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ON THE THEORY OF THE RADIATIVE INSTABILITY OF THE RELATIVISTIC CHARGED RINGS

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ON THE THEORY OF THE RADIATIVE INSTABILITY OF THE RELATIVISTIC CHARGED RINGS

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We consider the radiative instability in the charged beams of high density rotating in the magnetic field. Investigations of this problem are of some interest for the new methods of acceleration as well as for generation of microwaves. Radiation by such beams has been observed in a number of papers /1,2,3/.

One can assume that the coherent synchrotron radiation on long waves much larger than the cross section dimensions of the ring might be the most dangerous one. In this case it is possible to use the kinetic method /4,5/ suitable for thin beams. Due to the high intensity of the radiation the non-linear effects which can strongly restrict the fluctuation amplitudes must be considered. As to the linear theory it can estimate the initial increments corresponding to the transition time of the system into the non-linear stage.

We are interested in the longitudinal instability of the relativistic electron ring placed inside the infinitely long and perfectly conducting tube. To describe the particle motion in the initially uniform ring with fluctuation appearing there we use the kinetic approach developed in papers[4,5]. The motion of particles is examined in the phase space of the azimuthal angle and the canonical moment. Perturbations are supposed to be small enough. The linear approximation is used in the equation. The field is averaged over the cross section of the ring, i.e. only the perturbations with the wave lengths much larger than the small dimesnions of the ring are regarded.

The electromagnetic field connected with the fluctuation is presented as a superposition of chamber natural oscillations with the separation of the waves into longitudinal and transverse[6]. As a result of the solution of the self-consistent equation the dispersion formula is obtained:

$$1 = \frac{ie^2 \Omega N}{(2\pi)^2 n} Z(\Omega, n) \int \frac{\frac{d\psi_0}{dw}\, dw}{\Omega - n\omega(w)}, \tag{1}$$

where e is the electron charge, Ω, n is the frequency and number of perturbation harmonic, $\omega(w)$ is the frequency of particle rotating in a ring, w is the canonical moment, ψ_0 is the initial function of the distribution of particles in a beam, N is the total number of particles, value $Z(\Omega,n)$ is the chamber impedance with the beam and it is equal to:

$$Z(\Omega,n) = \frac{16\pi^3 R^2 i n^2}{a_z \Omega} \sum \frac{J_n^2(\lambda_{np} R/b)}{\lambda_{np}^2 J_{n+1}^2(\lambda_{np})} - \frac{8\pi^3 R^2 b i}{a_z^2 \Omega} \sum_p \Big\{ \frac{n^2 J_n^2(\lambda_{np} R/b)}{\lambda_{np}^2 J_{n+1}^2(\lambda_{np})} \cdot \frac{\sin\phi/2 + i\cos\phi/2}{\sqrt{\frac{\lambda^2 b^2}{c^2} - \lambda_{np}^2}} \times$$

$$\times \left(1 - e^{2i\frac{a_z}{b}\sqrt{\frac{\Omega^2 b^2}{c^2} - \lambda_{np}^2}}\right) +$$

$$+ \frac{R^2 J_n'^2(\lambda'_{np} R/b)\, \Omega^2}{c^2 J_n^2(\lambda'_{np})(1 - n^2/\lambda'^2_{np})} \cdot \frac{(\sin\phi/2 + i\cos\phi'/2)\left(1 + 2i\frac{a_z}{b}\sqrt{\frac{\Omega^2 b^2}{c^2} - \lambda'^2_{np}} - e^{2i\frac{a_z}{b}\sqrt{\frac{\Omega^2 b^2}{c^2} - \lambda'^2_{np}}}\right)}{\left|\sqrt{\frac{\Omega^2 b^2}{c^2} - \lambda'^2_{np}}\right| \left(\frac{\Omega^2 b^2}{c^2} - \lambda'^2_{np}\right)} \Big\}$$

