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# A COMPUTATIONAL STUDY OF THE CORRELATION FUNCTIONS OF WEAKLY TURBULENT PLASMA

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# A COMPUTATIONAL STUDY OF THE CORRELATION FUNCTIONS OF WEAKLY TURBULENT PLASMA

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The series of experimental works, in which correlation effects in a turbulent plasma were investigated, has appeared in recent years[1-3]. The fact, that correlation functions are one of the most important characteristics of the plasma turbulence, is of interest to such investigations. It is so that the studying of these functions enables us to know both the frequencies of turbulent pulsation and the level of their energy.

The theory now enables us to obtain in a consistent manner the equation describing the correlation effects if the following assumptions hold[4]

1) there has to be quasistationary and weak turbulence,
2) the spectrum is known, i.e. the distribution of energy pulsation density with respect to wave numbers $k$,
3) the relative increase of the correlation curve width has to be small ($\Delta\omega/\omega^{\ell} \ll 1$, where $\Delta\omega$ is the width of the correlation curve, $\omega^{\ell}$ is the frequency of linear plasma oscillations).

As is known the nonlinear dispersion equation then is of the form

$$\epsilon^{\ell}(\kappa)I^{\ell}_{\kappa} = \frac{4\pi i}{\omega}\{ I^{\ell}_{\kappa}\int\Sigma(\kappa,\kappa_1)I^{\ell}_{\kappa_1}d\kappa_1 - $$

$$-8\pi i\int\frac{|S(\kappa,\kappa_1,\kappa_2)|^2}{\omega\,\epsilon^{\ell}(-\kappa_1-\kappa_2)}I^{\ell}_{\kappa_1}I^{\ell}_{\kappa_2}d\lambda \qquad (1)$$

$$\kappa=\{\vec{k},\omega\}\ ,\ d\lambda=\delta(\kappa-\kappa_1-\kappa_2)d\kappa_1 d\kappa_2 \ .$$

Here $I^{\ell}_{\kappa}$ is the sought correlation function of stochastic electric fields, that is determined with the relation

$$< E^{\ell}_{\kappa'}\, E^{\ell *}_{\kappa} > = I^{\ell}_{\kappa}\,\delta(\kappa-\kappa'),$$

$\epsilon^{\ell}$ is the dielectric permeability of "quiet" (linear) plasma, $S(\kappa,\kappa_1,\kappa_2)$ and $\Sigma(\kappa,\kappa_1)$ are concerned with the nonlinear current of the second and third order with respect to $E_{\kappa}$ [5]. Let us define the nonlinear dielectric permeability $\epsilon^{N}(\kappa)$ (which is the coefficient at $I^{\ell}_{\kappa}$ in the induced process terms) via relation

$$\epsilon^{N}(\kappa)=\frac{4\pi i}{\omega}\int\Sigma(\kappa,\kappa_1)I^{\ell}_{\kappa_1}d\kappa_1, \qquad (2)$$

then

$$(\epsilon^{\ell}(\kappa)+\epsilon^{N}(\kappa))I^{\ell}_{\kappa}=\frac{32\pi^2}{\omega^2}\int\frac{|S(\kappa,\kappa_1,\kappa_2)|^2 I^{\ell}_{\kappa_1}\Gamma^{\ell}_{\kappa_2}d\lambda}{\epsilon^{\ell}(-\kappa_1-\kappa_2)} \ . \qquad (3)$$

From (3) follows, if $\omega\to\omega^{\ell}$, the denominator goes to zero. This means that the equation (3) is not more valid and cannot describe correlation effects near the resonance $\omega=\omega^{\ell}$. As is shown in [4] the correct equation based on summarizing the perturbation series is of the form

$$(\epsilon^{\ell}(\kappa)+\epsilon^{N}(\kappa))I^{\ell}_{\kappa}=\frac{32\pi^2}{\omega^2}\int\frac{|S(\kappa,\kappa_1,\kappa_2)|^2 I^{\ell}_{\kappa_1} I^{\ell}_{\kappa_2}}{\epsilon^{\ell}(-\kappa)+\epsilon^{N}(-\kappa)}d\lambda,$$

i.e. may be obtained in formal way by replacing $1/\epsilon^{\ell}(\kappa)$ in the righ-hand side of eq. (3) by $1/(\epsilon^{\ell}(\kappa)+\epsilon^{N}(\kappa))$. As $\epsilon(\kappa)=\epsilon^*(-\kappa)$ we derive finally

$$I^{\ell}_{\kappa}=\frac{32\pi^2}{\omega^2}\int\frac{|S(\kappa,\kappa_1,\kappa_2)|^2 I^{\ell}_{\kappa_1} I^{\ell}_{\kappa_2}}{|\epsilon^{\ell}(\kappa)+\epsilon^{N}(\kappa)|^2}d\lambda. \qquad (4)$$

The integrand of (4) has $\delta(\kappa-\kappa_1-\kappa_2)$, which corresponds to the decay process. The decay process of three waves is possible to be strong forbidden, for example, $\omega^{\ell}\neq\omega^{\ell}_1+\omega^{\ell}_2$ is valid for Langmuir pulsations at any their phase velocities. Therefore, to get the equation for correlations of Langmuir fields one has to take into account the 4-plasmon interactions. We would stress that these interactions play the principal role in forming the quasistationary plasma turbulence spectrum /6/.

