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ANALYTICAL CONSIDERATIONS OF THE PROTON BEAM EXTRACTION FROM THE DUBNA SYNCHROPHASOTRON BY EXCITING THE RESONANCE $Q_R = 2/3$

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ANALYTICAL CONSIDERATIONS OF THE PROTON BEAM EXTRACTION FROM THE DUBNA SYNCHROPHASOTRON BY EXCITING THE RESONANCE $Q_R = 2/3$

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1. Introduction

The resonant extraction of the proton beam from the 10 GeV Dubna synchrophasotron has been successfully achieved/1–4/ by exciting the resonance

$$Q = \frac{1 \text{ radial betatron oscillation}}{2 \text{ revolution}} .$$

In this note, the resonance $Q_R = 2/3$ is going to be considered, and a rough analytical approach to the problem will be attempted.

As the Dubna synchrophasotron is a weak focusing machine, the most appropriate/5–8/ perturbation seems to be the sextupolar second harmonic one. This 2nd-harmonic function of the azimuth θ should be created by summing two rectangular functions covering two opposite quadrants, as in Fig. 1.

The Fourier development of this function is

$$F(\theta) = \frac{2}{\pi} \left(1 + \sum_{k=0}^{\infty} \frac{\sin 2(2k+1)\theta}{2k+1} \right) \tag{1}$$

which yields as second harmonic $(k = 0)$ component:

$$F(\theta) \approx \frac{2}{\pi} (1 + \sin 2\theta) . \tag{2}$$

The radial motion equation is

$$\frac{d^2x}{d\theta^2} + \theta_R^2 x = - \frac{R_0}{B_0} \Delta B_z(x) F(\theta) , \tag{3}$$

where: $x = R - R_0$, R_0 – equilibrium orbit radius, B_0 – guide field on the equilibrium orbit. $F(\theta)$ is given by (2) and $\Delta B_z(x) \simeq b_0 - b_2 x^2$.

Performing the appropriate substitutions and manipulations it is possible to have

$$\frac{d^2x}{d\theta^2} + \left(Q_R^2 - \frac{2}{\pi}\frac{R_0 b_2}{B_0}x\right)x = -\frac{2}{\pi}R_0\frac{b_0}{B_0} - \frac{2}{\pi}\frac{R_0}{B_0}(b_0 - b_2 x^2)\sin 2\theta \qquad (4)$$

Confining the attention on the first member of equation (4) and considering the realistic radial field index distribution (see Fig. 2) as:

$$n(x) = n_0 + n_1 x + n_3 x^3 \qquad (5)$$

one can write:

$$Q_R^2 - \frac{2}{\pi}\frac{R_0 b_2}{B_0}x = 1 - n_0 - \left(n_1 + \frac{2}{\pi}\frac{R_0 b_2}{B_0}\right)x - n_3 x^3 \qquad (6)$$

i.e., the extra-term coming from the constant $2/\pi$ of formula (2) can only alter the slope of the field - index, without influencing the tuning of the machine which depends only on n_0.

If powers of x higher than one are neglected, equation (4) becomes:

$$\frac{d^2x}{d\theta^2} + Q_R^2 x = -\frac{2}{\pi}R_0\frac{b_0}{B_0}(1 + \sin 2\theta),$$

which yields a harmonic oscillation about a new closed orbit described as follows:

$$x_{c.o.} = -\frac{2}{\pi}\frac{R_0}{Q_R^2}\frac{b_0}{B_0}\left(1 - \frac{Q_R^2}{4 - Q_R^2}\sin 2\theta\right), \qquad (7a)$$

$$x'_{c.o.} = \frac{4}{\pi}\frac{b_0}{B_0}\frac{1}{4 - Q_R^2}\cos 2\theta \qquad (7b)$$

(henceforth the dash ' stands for $\frac{1}{R_0}\frac{d}{d\theta}$).

Finally, it remains to be discussed the effect of the term $\frac{2}{\pi}\frac{R_0 b_2}{B_0}x^2 \sin 2\theta$. Indeed, if this term also is considered separately as the others, equation (4) is reduced to

$$\frac{d^2x}{d\theta^2} + Q_R^2 x = \frac{1}{2}\frac{4}{\pi}\frac{R_0 b_2}{B_0}x^2 \sin 2\theta \qquad (8)$$

which can be integrated by using the Krylov-Bogólubov method/9/, finding well known and deeply discussed results/6/. (Incidentally, equation (8) is nothing but equation (2.1) of Ref. 6 with the non-linear forcing term halved).

However, being Q_R not very far from 2/3, the solution is

$$x = a(\theta)\sin\left(\frac{2}{3}\theta + \phi(\theta)\right) \qquad (9)$$

with $\phi(\theta)$ approaching/5,6/ as a $\pi/6$ is growing up to infinity, and with this amplitude growth faster/10/ than exponential.

Moreover, non-linear oscillations, described by formula (9), take place about the closed orbit (7a, 7b), according to an approximation already used in Ref. 6 and proved corrected in Ref. 11.

No worries for all the non-linear terms of relation (6): it will be proved in Sect. 3 that their contributions to the Krylov-Bogoliubov procedure will be null.

2. Building up of the Sextupolar Perturbation

The practical construction of the non-linear perturbation may imply, if particular care is not taken, the arising of some unwanted effects, which can completely harm the process of the slow resonant extraction.

In fact, being the perturbation built up by feeding currents into a few couples of wires of the pole-face windings, it is possible to have a perturbing field like either the one of Fig. 3 or the one of Fig. 4.

Fig. 3 refers to a perturbing field built up by feeding 250 A into two couples of wires located at -40 cm, -20 cm, and -250 A into other two couples of wires located at 20 cm, 40 cm.

Corresponding data of Fig. 4 are, 500 A fed into a couple of wires at -20 cm, and -500 A fed into a couple of wires at 20 cm. (Notice that for $x = 0$ both fields are roughly equal to 30 Gauss).

Solid lines of both Fig. 3 and 4, refer to a rather classical/12-14/ evaluation of the perturbing field (wires of infinite length, pole faces of infinite size and pole magnetic permeability of infinite value); dashed line of Fig. 3 is the simplest analytical expression $(\Delta B_z(x) = b_0 - b_2 x^2)$ fitting the above evaluation.

Thus, as it is evident observing Fig. 3, the perturbation has almost the due parabolic shape in a radial region as wide as 80 cm. Instead, within the same radial region, the situation of Fig. 4 can be characterized by an analytical expression of the type

$$\Delta B_z(x) = b_0 - b_2 x^2 + b_4 x^4 \qquad (10)$$

(A decupolar perturbation is added to the usual sextupolar one).