In the formula for the impedance: c is the velocity of light in the free space, R is the mean radius of a ring, a_z is the small dimension of a ring along the tube axis, b is the tube radius, λ_{np} and λ'_{np} corresponding roots of the Bessel functions J_n and J'_n, angle $\phi(\lambda_{np})$ is the argument of the complex number $\frac{\Omega^2 b^2}{c^2}-\lambda^2_{np}$ and $\phi'(\lambda'_{np})$ is determined analogically. Investigations of eq.(1) together with (2) make it possible to distinguish various types of instabilities with the various distances between the ring and the wall. First of all there might appear the case where the condition of the transverse wave radiation does not take place, then the instability of the negative mass only appears to be under discussion. In this case a certain selected harmonic of a mean rotation frequency approximately coinciding with the frequency of a perturbation wave differs greatly from any natural frequency of the chamber corresponding to this harmonic. In particular this condition takes place when the ring is near to the wall. In the nonrelativistic case the impedance appears to have always the capacity character $Z=iA$ and the instability of the negative mass type can appear at any distance from the wall. In the relativistic case the impedance can be both capacity impedance and inductive one ($Z=-iA$), in the latter case the instability does not appear at all./7/. For the ring with dimension R much larger than the distance from the ring to the wall ($b-R \ll R$) even in the case of capacity impedance a substantial decrease of the increment proportional to the value $\sqrt{\frac{b-R}{b}}$ occurs. For the thin ring the increment additionally decreases in $\sqrt{\frac{a_z}{b-R}}$ times as compared with E layer. When fixing a harmonic number of perturbation and changing the distance between the ring and the wall it is possible to note that alternation of the regions where the impedance possesses a clearly seen character of capacity, induction and activity.

The regions with active or complex impedance with a large real part correspond to the resonances on the natural frequencies of the chamber and the radiation of the electromagnetic energy. Here we can distinguish two cases:

1. Radiation occurs practically at one natural oscillation when the ring is close enough to the wall and the time of the instability development is much larger than the time the signal passes from the ring to the wall.

2. The resonances combine all together into a continuous spectrum and the radiation occurs in the same way as in the free space with the corresponding condition of time opposite site to the first variant.

In the first case the solution of the dispersion equations for the ring with a small spread of energies in the uniform magnetic field gives the instability increment

$$\frac{\mathrm{Im}\,\Omega}{n\,\omega_0} = -\left(\frac{\nu}{\gamma}\;\frac{2\pi f(n)}{n^2}\right)^{2/5} \qquad (3)$$

$f(n)$ – is the function growing as $n^{4/3}$ to $n \approx \gamma^3$ and then exponentially decreasing. With the increase of the spread the increments decrease substantially. The results keep unchanged in a wide class of distribution functions.

In the free space the real part of the impedance corresponds to the well-known expression for the synchrotron radiation, i.e. generalization of the Shott formula. A thershold of the radiation instability can be seen in the analysis of the dispersion equation:

$$\frac{\nu}{\gamma} \approx a\,n^{2/3}\,\epsilon^2 . \qquad (4)$$

Here a – is the unity coefficient, $\epsilon = \frac{\Delta\omega}{\omega}$ is the relative spread of the rotating particle frequencies in a beam. Occurance of the instability threshold for the initial harmonics is connected with the collective radiative Landau damping competing with the hydrodynamical radiation instability. Because of the threshold the instability development increases the spread of the energies and the instability disappears. Estimation of the time characterizing the spread increase at the initial stage can be obtained from the hydrodynamical equations, averaged over the particle orbits. Hence:

$$t = \frac{R}{2\, c n^{2/3} \sqrt{\nu/\gamma}} \ell n \left[\frac{1}{n_{max}} \left(\frac{N}{\Delta N_n}\right)^2 \left(\frac{\Delta\omega}{\Delta\omega_{np}}\right)^2 \right] \qquad (5)$$

ΔN_n is the initial fluctuation of the particles on the maximum harmonic, n_{max} is the maximum number of harmonic, satisfying approximation of the thin beam.

However time (5) determines only the intial rate of the instability growth. Non-linear effects appear to be essential long before the approach of the critical spread. Considering non-linear interactions of the harmonics the maximum field amplitude of n harmonic is this:

$$\epsilon_n \sim \frac{\nu}{\pi^3} n^{1/3} \frac{mc^2}{eR} = \epsilon_{sur}\, n^{1/3} \frac{a}{R}. \qquad (6)$$

$$\epsilon_{sur} = \frac{eN}{2\pi R a}.$$

As $\epsilon_n \ll \epsilon_{sur}$, then it is possible to conclude that at the non-linear stage the beam parameters change insignificantly and the radiation instability is not of a great danger for the ring.

In conclusion we should remark that in the experiments the coherent synchrotron radiation quickly disappears/2, 3/ without significant changes in the beam parameters. This fact is in accordance with the ideas given above.

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