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CALCULATION OF THE MAGNETIC FIELD
OF THE ISOCHRONOUS
CYCLOTRON SECTOR MAGNET
BY THE INTEGRAL EQUATION METHOD

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## Calculation of The magnetic field OF THE ISOCHRONOUS <br> CYCLOTRON SECTOR MAGNET BY THE INTEGRAL EQUATION METHOD

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At the Laboratory of Nuclear Problems, JINR, a possibility of the construction of a powerful neutron source with a neutron beam energy of 14 MeV and a flux of $10^{14}-10^{15} n /\left(\mathrm{cm}^{2} \cdot \mathrm{sec}\right)$ is investigated. The ring isochronous cyclotron with a 35 MeV deutron energy and a 10-100 mA beam current will be the main accelerator facility for producing the above-mentioned neutron flux. The radial-sector structure with a four-fold symmetry is conceived for the cyclotron.

The calculational results of the magnetic field of the cyclotron magnet are given in the report. Figure 1 shows the upper view of the calculational magnet model along with the sector current coll. The air gap height of the magnet is 5 cm , whereas the ratio of the maximum pole length to the gap height is about 30. The external diameter of the four sector system including the yoke is 8 m . The magnetic field value in the air gap is about 18 kGs . The field has been calculated using the computer code MAGSYS by solving a 3 -dimensional vector integral equation for the magnetic field induction ${ }^{\prime 2-3}$. The magnet body volume has been considered as a set of triangular prisms or parallelepipeds. The values of the induction vector and the magnetic permeability have been assumed to be constant inside the above said regions. The embedding method ${ }^{/ 4 /}$ has been used to solve the nonlinear algebraic system, which approximated the vector integral equation under above conditions. The resulting algebraic system is as follows:

$$
\begin{equation*}
B_{m n}-\sum_{i=1}^{M}\left(1-\frac{1}{\mu_{i}}\right) \sum_{j=1}^{3} f_{i j}^{m n} B_{i j}=\Psi_{m n} \tag{1}
\end{equation*}
$$



Fig. 1. A display representation of the sector magnet (the upper view).
where $m=1,2,3, \ldots M, n=1,2,3 . \quad \mu_{i}=\phi\left(\sum_{i=1}^{3} 1 B_{i j}^{2}\right) \quad$ is a magnetic permeability, $f_{i j}^{m n}$ is a field induction independent function, $\Psi_{m n}$ is a given function of coordinates. An abbreviated form of equation (1) is given by

$$
\begin{equation*}
\mathrm{A} \overrightarrow{\mathrm{x}}=\overrightarrow{\mathrm{b}} \tag{2}
\end{equation*}
$$

where the elements of the matrix A are functions of the solution vector $\vec{x}$. Consider, that the relation

$$
\begin{equation*}
\mu(t)=1+[\mu(1)-1] \cdot t \tag{3}
\end{equation*}
$$

takes place under the condition $0 \leq t \leq 1$. Then the vector $\vec{x}$ is a function of the parameter $t$ and the solution of equation (2) is reduced to that of the Cauchy problem with the initial value $\vec{x}(0)=\vec{b}$. The calculation has been made with a constant step of the parameter $t$ and with the recalculation of the initial $\quad \vec{x}_{0}$-value for the $\mathrm{t}=\mathrm{t}_{\mathrm{n}} \quad$ according to the expression

$$
\begin{equation*}
\vec{x}_{0}\left(t_{n}\right)=2 \vec{x}\left(t_{n-1}\right)-\vec{x}\left(t_{n-2}\right) \tag{4}
\end{equation*}
$$

The solution was improved at every $t_{n}$ using the iterative formula

$$
\begin{equation*}
\overrightarrow{\mathrm{x}}_{\mathrm{k}+1}=\overrightarrow{\mathrm{x}}_{\mathrm{k}}+\alpha\left(\mathrm{A}_{\mathrm{kn}} \overrightarrow{\mathrm{x}}_{\mathrm{k}}-\overrightarrow{\mathrm{b}}\right), \tag{5}
\end{equation*}
$$

where the $a$-value is given by

$$
\begin{equation*}
a=-\gamma \frac{\left(A_{k n} \vec{x}_{k}-\vec{b}, A_{k n}\left[A_{k n} \vec{x}_{k}-\vec{b}\right]\right)}{\left\|A_{k n}\left(A_{k n} \vec{x}_{k}-\vec{b}\right)\right\|^{2}} \tag{6}
\end{equation*}
$$

to obtain the maximum convergence rate of the process. In expression (6) it is assumed that

$$
\begin{equation*}
\left\|A_{k+1, n}\right\| \approx\left\|A_{k, n}\right\| . \tag{7}
\end{equation*}
$$

Condition (7) was provided with the help of the factor $y \leq 0.2$. In the calculations it sometimes occured that the magnitude of $\alpha$ tended to zero at some values of $\mathrm{t}_{\mathrm{n}}$. In this case instead of eq. (5) we used the expression

$$
\begin{equation*}
\overrightarrow{\mathrm{x}}_{k+1}=\overrightarrow{\mathrm{x}}_{\mathrm{k}}+\overrightarrow{\mathrm{D}}_{\mathrm{k}} \tag{8}
\end{equation*}
$$

where the vector $\vec{D}_{k}$ could be defined from the conditions

$$
\begin{align*}
& \left(\vec{D}_{k}\left\lceil A_{k n} \vec{x}_{k}-\vec{b}\right\rfloor\right)=0  \tag{9}\\
& \left|\vec{D}_{k}\right|=\left|A_{k n} \vec{x}_{k}-\vec{b}\right| \tag{10}
\end{align*}
$$

From our point of view, the main advantage of the used method for solving the algebraic system is a possibility to have a converging process in all calculations performed by the authors. The convergence takes place even for the magnet with a "bad"/ partition of the magnet volume into subregions. Other advantages are a possibility to keep in the computer core storage only a part of the system matrix and central processor execution time which is comparable with that of the GFUN3D code ${ }^{1 / 2}$ when running jobs at the CDC 6500 .


Fig. 2. A "hill" field performance.


Fig. $3 a$



The sector magnet was calculated with a various number of the subregions of constant magnetization. The maximum number was 192 subregions which was equivalent to the $576 \times 576$ algebraic system. The best results obtained for the triangular prism partition (Fig. 1) are presented by dashed lines in Figs. 2-4. The solid lines present the magnetic field measurement data for the 1 : 7 model of the magnet. As one can see in Fig. 4 the difference between the experimental and calculated data is about $0.5 \%$ for the azimuthally mean field, $5 \%$ for the flutter, $2 \%$ for the amplitude of the main harmonic in the radial range $R=15-28 \mathrm{~cm}$. Near the minimum and the maximum radial position of the observation point the deviations of the calculated curves from experimental ones are largest due to the strong nonuniformity of the pole field in the vicinity of these regions. The assumption of the piece-wise constant magnetization distribution in these parts of the pole are least valid.


Fig. 4. Results of the harmonic analysis of the field.

The evaluation of the magnet field using another method and a magnet model design based on it has made it possible to obtain the discrepancy between the measured mean field $\mathrm{B}_{0}$ and the required isochronous field $B_{R}$ in the range of $10 \%$ (Fig. 4). As seen from Fig. 4 the application of the described calculations could improve considerably the accuracy of the $B_{0}$ evaluation apart from the pole ends.

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