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CRITICAL CURRENT OF SUPERCONDUCTING MAGNETS WOUND FROM AN ANISOTROPIC SUPERCONDUCTING CONDUCTOR

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Гласник И., Ергел М., Кабат Д.

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Критический ток сверхпроводящих магнитов, намотанных из анизотропного сверхпроводника

Приведен общий алгоритм определения критического тока I_c сверхпроводящих магнитов, намотанных из анизотропного сверхпроводника. Для цилиндрических соленоидов рассчитан и в форме графиков приведен коэффициент $K_\phi(\alpha,\beta)=B_\phi^{max}/B_0$, при помощи которого можно определить I_c. Выведены формулы, по которым для соленоидов можно вычислить I_c в случае, когда характеристику сверхпроводника $I_c - B|_\psi$ можно описать аналитически.

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Critical Current of Superconducting Magnets Wound from an Anisotropic Superconducting Conductor

A critical current calculation procedure for a superconducting magnet wound from an anisotropic superconductor is shown. For axisymmetric solenoids with rectangular cross section and uniform current density the coefficient $K_\phi(\alpha,\beta) = B_\phi^{max}/B_0$ was calculated and presented in graphical form in the plane for the angles $\phi=30°$, $60°$, $90°$. Using the coefficient $K_\phi(\alpha,\beta)$ it is possible to determine the critical current for a given angle ψ. In case of the $I_c - B|_\psi$ characteristics of the superconductor expressed analytically, the term to determine $I_{c\psi}$ was derived. The critical current of the solenoid is determined by the lowest value of the critical currents $I_{c\psi}$. Some results of our experimental investigations, showing typical examples of a critical current anisotropy are presented.

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CRITICAL CURRENT OF SUPERCONDUCTING MAGNETS WOUND FROM AN ANISOTROPIC SUPERCONDUCTING CONDUCTOR

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1. INTRODUCTION

The critical current of a superconducting tape in transverse magnetic field generally does not depend only on the magnitude of the magnetic induction vector $\bar{B}$, but also on the angle ψ between $\bar{B}$ and normal $\bar{n}$ to the plane of the tape (see Fig. 1). Such anisotropy of the critical current was observed on Nb_3Sn tapes [1-3], V_3Ga tapes [4] and NeTi superconductor ribbons as well [5,6]. In paper [3] we pointed out that this anisotropy can influence the critical current of a superconducting magnet as well as the efficiency of the use of a non constant current density in the winding. This effect depends on the degree of an anisotropy and field distribution in the magnet winding. Besides that, the field component perpendicular to the tape plane induces the diamagnetic screening currents which can result in catastrophic flux jumps.

Walstrom and Lubell [7] calculated the radial field components for axisymmetric rectangular cross section coils with uniform current density. They calculated the coefficient $A(\alpha,\beta)=B_r^{max}/B_z^{max}$ that was undertaken as part of a general study of stability in superconducting solenoids.
They also pointed out the possible influence of the anisotropy of the superconductor on