In accordance with the foregoing we can get the equation sought for $I^{\ell}_{\kappa}$ (Langmuir correlation)

$$I^{\ell}_{\kappa}=\frac{32\pi^2}{\omega^2}\int\frac{|\tilde{\tilde{\Sigma}}'(\kappa,\kappa_1,\kappa_2,\kappa_3)|^2 I^{\ell}_{\kappa_1} I^{\ell}_{\kappa_2} I^{\ell}_{\kappa_3}}{|\epsilon^{\ell}(\kappa)+\epsilon^{N}(\kappa)|^2}d\lambda' \qquad (5)$$

$$d\lambda'=\delta(\kappa-\kappa_1-\kappa_2-\kappa_3)d\kappa_1 d\kappa_2 d\kappa_3 .$$

Here $\epsilon^N$ includes both the effects described by (2) and the induced part of the 4-plasmon interactions (the definition of $\tilde{\tilde{\Sigma}}'$ and the detailed derivation of eq. (5) see /4/).

Here examining (5) under conditions set forth above we would also suppose that dispersion of oscillations is defined by thermal motion, i.e. $\Delta\omega < k^2 v_e^2/\omega_{pe}$, where $v_e$ and $\omega_{pe}$ are the thermal velocity and Langmuir frequency of electrons, respectively. In this case it can be easily shown /6/ that the kernals of integral eq. (5) do not depend on $\omega$ in the approximation under consideration.

Therefore we can integrate over $\omega_1$, $\omega_2$, $\omega_3$ in the right-hand side of (5), using the normalization

$$\int I^{\ell}_{k,\omega}\, d\omega = W_k . \tag{6}$$

Here $W_k$ is the quasistationary Langmuir turbulence spectrum defined by imaginary part of eq. (1) with allowance for the 4-plasmon interaction terms. The spectrum of the isotropic plasma turbulence was examined in /6/. We use later the results of these works. Let us suppose the solution of eq. (5) is

$$I^{\ell}_{k,\omega} = \Phi(k,\omega) W_k . \tag{7}$$

Substituting (7) into (5) and using (6) we get

$$\Phi(k,\omega) = \frac{1}{\pi} \frac{\gamma^s}{\omega^2 (\mathrm{Re}\,\epsilon^{\ell} + \frac{\omega^N}{\omega})^2 + (\gamma^1)^2} , \tag{8}$$

here

$$\gamma^s = a \int R(k, k_1, k_2, k_3) k^2 \frac{W_{k_1} W_{k_2} W_{k_3}}{W_k} dk_1 dk_2 dk_3 .$$

$$\gamma^{l} = -\nu_{e} - \gamma_{k} + b\int Q(k,k_{1})W_{k_1}dk_{1} +$$

$$+ a\int R(k, k_{1}, k_{2}, k_{3})(k_{1}^{2} W_{k_2} W_{k_3} - k_{2}^{2} W_{k_1} W_{k_3} - k_{3}^{2} W_{k_1} W_{k_2}) dk_{1} dk_{2} dk_{3},$$

$a$ and $b$ are the numerical coefficients depended upon the plasma parameters chosen, $\omega^{N} = \omega_{pe} \frac{\pi}{6} \frac{W}{n_0 T_e}$, $W = \int W_k dk$ is the total energy of turbulent pulsations per $cm^3$.

By virtue of the quasistationary

$$\frac{d}{dt} W_{k} = (\gamma^{l} + \gamma^{s}) W_{k} = 0 , \qquad \text{i.e.} \quad \gamma^{s} = -\gamma^{l} .$$

The function (8) was found using the BESM-6 computer for the following parameters (taking place in astrophysical conditions)

$$\frac{W}{n_0 T_e} = 10^{-6}, \ \omega_{pe} = 5.6 \times 10^{3}, \ T_e = T_i, \ v_e = 4.8 \times 10^{7}, \ \nu_e = 2.4 \times 10^{-6}.$$

Besides we use the spectrum $W_k$ computed in /6/. We have shown later (Fig.1) plots of $\Phi(k,\omega)$ at $x = 0.2, 0.4, 0.6, 3.2$.

