ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E9 - 6385

ЛАБОРАТОРИЯ ВЫЧИСЛИТЕЛЬНОЙ ТЕХНИКИ
И АВТОМАТИЗАЦИИ

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COMPUTER INVESTIGATION OF NONLINEAR DYNAMICAL PROBLEMS OF PLASMA THEORY

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COMPUTER INVESTIGATION OF NONLINEAR DYNAMICAL PROBLEMS OF PLASMA THEORY

Report on the Ist International Conference on Computational Physics (Geneva, April 10-14, 1972)

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E9 - 6385

Численное исследование нелинейных динамических задач теории плазмы

Представлены результаты численного исследования трех нелинейных задач физики плазмы.

Препринт Объединенного института ядерных исследований.
Дубна, 1972

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E9 - 6385

Computer Investigation of Nonlinear Dynamical Problems of Plasma Theory

The results of a numerical investigation of three plasma theory nonlinear problems are given: 1) the evolutions of the Langmuir wave spectra due to their induced scattering on plasma ions and four-plasmon interactions; 2) the correlation function structure of quasi-steady isotropic Langmuir turbulence; 3) the dynamics of radiation instability of relativistic electron rings up to a saturation.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1972

I. The spectrum of quasi-steady isotropic Langmuir turbulence has been obtained via computer by Makhankov et al./1/. There naturally appears a question about the dynamics of reaching this steady spectrum, i.e. about evolution of quite arbitrary initial spectrum located in the region of large wave number $k > k_* = \frac{1}{3} \frac{\omega_{pe} v_i}{v_e^2}$ (the usual notations). The problem is reduced to a study of behaviour of the spectral energy density of l-wave $W_k(t)$ in time. This yields

$$\frac{\partial W_k}{\partial t} = W_k \int Q(k,k_1) W_{k_1} dk_1 + \int dk_1 dk_2 dk_3 R(k,k_1,k_2,k_3) \times$$
$$\times (k^2 W_{k_1} W_{k_2} W_{k_3} + k_1^2 W_k W_{k_2} W_{k_3} - k_2^2 W_k W_{k_1} W_{k_3} - k_3^2 W_k W_{k_1} W_{k_2}), \tag{I}$$

for the definition of $Q(k,k_1)$ and $R(k,k_1,k_2,k_3)$ see /1/. It is not difficult to verify that the quantity $W_0 = 4\pi \int W_k(t) k^2 dk$ is the constant in the evolution process (the energy conservation law).

Three-dimensional integrals were calculated by the repeated integration. The time derivative was approximated by the simple explicit scheme of the first order accuracy. The deviation of the value W_0 (evolution constant) is less than 3% in the end of the calculation if the time step $\Delta\tau = 10^{-3}$ (where $\tau = \frac{\omega_{pe}}{10} \frac{W_0}{n_0 T_e} t$). The following initial data considering nearly all physical conditions have been studied:

a) a broad wave spectrum (parcel) $\Delta k \lesssim k$ and $k_0 \gg k_*$,
b) a narrow wave spectrum $\Delta k \ll k$ and $k_0 \gg k_*$,
c) a wave spectrum $\Delta k \lesssim k$ and $k_0 \gtrsim 2k_*$,

k_0 is a center of a wave spectrum (see Fig. 1).
Results of the calculation show (see Figs. 2,3) if an initial spectrum is broad enough (case a) the evolution leads it quickly to become too narrow and the wave energy locates near the left-hand side of the initial spectrum. Thus we come to the conditions of b) type. In this

case the following process of transformation occurs: the initial spectrum growing more narrow becomes nearly a line-spectrum, unmoving along k-axis, simultaneously the accumulation of wave energy proceeds in the region corresponding to the maximum of nonlinear growth rate.

Then the energy from the first spectrum transforms into another quite narrow one. This is the two-level mechanism of energy transformation (through a set of satellites) first predicted in work /2/ .As an energy level at the left-hand side of the appearing spectrum being larger than that of the initial one the above mechanism of energy transformation goes to be broken and the new packet moves quicker than energy needed to create a new satellite accumulates. When the wave packet center has gone to a region of $k \sim 3k_*$ at the left of it there arises a broad packet overlaping with the main one and energy transforms from the first packet to another with their simultaneous displacement to small k , where the four-plasmon interaction and pair collisions should be taken into account. As a result of all these factors proceeding there can appear a quasi-steady spectrum.