The effect of the extra-term $b_4 x^4$ has been extensively studied/15/, considering the equation

$$\frac{d^2x}{d\theta^2} + (1-n)\, x = \frac{1}{2}\left(\frac{dn}{dx}\right) x^2 \sin 2\theta - k\, x^4 \sin 2\theta \,, \qquad (11)$$

$$\left(\left(\frac{dn}{dx}\right) = \text{constant}\right)$$

which yields, following the procedure of Ref. 6, a solution like (9) but with a relation between $\phi(\theta)$ and $a(\theta)$ given by the following expression:

$$\cos 3\phi = \frac{A_0}{a^3} + \frac{3}{2}\frac{a}{\alpha} - \frac{9k\delta}{(dn/dx)^2}\, a \,, \qquad (12)$$

where A_0 is a constant depending upon the initial conditions, $\alpha = 8\delta/(dn/dx)$ and $\delta = n - n_{res}$.

Clearly, for $k = 0$ and any value of δ, equations (11) and (12) become respectively equations (1.2) and (1.4) of Ref. 6.

Fig. 5 shows some possible curves, in the $(\cos 3\phi, a/\alpha)$ plot, derived from equation (12), while Fig. 6 does the same for $k = 0$, being thus very similar to Fig. 1 of Ref. 6.

At this stage, it is necessary to anticipate the experimental procedure.

i) The perturbation is established with such a strength that the biggest amplitude a of the circulating beam is still smaller than α. Consequently, the radial betatron oscillations remain bounded in both cases (curves of Fig. 5 and 6, closer to the $\cos 3\phi$ axis, on the left of the thick curve).

ii) The size of α is slowly reduced to zero by displacing, with a low rate, the initial value of the field index till the appropriate resonant value n_{res} is reached (see next Section for further discussions). This causes all the representative points jumping gradually on the right of the thick curve of both Fig. 5 and 6. But now there are two very different behaviours:

a) $k \neq 0$, no oscillation amplitude succeeds in growing up to infinity, as it is evident in Fig. 5.

b) $k = 0$, Fig. 6 illustrates just what has been discussed in Sect. 1 about formula (9), i.e. "regular" increasing of the amplitudes.

Anyway, in order to avoid this further bounding of the oscillations in the region of interest (roughly given by twice the radial distance between the equilibrium orbit and the first elements of the extraction channel), it suffices to create a parabolic bump wide enough: like that of Fig. 3, for instance.

Discussing/15/ solutions of equation (11) in a (x, x') plot, graphs like those of Fig. 7 are found.

3. Tuning of the Synchrophasotron

Once the perturbation has been turned on, the actual slow resonant extraction of the circulating beam can be attained by slowly tuning the machine in the resonance $Q_R = 2/3$, i.e. by slowly varying the field index till an appropriate value n_{res}, according to what has been mentioned in Sect. 2.

This variation is supposed to take place inside two quadrants only, as it has been done/1/ for the half-integer resonant extraction.

Since two opposite quadrants are involved in the azimuthal construction of the perturbation (Sect. 1), it is suggested to use the wires of the other couple of quadrants for varying the field index, being convenient not to mix up the same quadrant wires with different aims.

It can be shown that, if two opposite quadrants have a field index n and the other two a field index $\bar{n}$, the relation among n, $\bar{n}$, Q_R and L (straight section length) is

$$Q_R = \left(1 + \frac{L}{\pi R_0}\right) \frac{\sqrt{1-n} + \sqrt{1-\bar{n}}}{2} \qquad (13a)$$

which must be born in mind in the considerations.

Notice that formula (13) becomes the very well known relation

$$Q_R = \left(1 + \frac{L}{\pi R_0}\right) \sqrt{1-n} , \qquad (13b)$$

when $n = \bar{n}$.

The practical way of varying the field index in these quadrants consists of creating a field index bump of the appropriate heigth, but having a width which must be assessed with particular care.

In fact, if the very well shaped field index of Fig. 2 is altered by superimposing a too narrow bump, one can have a final configuration like that of Fig. 8.

Clearly, such a curve is no longer described by an equation like (5), being the following expression more appropriate:

$$n(x) = n_0 + n_1 x + n_2 x^2 + n_3 x^3 . \qquad (14)$$

Substituting (14) in the simplest perturbed equation one has:

$$\frac{d^2x}{d\theta} + (1 - n_0)\, x = n_1 x^2 + n_2 x^3 + n_3 x^4 + \frac{1}{2}\left(\frac{dn}{dx}\right) x^2 \sin 2\theta . \qquad (15)$$

Repeating the previous considerations, it is possible to obtain a solution like (9) followed by a relation very similar to (12):

$$\cos 3\phi = \frac{B_0}{a^3} + \frac{3}{2}\frac{a}{a} + \frac{9}{2}\frac{n_2}{(dn/dx)}\, a \qquad (16)$$

(Notice that odd powers of x of (14) do not contribute to (16)).

The sole differences between (16) and (12) are given by the constants depending on the initial conditions and by the coefficients of the linear terms.

Curves very similar to those of Fig. 5 are shown in Fig. 9, where a further bounding of the oscillations is evidenced.

An approximate criterion for minimizing this unwanted effect can be found in the following way.

i) Let B_0/a^3 be neglected with respect to other terms of (16). (A sort of asymptotic solution).

ii) Let $\bar{a}/a$ be the abscissa of the point where the asymptotic solution changes slope (its derivative is null).

Once at this stage, it suffice to have $\bar{a}$ much bigger than a; this yields:

$$n_2 \ll \frac{1}{12\delta} \left(\frac{dn}{dx}\right)^2 . \qquad (17)$$

An equation quite similar to (15) has been extensively studied/16,17/ by using (x, x') plots and FORTRAN programming.

Fig. 10 shows a sample of the results obtained/16/ (things here have been slightly modified).

Notice that the differences between graphs of Fig. 7 and 10 are very small, and Fig. 11 just illustrates how it is possible to transform curves of Fig. 7 (upper part of Fig. 11) into curves of Fig. 10 (lower part of Fig. 11).

Bearing in mind all these considerations, and recalling that the machine is turned in practice by lowering n, it is suggested to add a field index bump, like that of Fig. 12, to the unperturbed field index of Fig. 22

This bump can be obtained by feeding -255A into four couples of wires located at -30 cm, -10 cm, 10 cm, 30 cm, respectively.

The global result is shown in Fig. 13, where it is evident that no x^2 contribution has arisen, apart from some slight "ripples" which could be reasonably assumed as being harmless.

4. Conclusions

An attempt is done now for giving some hints on how to proceed in practice. As it has been discussed in Sect. 2, the perturbing field can be written as

$$\Delta B_z = b_0 - b_2 x^2 ,$$

where $b_0 = 30$ Gs, $b_2 = 1.45 \times 10^{-2}$ Gs/cm^2, for currents of 250 A.

Putting this value of b_0 into (7a), (7b), it can be obtained:

$$x_{c.o.} = -9.65 \text{ cm} + (1.20 \text{ cm}) \sin 2\theta$$

$$x'_{c.o.} = (0.85 \text{ mrad}) \cos 2\theta .$$

having considered $R_0 = 2.8 \times 10^3$ cm and $B_0 = 1.262 \times 10^4$ Gs.

