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COULOMB COLLISIONS OF PARTICLES IN A TURBULENT PLASMA

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Further Evolution of the Theory

The kinetic approach evolved above is quite complicated and not so comfortable for practical calculations. Let us show here that some phenomenological system of hydrodynamic equations (in which the friction force has different numerical coefficients when $\omega > \nu_e$ and $\omega < \nu_e$) can be of use in concrete studies. The basis of this system is the kinetical investigation carried out above. According to [1] we obtain here a relatively simple system of hydrodynamic equation adequately describing the nonlinear phenomena in a plasma in the frequency region of interest. The relative simplicity of this system enables us to study complex problem concerning the nonlinear spectra of a turbulent plasma in the range of frequent Coulomb collisions [2,3].

Besides that on the basis of this system of equations the new types of the aperiodical low frequency (L.F.) instabilities can be obtained when high frequency oscillations change the electromagnetic properties of a plasma essentially in the low frequency range. The physical sense of this situation being possible is that the nonlinear shifts of the frequencies are turned out to be bigger than the proper frequencies of oscillations (in the L.F. region).

Therefore the dispersion properties of a plasma are determined by the properties of nonlinear shifts due to intensive H.F. turbulence.

It was shown in [1] that expanding well-known hydrodynamic equations to be hold at $\omega < \nu_e$ with respect to small parameters $\frac{\nu_e}{\omega_1}$, $\frac{\omega}{\omega_1}$ (ω_1 is high frequency) one can get

$$\left(-i\omega + i\frac{k^2 v_e^2}{\omega}\right) V^R_{ke} = -0.51\,\nu_e U^R_k - 1.71\, i k v_e^2 \frac{T^R_{ke}}{T_{oe}} + \frac{e}{m_e} E^R_k,$$

$$\left(-\frac{3}{2} i\omega + 3.16\frac{k^2 v_e^2}{\nu_e} + \delta\right)\frac{T^R_{ke}}{T_{oe}} = -ikV^R_{ke} - 0.71\, ik\, U^R_k + \delta\frac{T^R_{ki}}{T_{oe}} +$$

$$+ \frac{m_e \nu_e}{T_{oe}} \int \langle \vec{V}_{k_1} \vec{V}_{k_2} \rangle \delta(k - k_1 - k_2)\, dk_1 dk_2, \tag{8.1}$$

$$\left(-i\omega + 1.28\frac{k^2 v_i^2}{\nu_i} + i\frac{k^2 v_i^2}{\omega}\right) V^R_{ki} = -ikv_i^2 \frac{T^R_{ki}}{T_{oi}} + 0.71\, ikv_i^2 \frac{T^R_{ke}}{T_{oi}} +$$

$$+ 0.51\frac{m_e}{m_i}\nu_e U^R_k - \frac{e}{m_i} E^R_k, \quad \delta = 3\frac{m_e}{m_i}\nu_e$$

$$\left(-\frac{3}{2} i\omega + 3.9\frac{k^2 v_i^2}{\nu_i} + \delta\right)\frac{T^R_{ki}}{T_{oi}} = -ikV^R_{ki} + \delta\frac{T^R_{ke}}{T_{oi}}.$$

Let us solve (8.1) with respect to V^R_{ke} and V^R_{ki}

$$V^R_{ke} = -\frac{ik\,\nu_e A_e}{\kappa_1 \Omega_{e1} \omega_{e1}} \int \vec{V}_{k_1} \vec{V}_{k_2} \delta(k - k_1 - k_2)\, dk_1\, dk_2$$

$$V^R_{ki} = \frac{ik\nu_e A_i m_e}{\kappa_1 \Omega_{e1} \omega_{i1} m_i} \int \vec{V}_{k_1} \vec{V}_{k_2} \delta(k - k_1 - k_2)\, dk_1 dk_2. \tag{8.2}$$