2. Correlation functions being one of the most important characteristics of plasma turbulence have been measured in a lot of experiments. Knowledge of these functions enables us to know both the frequencies of turbulent pulsations and the level of their energy. Calculations of correlation functions of isotropic Langmuir turbulence have been made for some values of k and based on the equation obtained in /3/ .

Results show (see Fig. 4) that the width of the correlation curve increases with increasing k up to a maximum at $k = k_0$, which corresponds to the maximum of the turbulence spectrum $W(k_0) = W_{max}$ /1/. Thereafter the width of curve decreases with increasing k .

The shift of the curve maximum as a function of k is determined mainly by the linear dispersion, therefore it increases with increasing k .

The calculation of correlation functions of the plasma-beam experiment (rf./4/) has been proceeded. Sufficiently good coincidence between computation and experimental data was obtained to reach if the level of turbulence is $(W/n_0T_e) \sim 10^{-3}$ that corresponds to other measurements.

3. There are many common topics (problems) of turbulent plasma theory and explosively evolved by now theory of collective acceleration /5/. The study of electron-ion ring stability is one of the main problems. As it is well-known the radiative instability being of a hyd-

rodynamical nature is the most dangerous for the mentioned acceleration. Its investigation comes mathematically to solving the one-dimensional Vlasov equation for oscillations of electrons with "complex" effective mass. Therefore the instability is of a hydrodynamical type and is attended with an intensive coherent radiation of electromagnetic waves with their lengths being larger than the minor ring radius, i.e. $\lambda \gg a$.

The dynamics of this instability has been considered beginning with disregarding azimuthal perturbations ($\sim 10^{-5}$) of electron distribution function. There were taken into account 10 coherent harmonics. The Couchy problem for a set of 21 quasilinear (in a mathematical sense) equations was solved numerically. We used an implicit difference scheme, having verified its stability for parameters employed (most interesting from the point of view of the Dubna and Berkeley experiments). Results were obtained for three versions of initial data (two versions of initial perturbation energy distribution over harmonics (spectrum) a) and b); for the case a) two values of initial energy spread were taken (Fig. 5)).

The results of a linear theory (growth rates and thresholds) at the initial stage of instability development with a good accuracy have been confirmed. The system has a tendency, due to evolution, to distribute energy equally over all the harmonics because of their non-linear interaction (Fig. 6). At the moment of time near 3-3.5 reverse linear growth rate of the first harmonic, a nonlinear limitation of the amplitude increase occurs in all the cases considered then during a short time the field energy grows sharply (it may be understood as an exposive instability (Fig. 7)) and finally further behaviour of the wave energy in time is of oscillation nature with some average value (small enough [6]) that justifies the saturation of the instability observed.

The zero harmonic describes the particle distribution over energies in a ring. The calculation results show that at $\tau \approx 3.5$ a step of a "shock-wave" type in the energetic space passing into the region of a small energies appears. The front of this wave gradually flattens and a velocity of its moving decreases. This process is repeated, and a shape of the distribution function becomes approximately of a Gaussian type with energy spread to be of order of threshold one (see Fig. 8).

References

1. V.G.Makhankov, E.P. Zhidkov, V.N. Tsytovich and Che Ze Hen. Plasma Phys., 12, 191 (1970).
2. A.S.Kingsep and L.I. Rudakov. JETP, 58, 582 (1970).
3. V.G.Makhankov and V.N.Tsytovich. Nucl.Fusion, 10, 405 (1970).
4. E.A. Kornolov, Yu.B.Fainberg and O.F. Kovpik. JETP Lett., 4, 101 (1966).
5. V.I. Veksler et al. Report on IV IC on HEA Cambridge, USA, 1967.
6. A.G.Bonch-Osmolovskii et al. JINR Preprint, P9-5622, Dubna, 1971; Plasma Phys., (to be published).

Received by Publishing Department on April 17, 1972.