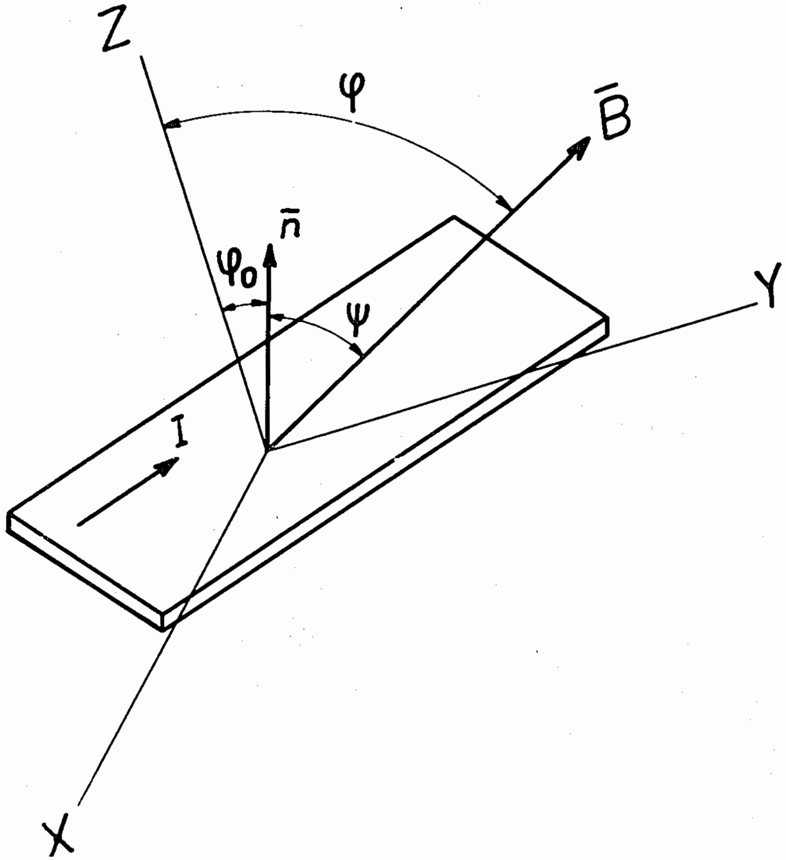

Fig. 1. Tape and/or flattened superconductor in transverse magnetic field.

the critical current of a superconducting solenoid, but they did not give any critical current calculation procedure.

The purpose of the present paper is to show such superconducting magnet critical current calculation procedure and also to

show that there are some possibilities of designing such magnets with respect to the type of winding and the angular dependence of its critical current. We also show the results of calculations, which enable one to determine maximum magnetic induction values B_{ψ}^{max} for an axisymmetric rectangular cross section coils with the uniform current density. By using B_{ψ}^{max} it is possible to determine the critical current of a magnet wound by the anisotropic superconductor. We present some results of our experimental investigations, showing typical critical current anisotropy of some technical superconductors. We also show determination of a solenoid critical current in the case of $I_c - B|_{\psi}$ superconductor characteristics presented analytically.

II. A CRITICAL CURRENT CALCULATION PROCEDURE FOR A SUPERCONDUCTING MAGNET WOUND FROM AN ANISOTROPIC SUPERCONDUCTOR

When calculating the critical current of a magnet wound from such an anisotropic superconductor, at first it is necessary to know the I_c-B characteristics of the superconductor with the angle ψ between $\bar{B}$ and $\bar{n}$ as a parameter. The usual way of presentation of these characteristics is the graphical one in the form of curves $I_c - B|_{\psi}$ for different angles ψ (see Fig. 3).

Furthermore for a given geometry of the superconducting magnet it is necessary to calculate the magnetic induction vector components throughout the volume of the winding for a unit average current density and therefore the $B_{\psi 1}$ values for different values of the angle ψ. Using the $B_{\psi 1}$ values we determine the maximum magnitudes of $B_{\psi 1}$ for different values of the angle ψ - $B_{\psi 1}^{max}$. Using the values $B_{\psi 1}^{max}$ we can determine a magnet load line $B_{\psi}^{max}(I)$ as a function of the magnet current I as:

$$B_{\psi}^{max}(I) = B_{\psi 1}^{max} I/S, \qquad (1)$$

where S is the cross section of one turn in the winding (equivalent cross section of the superconductor). By the symbol $I_{c\psi}$ we designate the critical current, determined by the point of intersection of the $B_{\psi}^{max}(I)$ load line and the corresponding $I_c - B|_{\psi}$ characteristics. The magnet critical current is determined by the minimum value of $I_{c\psi}$. The value of $I_{c\psi}^{min}$ for a given magnet geometry and given critical current anisotropy $I_c - B|_{\psi}$ of the superconductor may be influenced by the type of winding. There are some possibilities of winding the tape or flattened superconductor [8]. In the first case its larger side is practically parallel to the axis of the coil (see Fig. 2A and C), in the second one, it is perpendicular to the axis (see Fig . 2B and D) and in the third case the angle between the larger side and the axis varies with the winding (see Fig.2E).