Here we denote

$$\omega_{i1} = -i\omega + i\frac{k^2 v_i^2}{\omega} + 1.28\frac{k^2 v_i^2}{\nu_i} + \frac{k^2 v_i^2}{\Omega_i} - \frac{T_{oe}}{T_{oi}}\frac{k^2 v_i^2}{\Omega_{e1}}(0.71 - \frac{\delta}{\Omega_{i1}})(1+\frac{\delta}{\Omega_{i1}})$$

$$\omega_{e1} = -i\omega + i\frac{k^2 v_e^2}{\omega} + 1.71\frac{k^2 v_e^2}{\Omega_{e1}}(1+\frac{\delta}{\Omega_{i1}}), \quad v_i^2 = \frac{T_{oi}}{m_i}, \quad v_e^2 = \frac{T_{oe}}{m_e},$$

$$\Omega_{i1} = -\frac{3}{2}i\omega + 3.9\frac{k^2 v_i^2}{\nu_i} + \delta, \quad \Omega_{e1} = -\frac{3}{2}i\omega + 3.16\frac{k^2 v_e^2}{\nu_e} + \delta - \frac{\delta^2}{\Omega_{i1}},$$

$$\kappa_1 = 1 + (0.51\nu_e + 1.71(0.71 - \frac{\delta}{\Omega_{i1}})\frac{k^2 v_e^2}{\Omega_{e1}})(\frac{1}{\omega_{e1}} + \frac{m_e}{m_i}\frac{1}{\omega_{i1}}) \equiv \tag{8.3}$$

$$\equiv 1 + a(\frac{1}{\omega_{e1}} + \frac{m_e}{m_i}\frac{1}{\omega_{i1}})$$

$$A_e = 1.71 + \frac{m_e}{m_i}\frac{a}{\omega_{i1}}(1+\frac{\delta}{\Omega_{i1}})$$

$$A_i = 0.71 - \frac{\delta}{\Omega_{i1}} - \frac{a}{\omega_{e1}}(1+\frac{\delta}{\Omega_{i1}}).$$

From this

$$S(k, k_1, k_2) = -e n_0 \frac{ik\nu_e}{\kappa_1 \Omega_{e1}}(\frac{A_e}{\omega_{e1}} + \frac{m_e}{m_i}\frac{A_i}{\omega_{i1}})\frac{V_{k_1} V_{k_2}}{E_{k_1} E_{k_2}}. \tag{8.4}$$

By comparing electron and ion terms in (8.4) we have $\frac{1}{\omega_{ei}} \gg \frac{m_e}{m_i}\omega_{ii}$ and (8.4) coincides with (3.35)/7/at $\omega^2 \gg \frac{m_e}{m_i} k^2 v_e^2$. In the limit of sufficient low frequency $\omega \lesssim k v_i$ the ion contribution should be taken into account in the nonlinear polarizability S_1. The function S_2 last argument of which corresponds to low frequency is formally of the form obtained in §3. /7/.(see (3.46), where κ_1, ω_{ei} and others are now determined by formulae (8.3)).

The system (8.1) is now the basis for getting and studying the dielectric permeability of a weak-turbulent plasma in the region $\omega \ll \nu_e$. For this (as there was in the collisionless limit, see Tsytovich's review in this issue) the electric field is needed to be divided into regular part E^R and turbulent one E^T.

Following/2/ we can solve the set (8.1) that now determines the regular parts of E, V, n, T. The equations for the turbulent parts are

$$\frac{\partial n}{\partial t} + \operatorname{div} n\vec{V} = 0$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}\vec{\nabla})\vec{V} = \frac{e}{m}\vec{E}. \tag{8.5}$$

Let us expand both the regular and the turbulent functions sought to the first order with respect to the weak regular field amplitude (for example $V^T = V^{T(0)} + V^{T(1)}$, etc.). Then from (8.5)

$$V_k^{T(0)} = \frac{e}{m_e}\frac{\vec{E}^{T(0)}_k \vec{k}}{k\omega_k}$$

$$V_k^{T(1)} = \frac{1}{k\epsilon(\omega, \vec{k})} \int \frac{k_1'}{\omega_1'} \frac{(\vec{k}\vec{k}_2')}{k_2'} V_{k_1'}^{R} V_{k_2'}^{T(0)} \delta(k - k_1' - k_2') dk_1' dk_2' .$$

Substituting these expressions into the nonlinear term of the system (8.1) and using eq. $\frac{\partial E^R}{\partial t} = -4\pi j^R$ one obtaines

$$\epsilon^L(\omega, \vec{k}, W_{k_1}^{\ell}) = 1 + i \frac{m_e}{m_i} \frac{\omega_{pe}^2}{\kappa_1 \omega \omega_{i1}} + i \frac{\omega_{pe}^2}{\kappa_1 \omega \omega_{e1}} \frac{1 - \frac{m_e}{m_i} \frac{A_i \omega_{e1}}{A_e \omega_{i1}} \beta^{\ell}}{1 + \beta^{\ell}} \quad (8.6)$$

$$\beta^{\ell} = -\frac{i k^2 \nu_e A_e e^2}{8\pi^2 \kappa_1 \Omega_{e1} \omega_{e1} \omega m_e^2} \int \frac{d\vec{k}_1}{\omega - \vec{k}\vec{v}_g + i0} (\vec{k} \frac{\partial}{\partial \vec{k}_1}) N_{\vec{k}_1}^{\ell} . \quad (8.7)$$