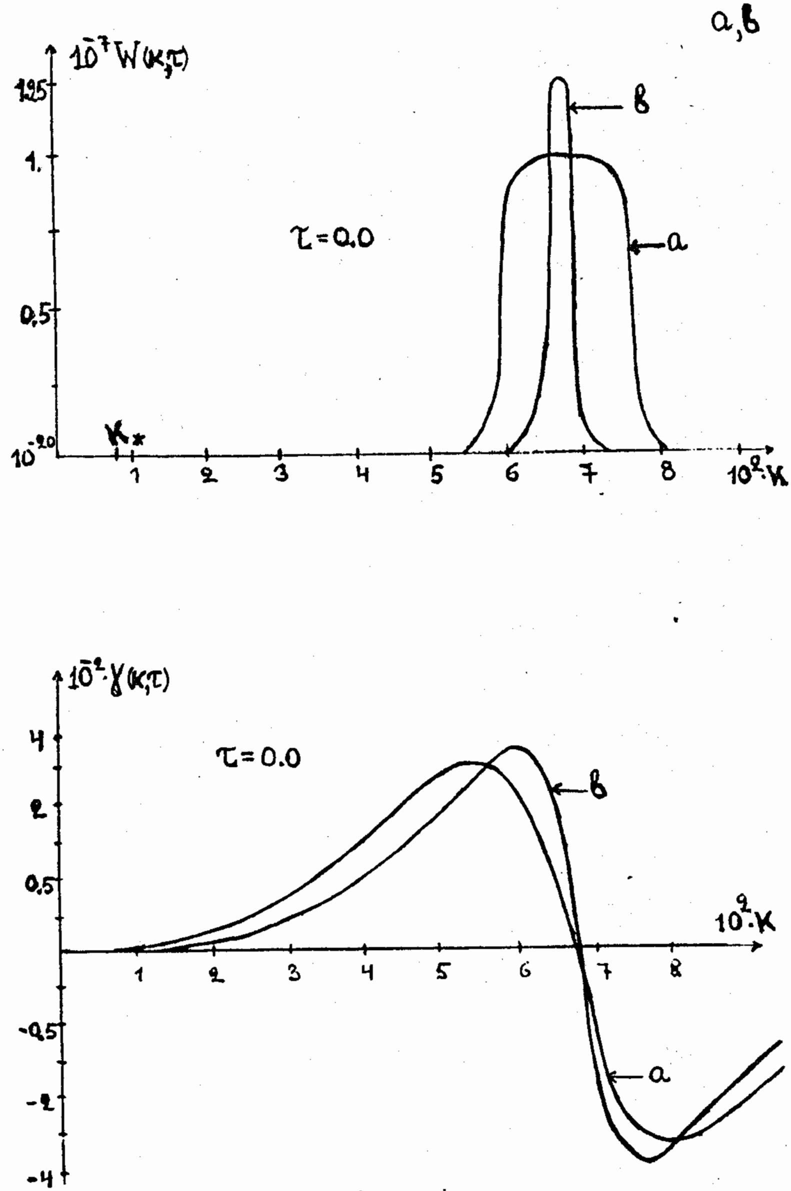
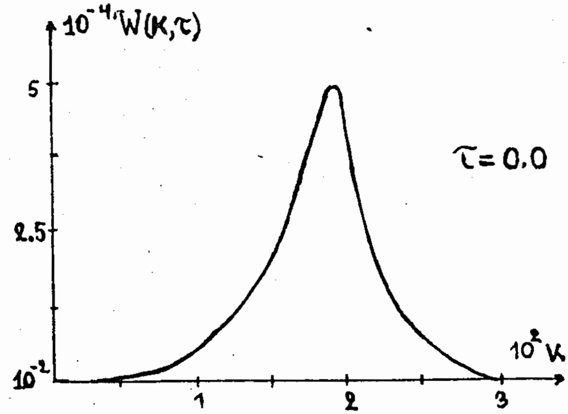


Fig. 1(a,b), 2(a,b). The initial spectra $W(v,0)$ and corresponding to them nonlinear growth rates.

c



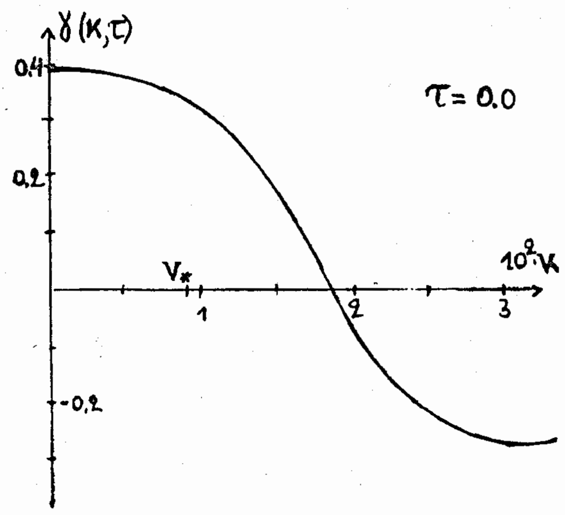


Fig. I(c), 2(c). The initial spectra $W(v,0)$ and corresponding to them nonlinear growth rates.