From the point of view of the magnitude of the magnet critical current I_{cM} it is advantageous to choose such type of winding for which $I_{c\psi}^{min} \equiv I_{cM}$ is the highest. On the other hand, if the type of winding is determined by other considerations, it is possible to increase the magnitude of the

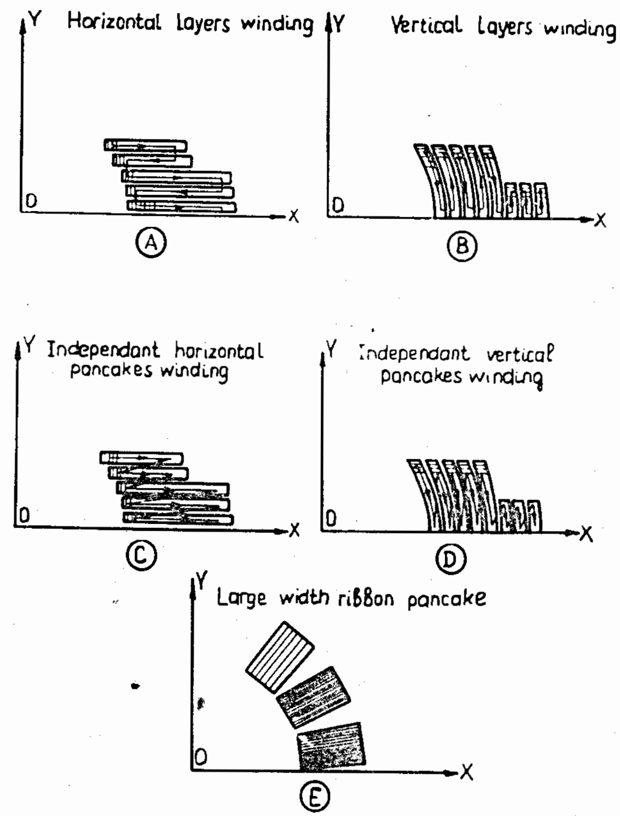

Fig. 2. Solenoid winding types (Y - coil axis). A) horizontal layers winding, B) vertical layers winding, C) independent horizontal pancakes winding, D) independent vertical pancakes winding, E) large width ribbon pancake.

I_{cM} by choosing the most convenient superconductor technology, because of the dependence of anisotropy on it.

III. CRITICAL CURRENT ANISOTROPY OF SOME TECHNICAL SUPERCONDUCTORS

To demonstrate the importance of remarks mentioned in the previous chapter, we wish to present in the following section some results of our experimental investigations, showing typical examples of critical currents anisotropy of tape and/or flattened superconductors used for construction of magnet systems and to point out how it depends on the structure of these materials, i.e., on the technology of preparation of the mentioned superconductors.

Three types of tapes and/or flattened superconductors have been used for our measurements, the vapor-deposited Nb_3Sn type prepared by CSCC of Canada (a), diffusion processed Nb_3Sn made by KABLO Bratislava (b) and, flattened multifilamentary NbTi conductor, produced by CSCC (c).

The I_c measurements were taken at 4.2K in constant transverse magnetic field in the whole range of angle ψ from 0° to 360°. The $I_{c\|}$ and $I_{c\perp}$ values (i.e., I_c ($\psi = 90°$) and I_c ($\psi = 0°$) respectively), of some samples have been measured in magnetic fields of 4T to 16T. The usual four-probe technique of I_c measurements has been used.

Figure 3 illustrates the dependence of the critical currents I_c on the magnetic induction B for various angles ψ between $\bar{B}$ and the normal $\bar{n}$ (see Fig. 1).

A typical polar angular diagram of critical current absolute value I_c for magnetic field for samples (a), (b) and (c) of Fig. 3 is shown in Fig. 4.