If this quite relevant inward displacement of the closed orbit has to be restrained, it is necessary to work with currents equal to or smaller than 250 A. Unless the sextupolar 2nd-harmonic pattern is built up the classical/5,6/ way, obtaining thus the very well known/6,11/ azimuthal function $(4/\pi) \sin 2\theta$. Certainly, this solution does imply relevant advantages: the constant displacement of the closed orbit is completely cancelled and a stronger perturbation is attained by feeding the same amount of current.

It is easy to show that

$$\left(\frac{dn}{dx}\right) = \frac{4}{\pi} \frac{b_2 R_0}{B_0} = 4.1 \times 10^{-3} \text{ cm}^{-1}$$

having performed the due substitutions.

(Notice that , in formula (6),

$$\frac{2}{\pi} \frac{b_2 R_0}{B_0} = 2.05 \times 10^{-3} \text{ cm}^{-1}$$

being comparable to

$$n_1 \approx 3.5 \times 10^{-3} \text{ cm}^{-1}$$

as it can be deduced graphically by Fig. 2).

Then, recalling the definition of a, it is possible to write

$$\frac{a}{\delta} = \frac{8}{(dn/dx)} = 2.\ 10^3 \text{ cm} . \qquad (18)$$

A few words must be spoken about the value to be assigned to δ.

In fact, while the operating field index is always $n = 0.67$, the field index n_{res}, which corresponds to the $Q_R = 2/3$ resonance, changes noticeably with respect to the nominal $n_{res} = 0.63$ (unperturbed machine).

First of all, setting Q_R equal to 2/3 in (13), one has $\bar{n} = n_{res} = 0.59$, instead of 0.63, valid when all the quadrants are involved in turning synchrophasotron.

Independently, it has been proved/11/ and confirmed/17,18/ that a perturbation like that of Fig. 3 depresses n_{res} far below the nominal value. Moreover, the introduction of the straight sections (so far neglected according to criteria of Ref. 5,6) causes a a further lowering/18,19/ of n_{res} . But, as it is clear from Figs. 12, 13, it is not a serious problem to displace tha field index value of big amounts.

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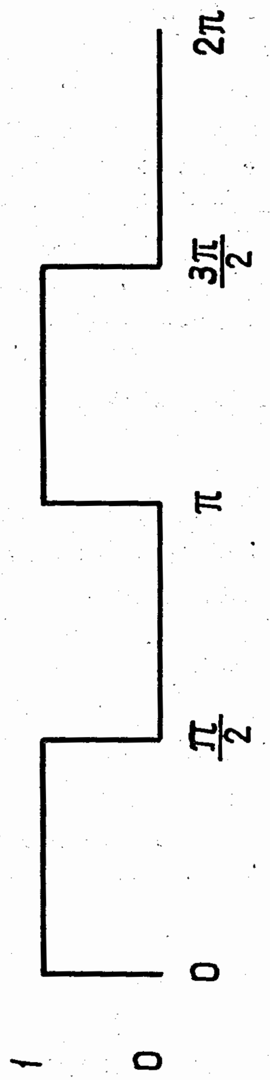

Fig. 1. Proposed azimuthal shape of the perturbation.

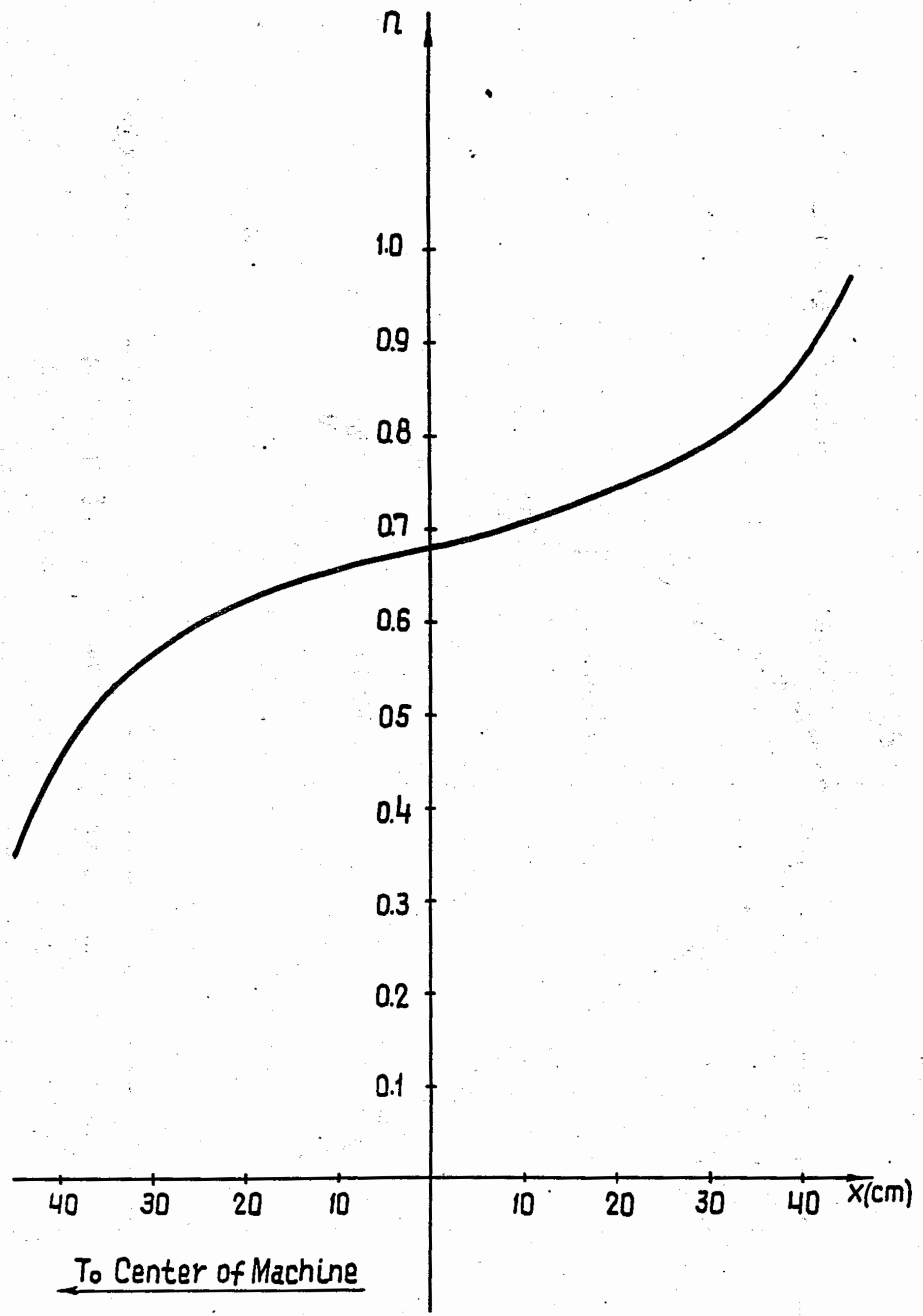

Fig. 2. Average radial field index distribution of the Dubna synchrophasotron. $x = R - R_0$, where R is the radius of a general orbit, and $R_0 = 28$ m is the equilibrium orbit radius.

