved by the numerical study of the system (3) at the following initial conditions: $S_I = 2.15,\ 2.45,\ 2.77$, $M_s = 0.3,\ 0.5,\ 0.2$, respectively. Then the interaction of solitons is accompanied by generation of ion-sound wave with the greater value S_I the greater their energy. When $T_e = T_i$ they will be quickly absorbed, heating ions. The dynamics of such an interaction is shown in Fig. 5 ($S_I = 2.77$, $M_s = 0.2$).

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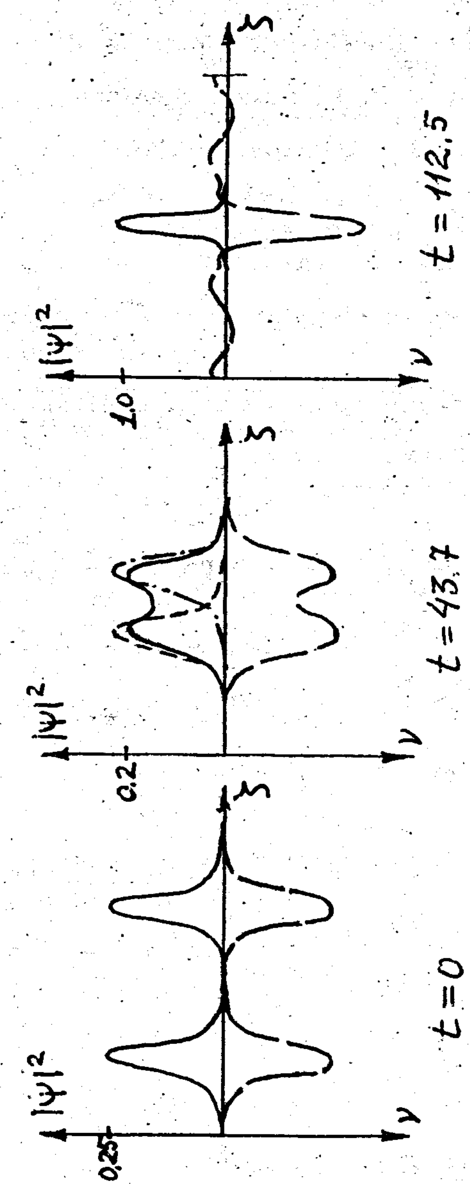

Fig. 5

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