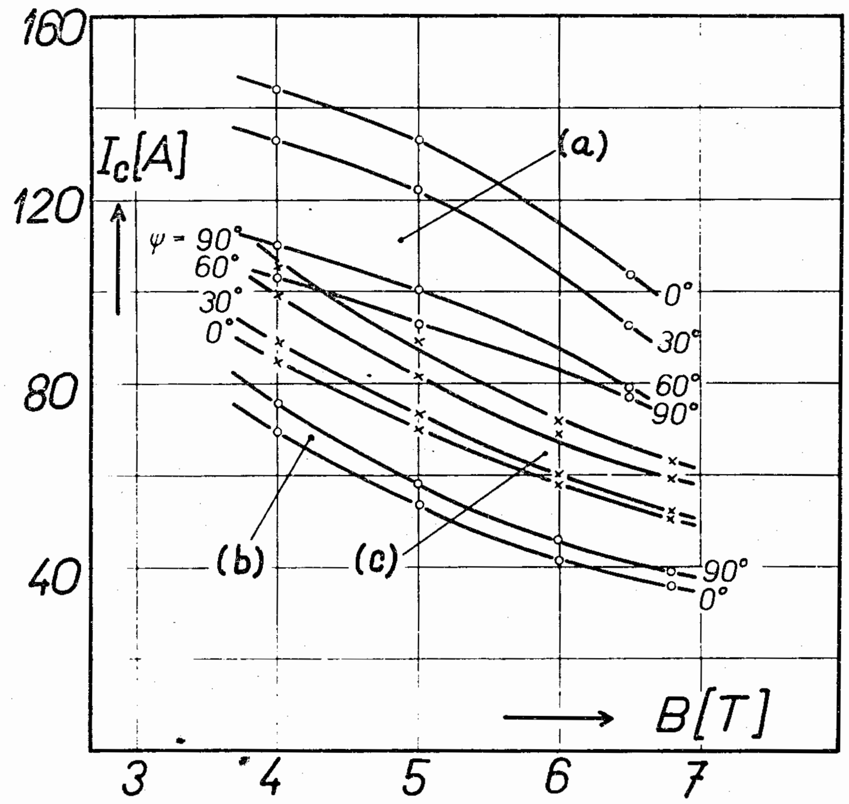

Fig. 3. The magnetic field dependence of I_c with the angle ψ as a parameter for following tape and/or flattened superconductors: (a) - vapor-deposited Nb_3Sn tape, CSCC, Canada, (b) - diffusion-processed Nb_3Sn tape, KABLO Bratislava (c) - flattened multifilamentary NbTi, CSCC, Canada.