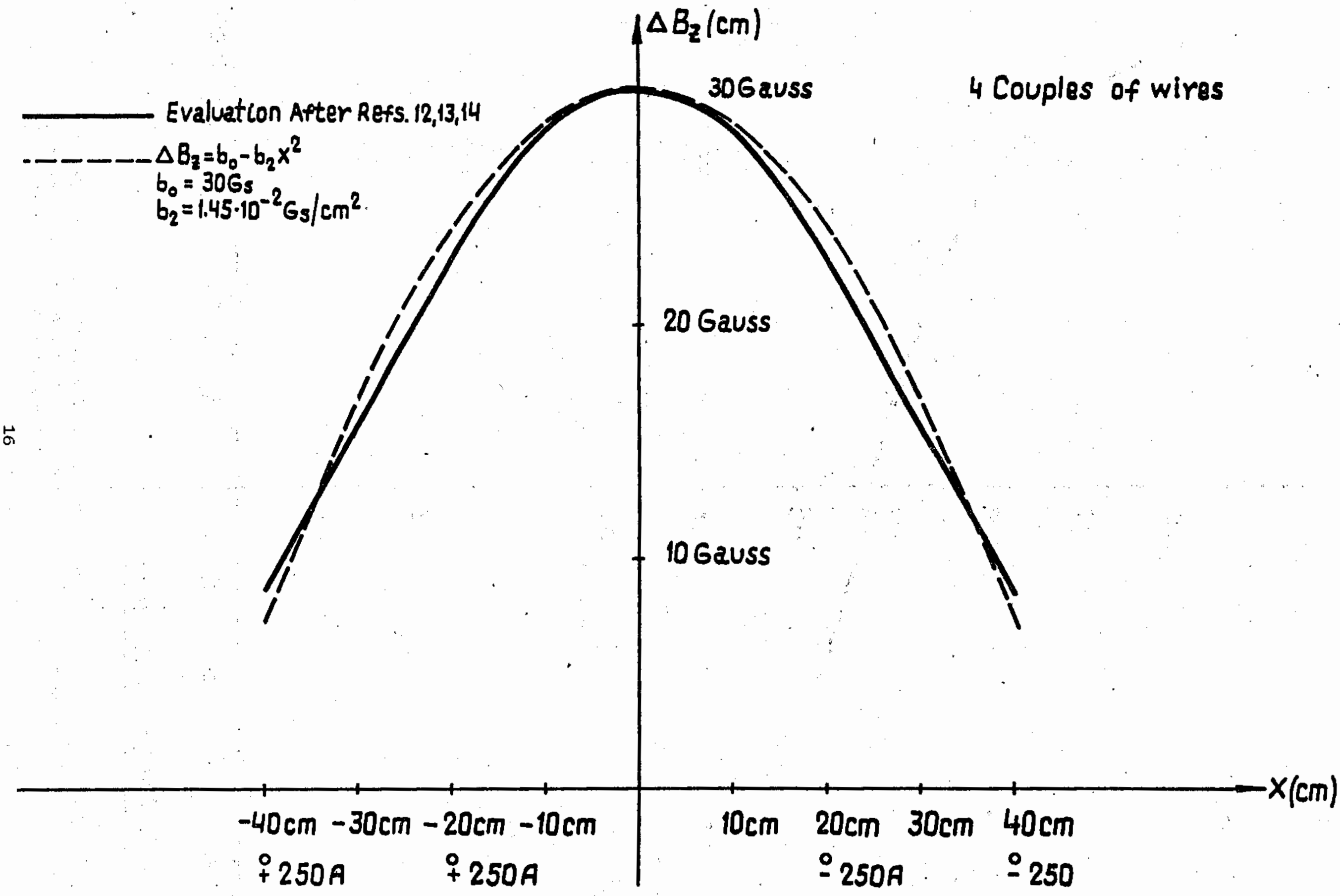

Fig. 3. Sextupolar perturbation created by 4 couples of wires fed by ± 250 A. Dashed line refers to the simplest analytical best fit.

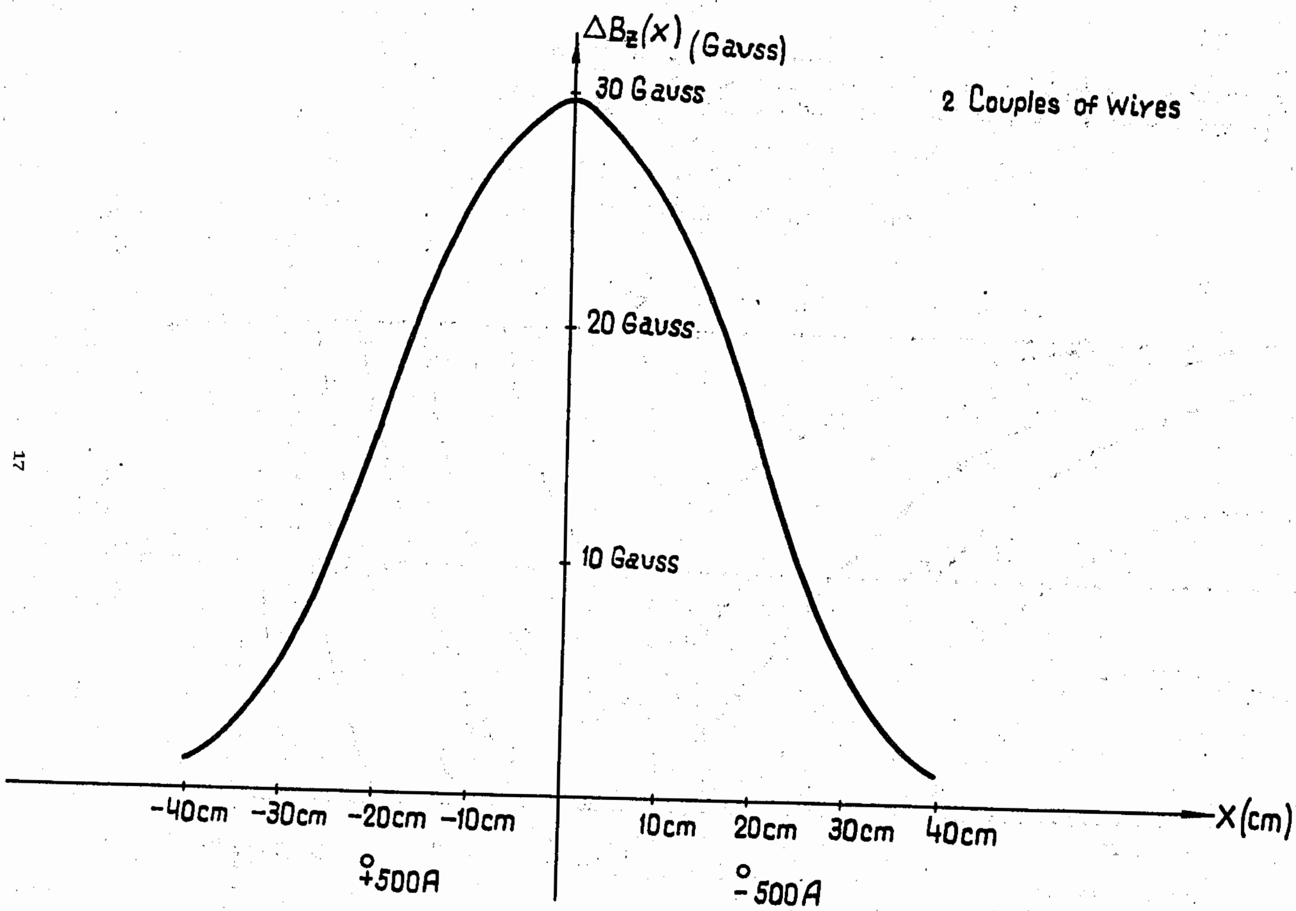

Fig. 4. Sextupolar perturbation created by 2 couples of wires fed by ± 500 A.