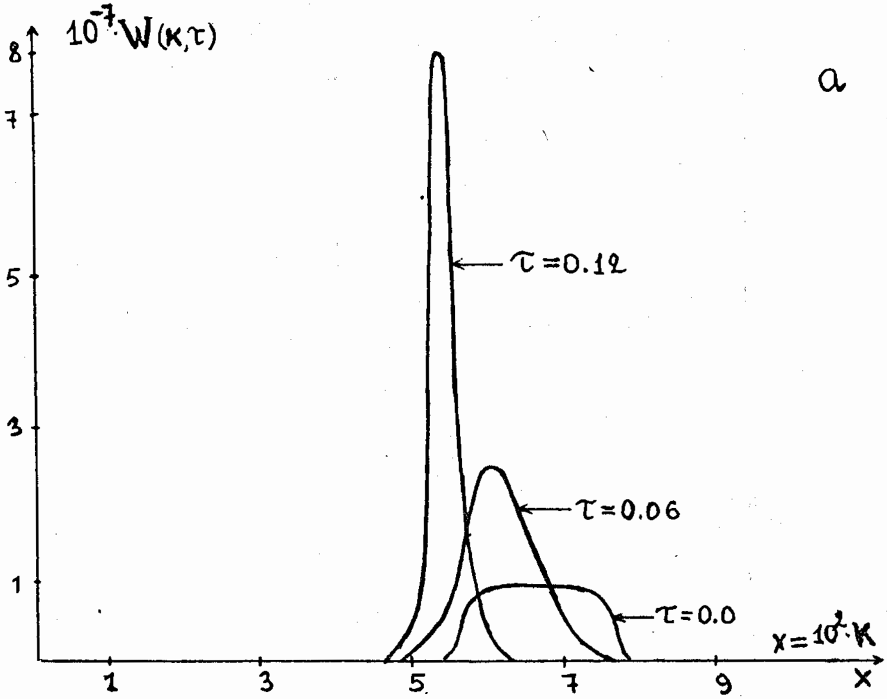
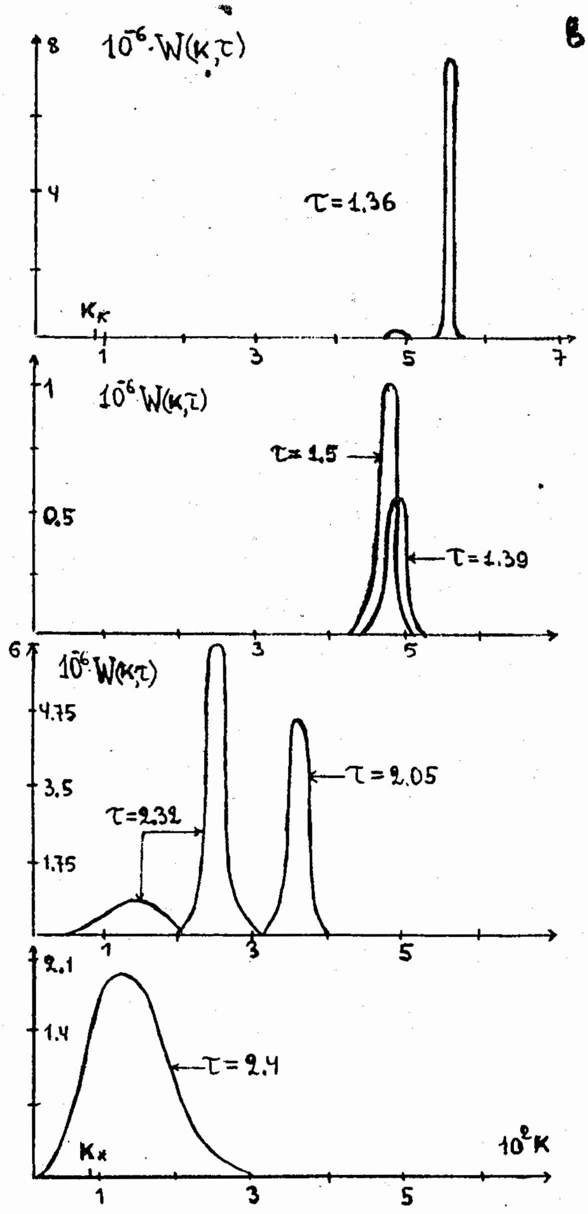


Fig. 3(a,b,c).The evolution of the l-wave spectra due to induced scattering on the plasma ions.