Here $\vec{v}_g = \frac{d\omega_{\vec{k}_1}}{d\vec{k}_1}$ is the group velocity of H.F. oscillations, $N_{\vec{k}_1}^{\ell}$ is the number of Langmuir wave quanta. The value $\epsilon^L(\omega, \vec{k}, W_{k_1}^{\ell})$ describes the dispersion properties of the plasma in the presence of quasisteady Langmuir turbulence when the unequality $\omega \ll \nu_e$ is performed, getting (8.6) we assume L.F. oscillations to be longitudinal.

The analogous expression for $\epsilon^L(\omega, \vec{k}, W^T)$ may be obtained if transverse wave beam passes through a plasma (e.g. Laser one) but in this case

$$\beta^T = \frac{-ik\nu_e^2 e^2 \omega_{pe}^2 A_e}{16\pi^2 \kappa_1 \omega_{e1} \Omega_{e1} \omega m_e^2} \int (1 + \frac{(\vec{k}_1, \vec{k}_1 - \vec{k})^2}{k_1^2 |\vec{k}_1 - \vec{k}|^2}) \frac{d\vec{k}_1}{\omega_{\vec{k}_1} \omega_{\vec{k}_1 - \vec{k}}} \frac{N_{\vec{k}_1}^T - N_{\vec{k}_1 - \vec{k}}^T}{\omega - \Delta\omega_{\vec{k}, \vec{k}_1}} \quad (8.8)$$

Let us examine briefly the equation

$$\epsilon^L(\omega, \vec{k}, W^{\ell}_{k_1}) = 0. \tag{8.9}$$

It was shown above that in the limit of very weak nonlinearity there exist two branches of oscillations described by these equation. In both cases the frequency ω_s may be estimated as $\omega_s \approx k v_i \sqrt{1+\frac{T_e}{T_i}}$. In a weak turbulent plasma principally new branches of oscillations (e.g. the so called "second" sound type) can appear. They become unstable in certain conditions x/.

As an example let us write down the solution of (8.9) in the region $\omega_s \nu_e \ll k^2 v_e^2$, where H.F. acoustic oscillations are in a "quiet" plasma

$$\omega^2 = \frac{1}{2}\omega_s^2 \pm \frac{1}{2}\left(\omega_s^4 + k^2 v_i^2 \nu_e^2 \frac{T_e}{T_i} \frac{W^{\ell}}{n_0 T_e}\right)^{1/2}, \tag{8.10}$$

here $\omega_s = k v_i \left(\frac{5}{3} + \frac{T_e}{T_i}\right)^{1/2}$ is the solution of the linear equation.

The expression (8.10) describes unstable oscillations and is obtained under assumption

$$\frac{W^{\ell}}{n_0 T_e} \gg \frac{k^2 v_e^2}{\nu_e} \frac{m_e}{m_i} \equiv \frac{W^{\ell}_{cr}(\nu)}{n_0 T_e}. \tag{8.11}$$

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x/ As to instabilities in a collisionless limit see /6/ and the References cited there.

In the opposite limit the solutions of (8.9) are stable. Comparing criterion (8.11) and that of instability of collisionless ($\omega \gg \nu_e$) L.F. oscillations /6/ we have

$$\frac{W^{\ell}}{n_0 T_e} > 12 \frac{v_e^2}{v_{ph}^2} \approx \frac{m_e}{m_i} \equiv \frac{W^{\ell}_{cr}}{n_0 T_e}$$

we can see "acoustic" oscillations may be excited at significantly weaker turbulence energies

$$\frac{W^{\ell}_{cr}(\nu)}{W^{\ell}_{cr}} = \frac{k^2 v_e^2}{\nu_e^2} \ll 1 .$$

Then in the same way one can investigate the equation $\epsilon^{L}(\omega, \vec{k}, W^{T}_{k_1}) = 0$ describing excitation of L.F. oscillations by a transverse wave beam /3/.

In this particular case a lot of restrictions associated with the conditions $\mathrm{Im}\,\omega \ll \omega_s$ drop off[x] and the corresponding growth rates can reach the order of collision frequency magnitude.