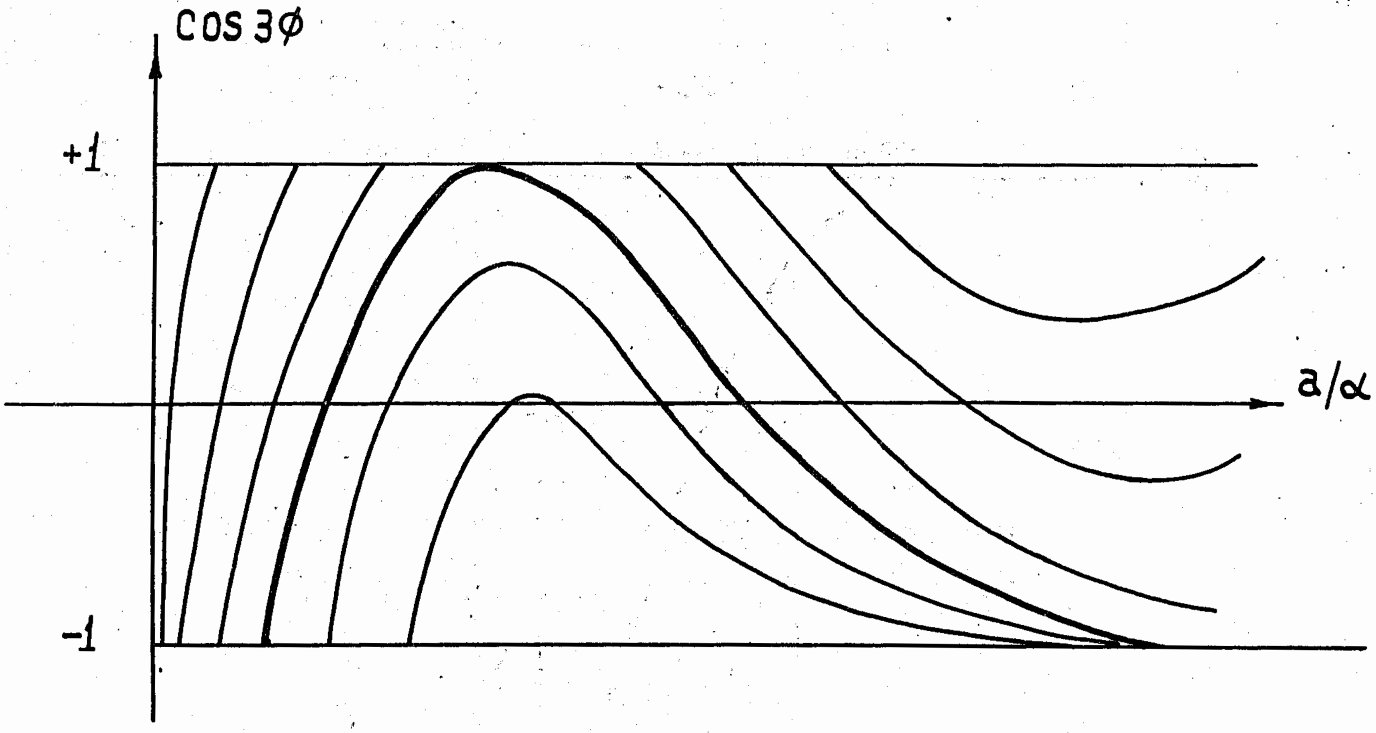

Fig. 5. Representation of the function

$$\cos 3\phi = \frac{A_0}{a^3} + \frac{3}{2}\frac{\alpha}{a} - \frac{9k\delta}{(dn/dx)^2}\,a$$

in the a/α, $\cos 3\phi$ plot. All the oscillations are bounded. Thick curve defines regions where oscillation bounding takes place in different ways.

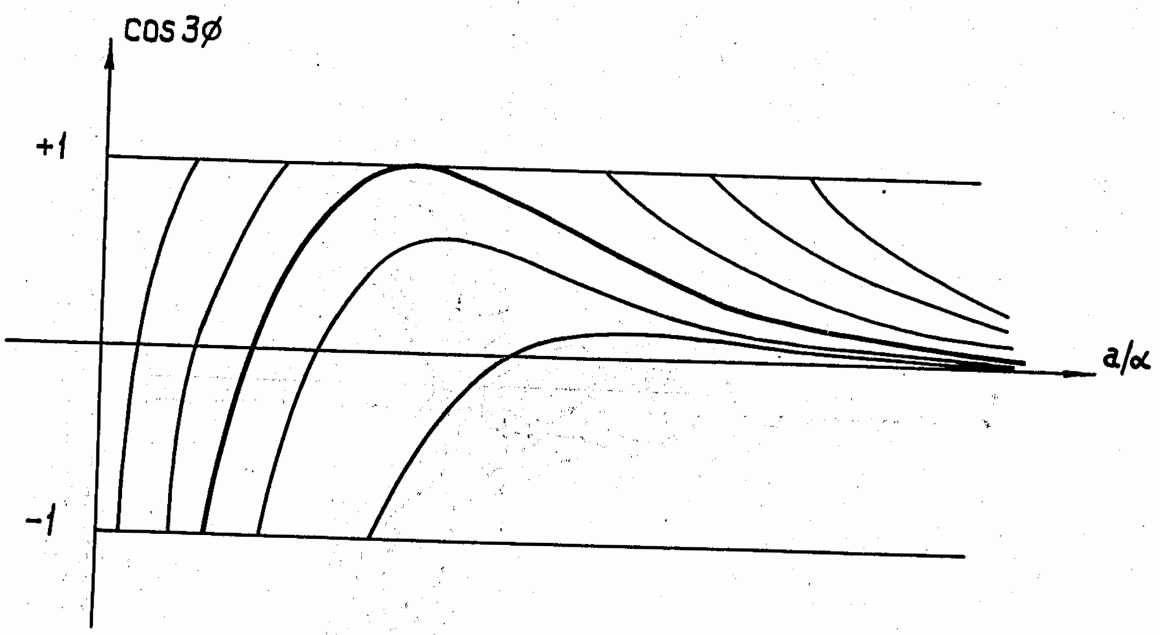

Fig. 6. Representation of the function

$$\cos 3\phi = \frac{A'_a}{a^3} + \frac{3}{2}\frac{\alpha}{a}$$

in the a/α, $\cos 3\phi$ plot. Thick curve parts bounded oscillations (on the left) from oscillations which grow up to infinity (on the right).