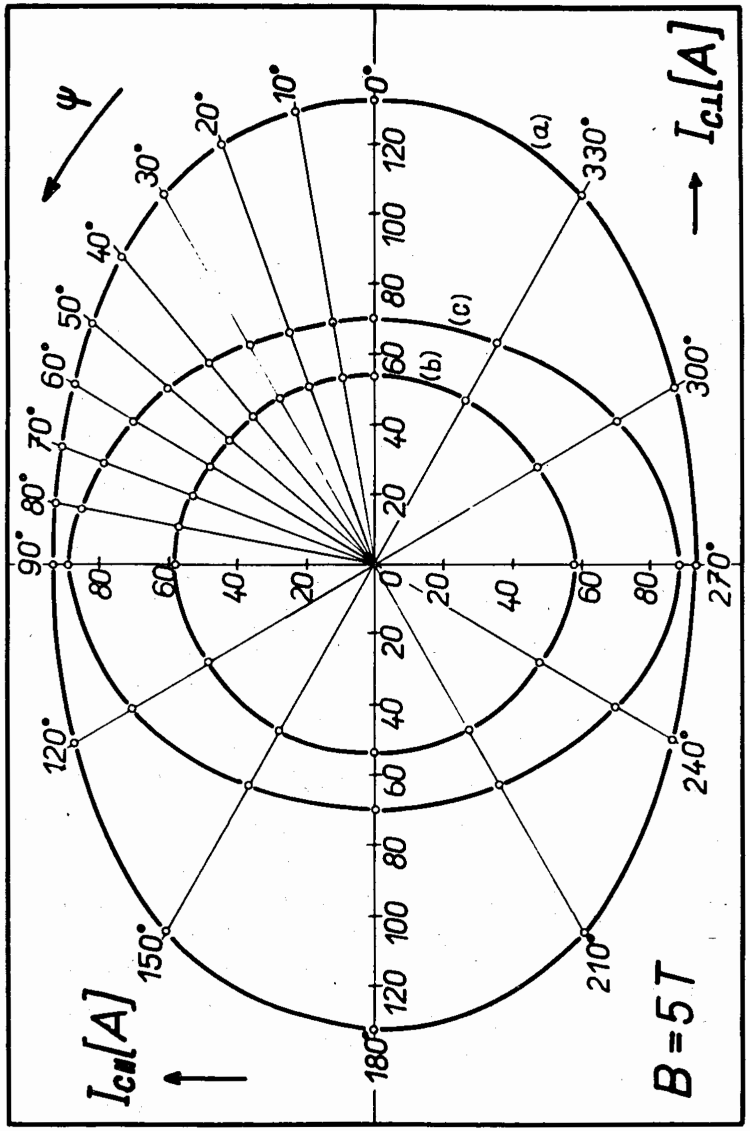

Fig. 4. Polar diagram of critical current absolute value I_c for magnetic field 5T, for superconductors of Fig. 3.

As we have already found out previously [9-11] the critical current anisotropy behaviour of various Nb_3Sn superconductors has its origin in the very structure of the mentioned material. In the process of vapor-deposition, the Nb_3Sn grains grow rapidly from the gaseous components following the surface structure of the stainless-steel substrate. The dimensions of grains in the direction of the growth perpendicular to the substrate surface may therefore be larger than the dimensions of grains parallel to the substrate surface. For diffusion-processed tapes, the diffusion of Sn takes place along the boundaries of Nb grains, which are on the contrary flattened by the rolling process and elongated in the direction of the rolling and, consequently, the Nb_3Sn grains are of a similar geometry.

The influence of differences in the structure of the mentioned materials on the critical current anisotropy is illustrated in an illuminating way on the normalized polar diagram of Fig. 5. The curve inside the unit circle applies for vapor-deposited Nb_3Sn (a), while the outside curves for diffusion-processed Nb_3Sn (b) and for the flattened NbTi (c). The shape of individual curves may be influenced by the technological procedure. In case, the gaseous impurities added into the reaction atmosphere are used, these impurities would limit the grain growth during the vapor-deposition formation process. For diffusion-processed Nb_3Sn tapes the reduction or deformation grade of rolling is of primary importance. The cri-