B
$10^{-6}\cdot W(K,\tau)$
8
4
$\tau = 1.36$
K_K
1
3
5
7
$10^{-6} W(K,\tau)$
1
0.5
$\tau = 1.5$
$\tau = 1.39$
3
5
6
$10^{-6} W(K,\tau)$
4.75
3.5
1.75
$\tau = 2.32$
$\tau = 2.05$
1
3
5
2.1
1.4
$\tau = 2.4$
K_x
$10^2 K$
1
3
5

Fig. 3 b .

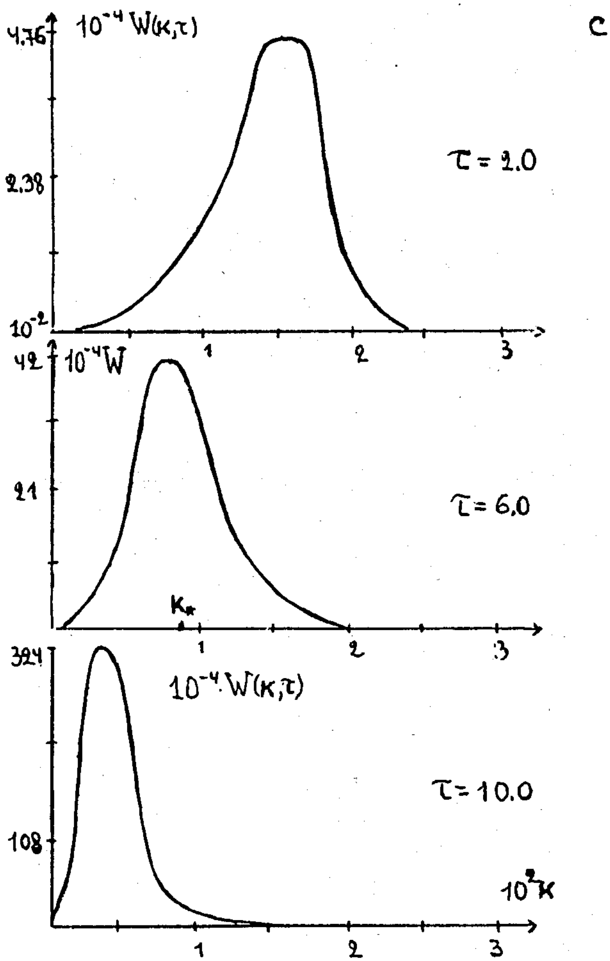

c
4.76
10^{-4} W(κ,τ)
2.38
10^{-2}
τ = 2.0
1
2
3
42
10^{-4}W
21
τ = 6.0
K_*
1
2
3
324
10^{-4}·W(κ,τ)
τ = 10.0
108
10^2κ
1
2
3

Fig. 3c.

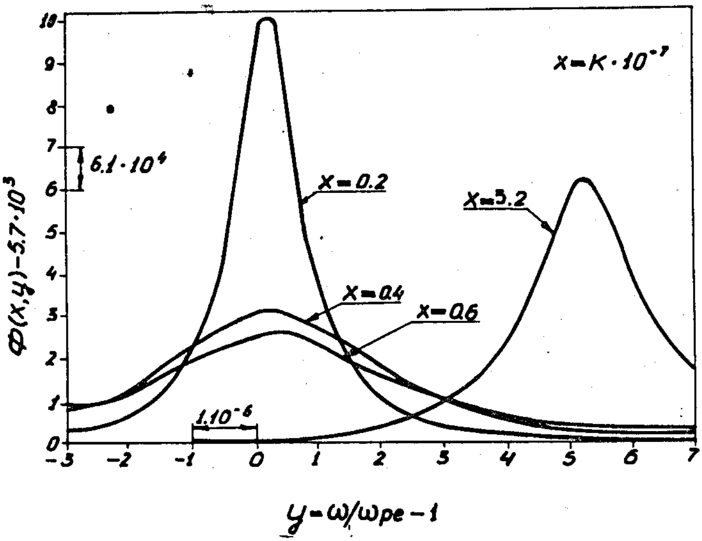


Fig. 4a. The correlation functions of the isotropic Langmuir turbulence.