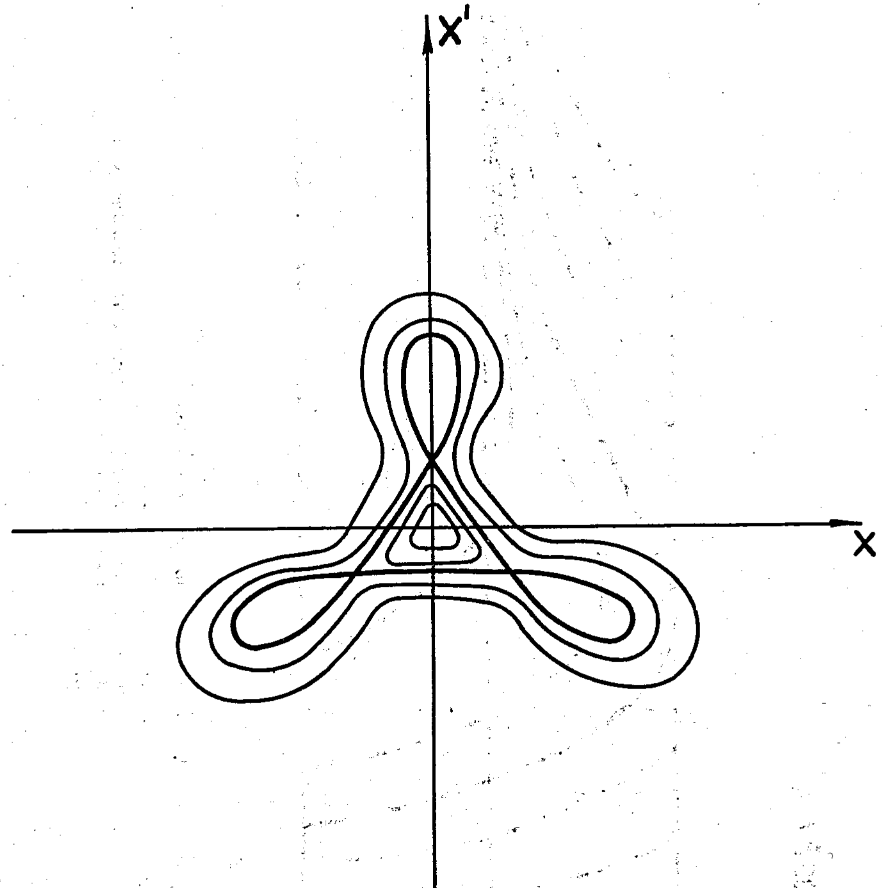

Fig. 7. Possible stroboscopic representation in the (x, x') plot of solutions corresponding to Fig. 5 $(x' = \frac{1}{R_0} \frac{dx}{d\theta})$

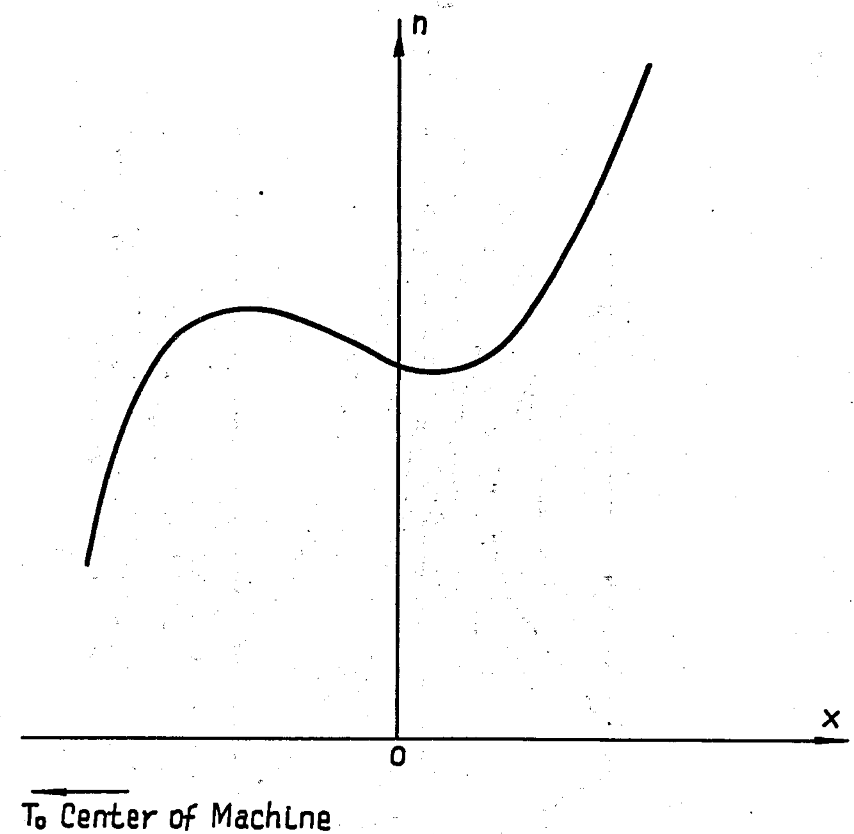

Fig. 8. Field index shape which must be avoided.

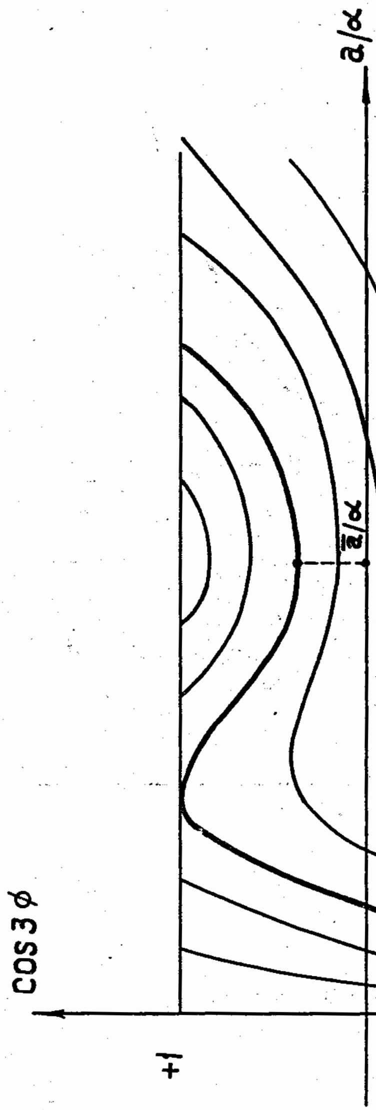

Fig. 9. Representation of the function

$\cos 3\phi = \frac{B_0}{a^3} + \frac{3}{2}\frac{a}{\alpha} + \frac{9}{2}\frac{n_2}{(dn/dx)}\,a$

In the a/α, $\cos 3\phi$ plot. All the oscillations are bounded. Thick curve defines regions where oscillation bounding takes place in different ways. $\bar{a}/\alpha$ gives a useful limiting point.