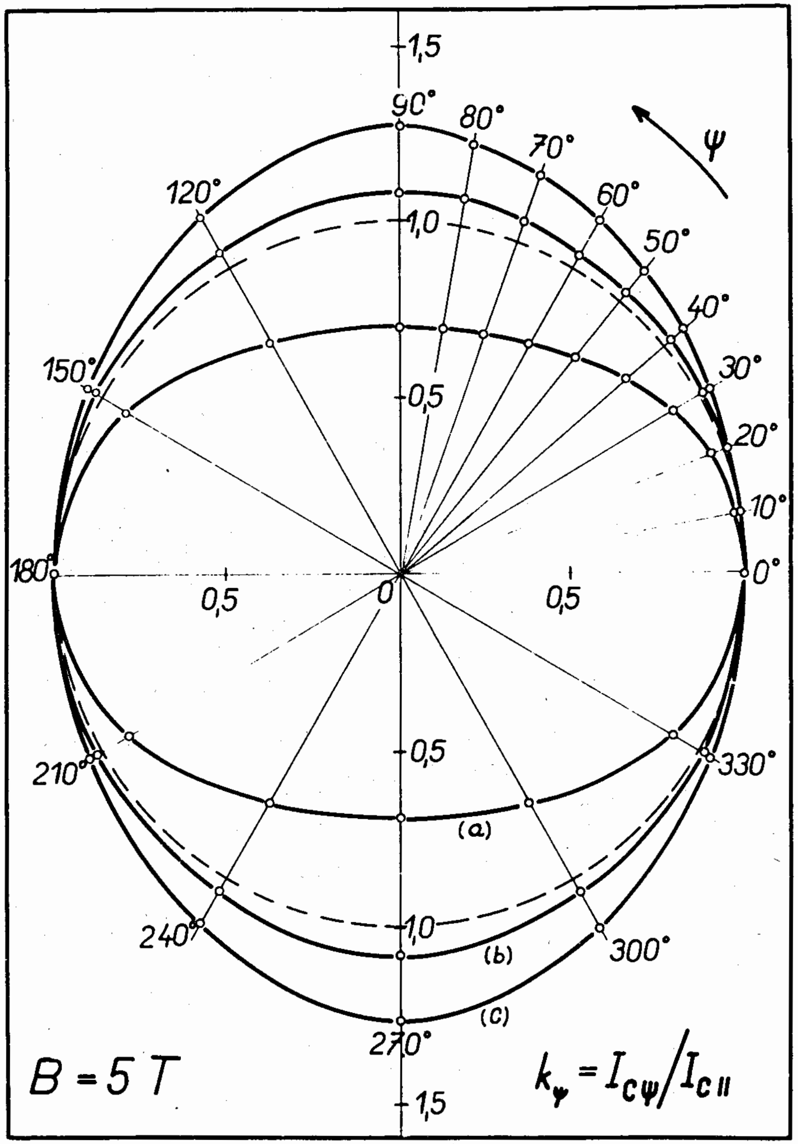

Fig. 5. Normalized polar diagram of the ratio $k = I_{c||}/I_{c\perp}$ for magnetic field 5T, for samples of Fig. 3 (or Fig. 4, respectively).

tical current anisotropy of flattened multifilamentary NbTi is similar to that of the diffusion-processed Nb_3Sn tapes. The mechanism seems to be quite understandable because the similar deformation process gives very probably a similar grain geometry and structure.

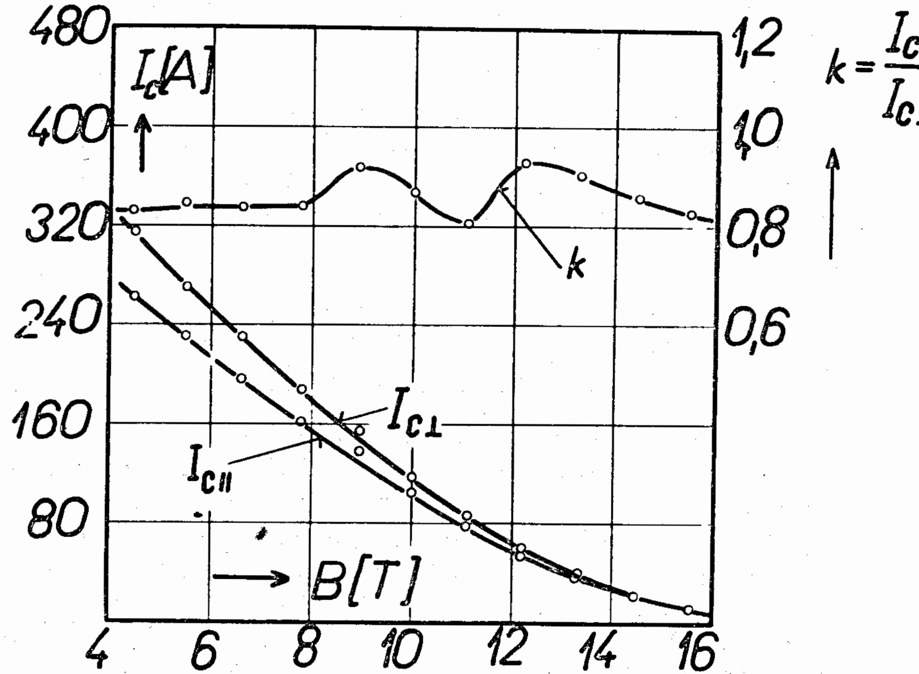

Fig. 6. The magnetic field dependence of $I_{c\perp}$, $I_{c||}$ and $k = I_{c||}/I_{c\perp}$ for sample of vapor-deposited Nb_3Sn – II. tape, CSCC, Canada.