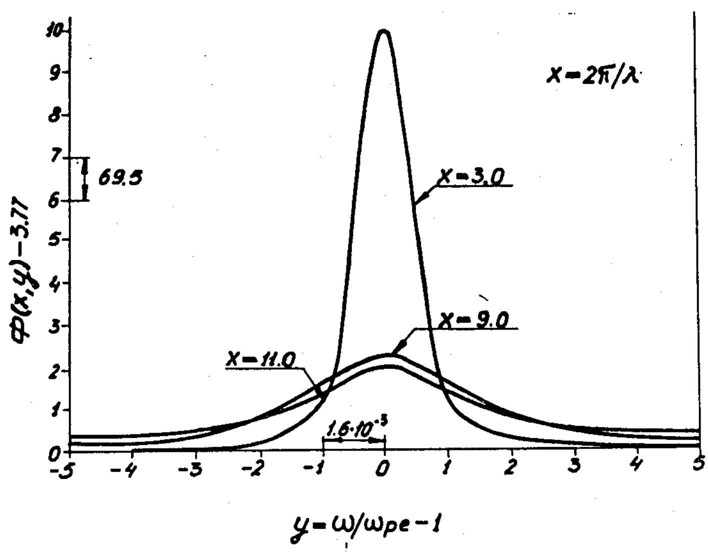


Fig. 4b. The correlation functions calculated for plasma-beam experiment.

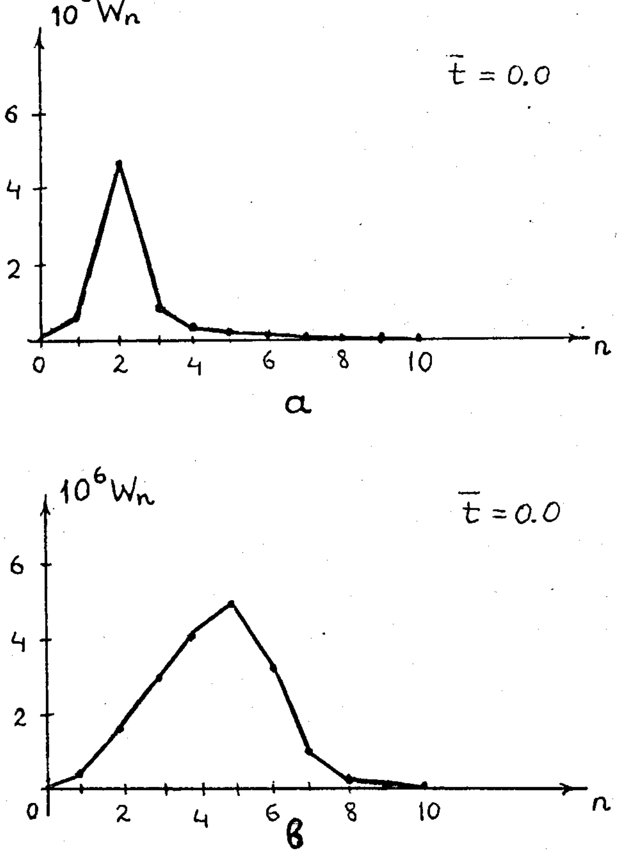


Fig. 5a, 5b. Two forms of initial distribution of the perturbation field energy over harmonic numbers. Plots of evolution of functions sought are shown in Figs. 6-8, case a.