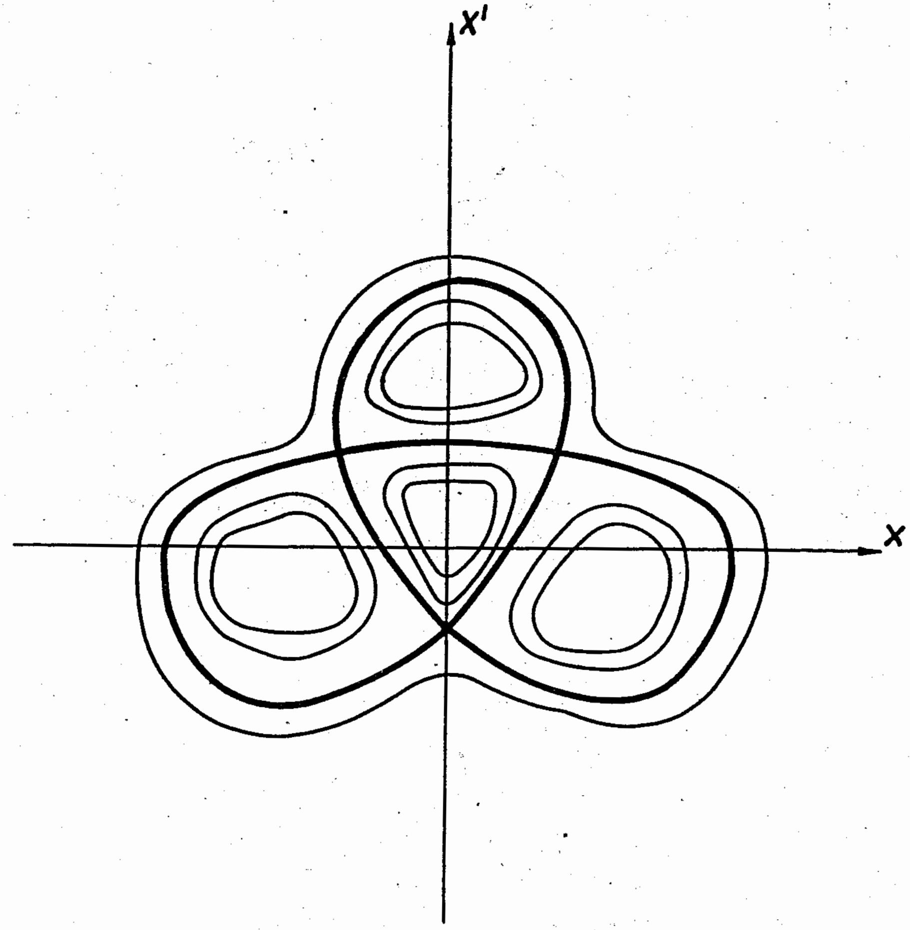

Fig. 10. Possible stroboscopic representation in the (x, x') plot of solutions from Fig. 9.

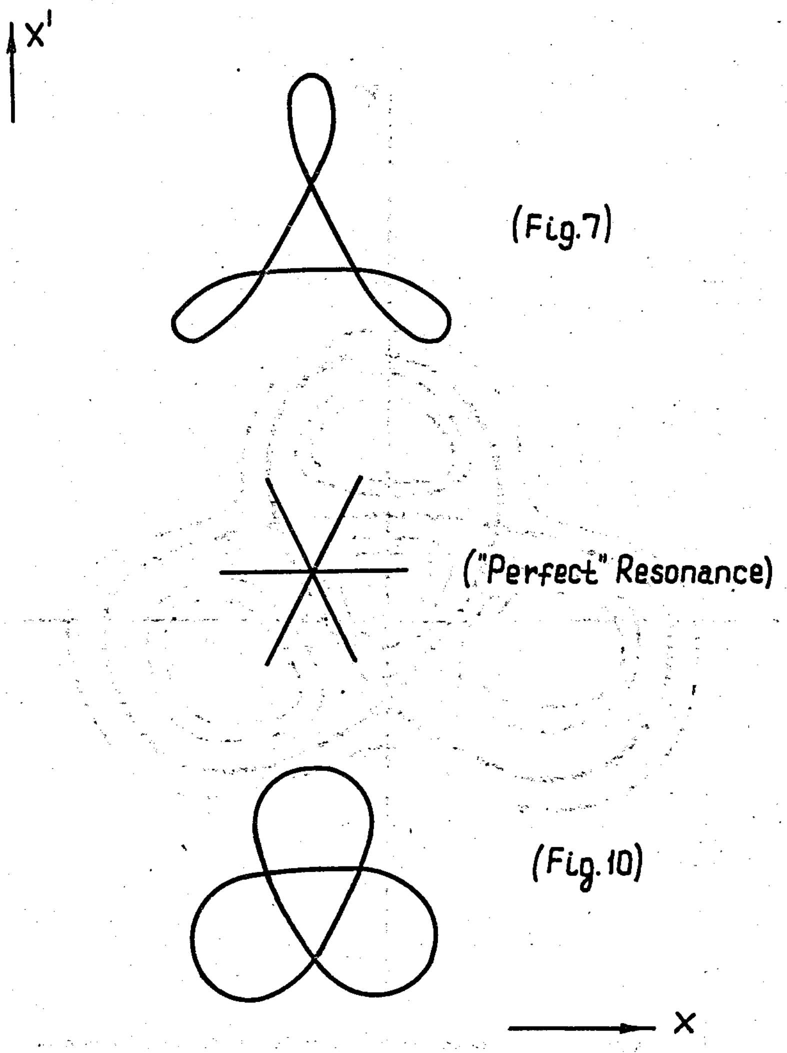

Fig. 11. Illustration of the equivalence between plots of Fig. 9 and Fig. 11.