There are excited (as in /4/) the oscillations passing nearly perpendicular to a transverse wave beam. As it is known the rate of Joule dissipation (due to pair collisions direct) may be estimated from the formula

$$\frac{d}{dt} W^{T} = -\nu_e \left(\frac{\omega_{pe}}{\omega_1}\right)^2 W^{T} .$$

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[x] This condition takes place if the excitation of acoustic oscillations by a transverse wave beam being concerned with a "decay" instability /4/ is considered (as it was proceeded in the section 7 for interaction Langmuir and acoustic oscillations). Moreover in /4/ it was pointed that the condition $\omega_s \gg \mathrm{Im}\,\omega$ could be broken even at a very weak intensity of H.F. weaves. Therefore the methods considered in /6/ are needed to apply to examine interaction transverse and acoustic waves.

As a result of excitation of L.F. perturbation the rate of dissipation of H.F. wave energy can considerably increase and be of order ν_e. This should be taken into account for the interpretation of experiments concerning with interaction lasers with a dense plasma[5].

It should be noted here that above expression for β and, in particular, an increase of "probability of H.F. and L.F. oscillation interaction (in the region $\omega^L \nu_e \ll k^2 v_e^2$) with a $(\nu_e / k_1 v_e)^2$ factor (k_1 is a wave vector of H.F. oscillations) are just for the waves which wave number being of $k_1 < \nu_e / v_e$. If the main part of the spectrum energy of H.F. oscillations is located in the region $k_1 \gg \nu_e / v_e$ it is needed to take into account the term $\pi^T_{e,\alpha\beta}(\partial V^T_{e,\alpha} / \partial x_\beta)$ proportional to $\frac{\partial V^T_e}{\partial x_\alpha} \frac{\partial V^T_e}{\partial x_\beta}$ (in the equation of energy transport) that yields the following renormalization of expression for β (or $S_1 S_2$)[x]

$$\beta^{\ell} = \int \Big[\frac{\vec{k}_1 \vec{k}_2}{k_1 k_2} - \mu \frac{k_1 k_2}{\nu_e^2} v_e^2 \Big\{ 2 \frac{(\vec{k}_1 \vec{k}_2)^2}{k_1^2 k_2^2} - \frac{[\vec{k}_1 \vec{k}_2]^2}{k_1^2 k_2^2} \Big\} \Big] \times \tag{8.12}$$

$$\times V^{(0)}_{k_1} V^{(1)}_{k_2} \delta(k - k_1 - k_2) \, dk_1 \, dk_2 \, ,$$

μ_1 is the numerical coefficient of the order 1. From (8.12) it follows that in the case of $k_1 > \nu_e / v_e$ instead of the factor $\frac{\nu_e^2}{k^2 v_e^2} \gg 1$ there appears larger coefficient of the form k_1^2 / k^2 in all the formulae describing an interaction of L.F. and H.F. oscillations.

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[x] This also follows from the formula (3.3) of paper[7] in which the term $I_{ee}(1,1)$ now should be taken into account.

At last let us briefly describe a qualitative picture of the nonlinear interaction of Langmuir waves. The Coulomb collisions lead to that a critical value of a phase velocity $v_{cr} = v_e N_D$ appears. Both the influence of pair collisions and the nonlinear effects in a plasma themselves are essentially different for oscillations of which phase velocities are larger or less than v_{cr}. In the range of $v_{ph} < v_{cr}$ the energy used to be transferred by induced scattering and decay processes into the region of larger v_{ph} (and the scale of a turbulence). Then the collision effects can be essential for only the dispersion of oscillations (see /7/ section 2) and connected with this for the rate of spectral transfer, nevertheless without changing its direction.

In the range of $v_{ph} > v_{cr}$ the situation is substantially different. Here the nonlinear effects either are very small in comparison with the pair collision absorption or make the energy mainly transfer into the side of smaller v_{ph} (i.e. a turbulence scale decreses analogously hydrodynamic one). This conclusion is confirmed by the allowance of 4-plasmon interactions.

Thus there appears some "wall" stopping the energy flow along the spectrum and so only a small part of turbulence energy can penetrate into the region $v_{ph} > v_{cr}$ (e.g. due to dissipative instability, see section 4 of paper /7/).

In the case of plasma with $T_e \gg T_i$ the picture is more complicated and substansially depends on the plasma macroscopic parameters (see /7/ sec. 5). One may say qualitatively in this case the energy of Langmuir turbulence will apparantly concentrate near $v_{ph_1} = v_e (T_i / T_e)^2 N_D$.

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