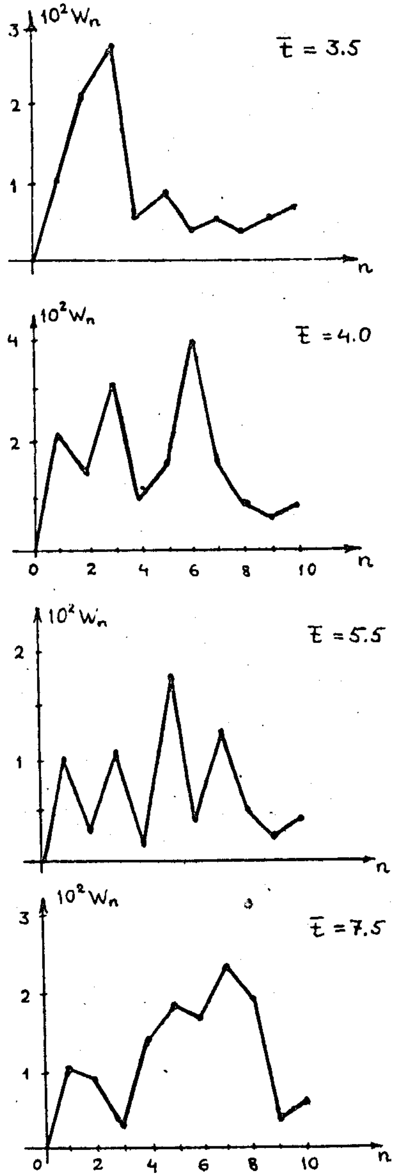


Fig. 6. The spectrum of the perturbation in various times.

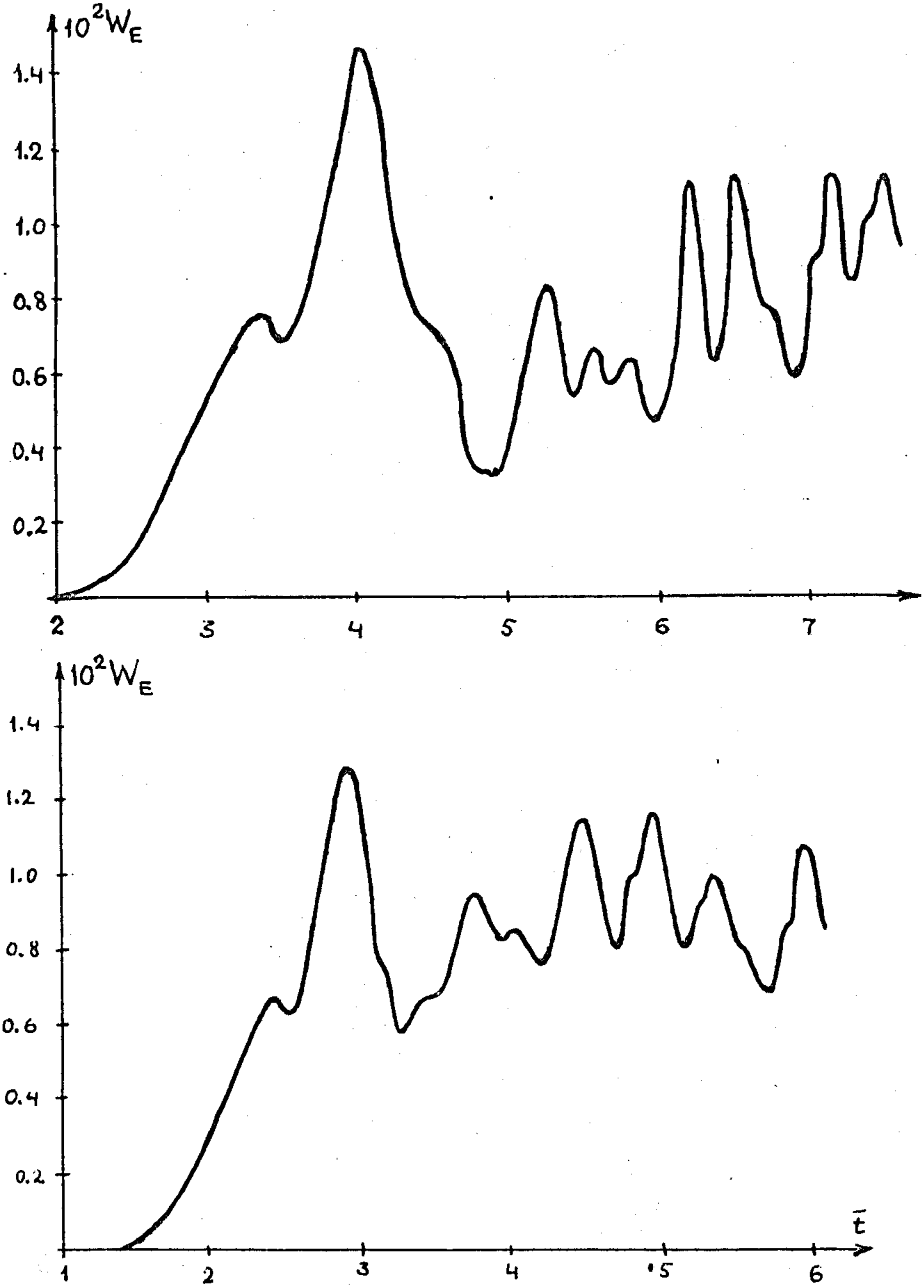


Fig. 7a, 7b. The total perturbation field energy as a function of time at initial conditions a and b, respectively.