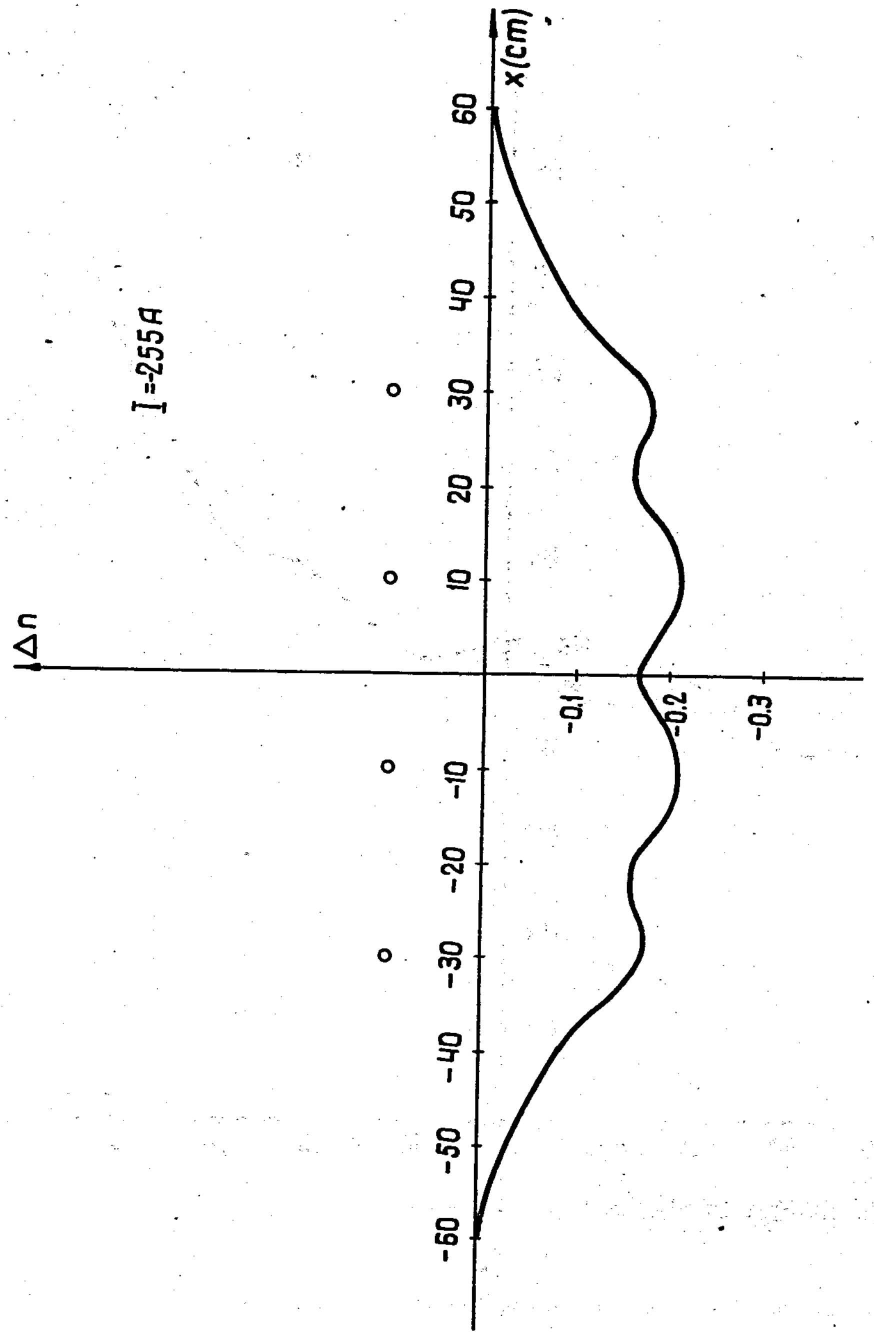

Fig. 12. Proposed field index bump as a correction for tuning the machine. Currents fed -255 A.

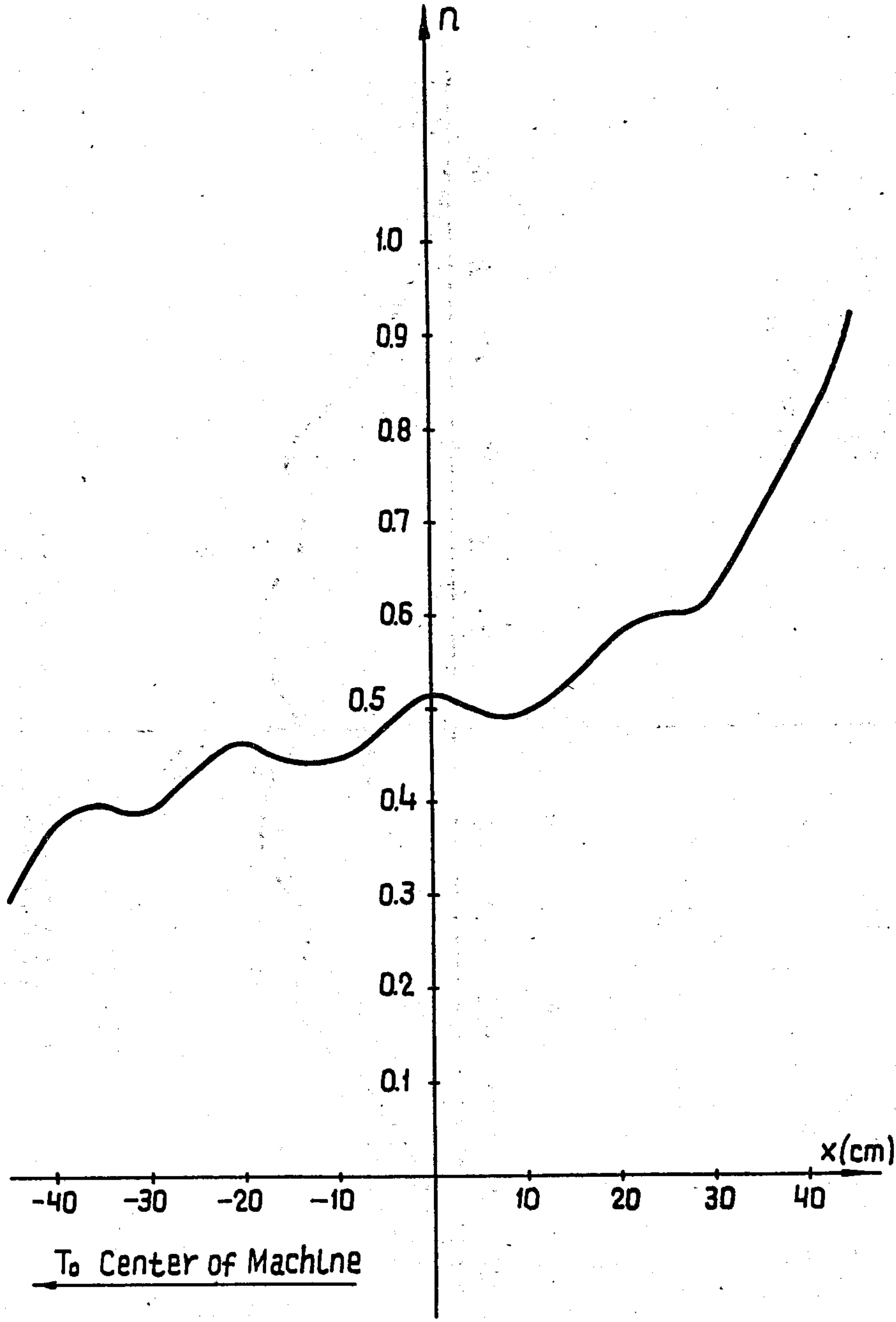

Fig. 13. Corrected field index, after having created the bump of Fig.12.