The results of calculation show that the width of the correlation curve increases with increasing $x$ and has a maximum at $x = x_0 = 0.6$, which the maximum of turbulence spetcrum corresponds to $(W(x_0) = \max W(x))$. Then the width of curve decreases with increasing $x$. We can see from this that the increasing of the correlation curve width can be smaller than that determined by linear theory $(\Delta\omega \approx \nu_e)$ at some values $x$.

The shift of the curve maximum as function of $x$ is determined mainly by the linear dispersion and is proportional to $k^2 v_e^2 / \omega_{pe}$. Therefore it increases with increasing $x$. It should be noted that the correlation associated with the thermal motion becomes very small at small $x$ and can be less than $\Delta y$ at the same $x$, i.e. in this case the condition $\Delta\omega_L < k^2 v_e^2/\omega_{pe}$ is broken. Then it is necessary to solve eq. (1.5) together with the equation for the spectrum in which kernals depend on frequency with the method above-mentioned.

Thus the correlation effects can affect the form of quasistationary turbulence spectrum. The correlation curve width may be qualitatively estimated at $\Delta\omega_N > k^2 v_e^2 / \omega_{pe}$ from the formula $\Delta\omega_N = \Delta\omega_L \left(\frac{k^2 v_e^2}{\Delta\omega_L \omega_{pe}}\right)^{1/2}$. And in the region of a very small $x$ ($k < \nu_e / v_e$) it is very important to take into account the Coulomb collisions in the kernals of equations describing the spectrum and correlation function.

The calculation results for correlation curves at $\frac{W}{n_0 T_e} = 10^{-5}$ and the same plasma parameters are of qualitative character due to the above-mentioned restriction.

Let us then consider as an example the calculation of correlation effects being at plasma-beam experiment /1/. Using the spectrum found experimentally we obtained the results presented in Fig.2.

Thus, sufficiently good coincidence of calculations and experimental data is observed if the level of turbulence is of $\frac{W}{n_0 T_e} \approx 10^{-3}$. More detailed comparison demands further experimental and theoretical study.

Thus, our results are:

1. There are obtained the plots and calculated the widths of correlation curves of weak turbulent plasma.
2. The correlation effects can affect essentially the form of the stationary turbulence spectrum even if the level of its energy is quite small.

3. The method above developed enables us

a) to associate (in a single way) the width of correlation function with the energy level of turbulence and can be used for diagnostic purposes via correlation measurements;

b) to find (reconstruct) the turbulence spectrum via correlation curves with the aid of the method described in [6] and equation

$$\Delta\omega\,(\text{exper}) = a\,\frac{k^2}{W_k}\int R(k, k_1, k_2, k_3)\, W_{k_1} W_{k_2} W_{k_3}\, dk_1\, dk_2\, dk_3 .$$

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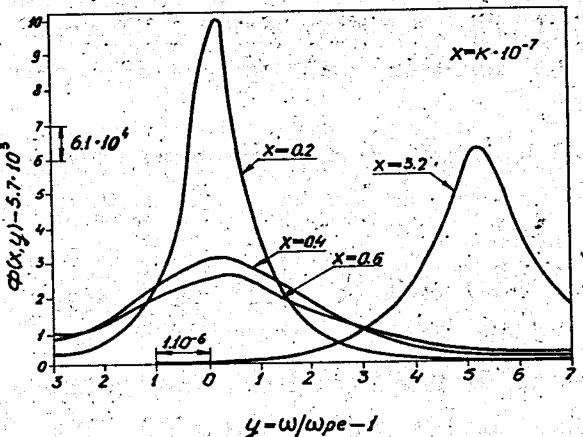


Fig.1. The correlation functions at different values $k$ corresponding to the points on the curve $W_k$ calculated in /6/ ($\frac{W}{n_0 T_e} = 10^{-6}$).

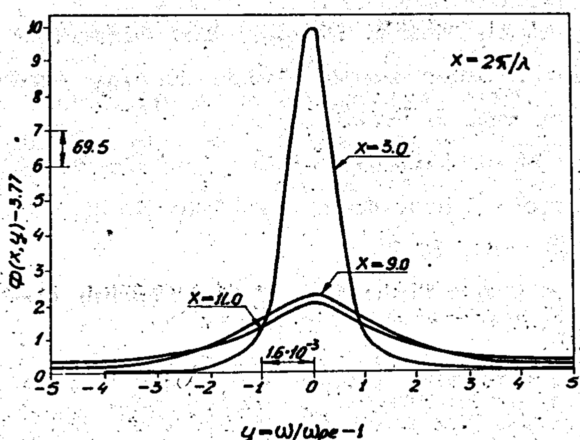


Fig.2. The correlation functions calculated for plasma-beam experiment at $\frac{W}{n_0 T_e} \approx 10^{-3}$.