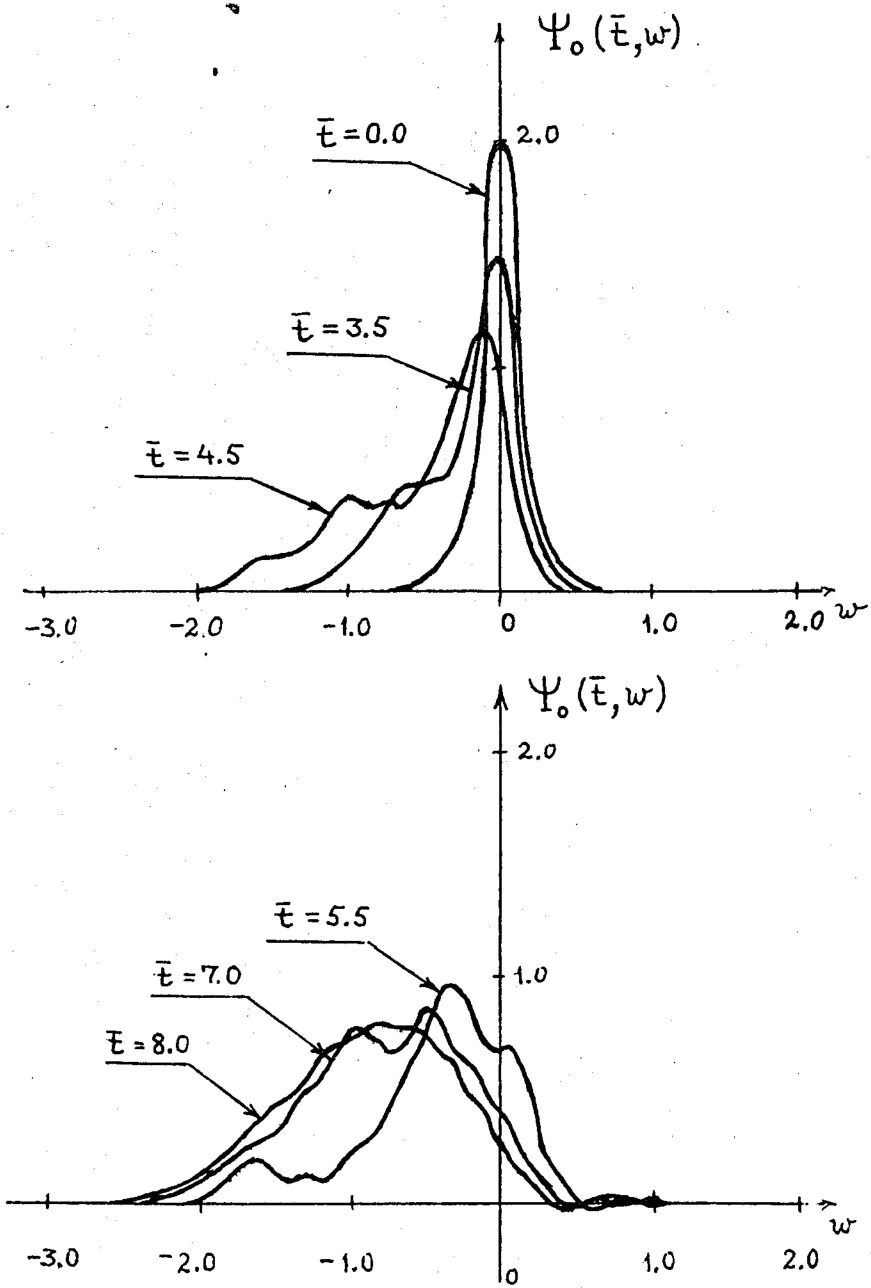


Fig. 8. The evolution of particle distribution function over w in a ring.