One important problem which still remains to be solved is the magnetic field dependence of anisotropy (see Fig. 6). On this diagram the results of I_c investigation in transverse magnetic field $B_{||}$ and $B_{\perp}$ in the region of 4T to 16T are plotted. The values of the ratio "k" ($I_{c\parallel}/I_{c\perp}$) for a given type of vapor-deposited Nb_3Sn sample change in the region 0.8 to 1.0. In our case it may be due to the presence of different types of pinning centers giving rise to various anisotropies as well as field dependences of critical currents in the materials mentioned above.

We have shown that the critical current anisotropy of tape and/or flattened superconductors depends on their structure only, and may be well controlled to some extent by the process of technological preparation.

IV. THE CRITICAL CURRENT OF AXISYMMETRIC SOLENOIDS WITH RECTANGULAR CROSS SECTION

A) The critical current of a superconducting axisymmetric solenoid with rectangular cross section and uniform current density, wound from isotropic superconductor is given by the point of intersection of the $I_c - B$ characteristics of the superconductor and the load line of the solenoid $B^{max}(I)$, constructed for the point of the solenoid winding with the maximum magnitude of magnetic induction:

$$B^{max}(I) = B_1^{max} I/S = a_1 F(\alpha,\beta) k_m(\alpha,\beta) I/S, \qquad (2)$$

where $F(\alpha, \beta)$ is the form factor of the winding:

$$F(\alpha,\beta) = \mu\beta \ln \frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{1 + \sqrt{1 + \beta^2}}. \qquad (3)$$

and $k_m(\alpha,\beta) = B^{max}/B_0$, where B_0 is the value of the magnetic induction at the centre of the solenoid. The values of $F(\alpha,\beta)$ and $k_m(\alpha,\beta)$ are tabulated, or presented in graphical form depending upon the normalized dimensions of the solenoid α and β - see Fig. 7 (see, for example, /12/).

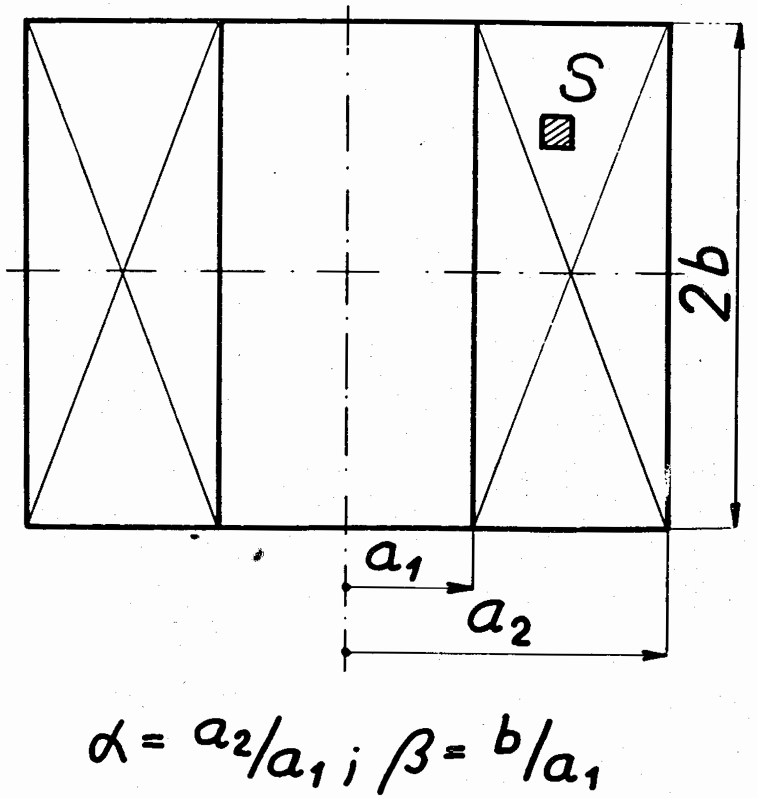

Fig. 7. Definition of parameters for axisymmetric solenoids of rectangular cross section with uniform current density.

B) If the superconductor is the anisotropic one, the procedure shown above (see part IV.A) must be repeated for all choosen values of the angle ψ. The magnitudes $B^{max}_{\psi 1}$ for a unit average current density can be expressed by the coefficient $K_\phi(a,\beta) = B^{max}_{\phi 1}/B_{01}$ which depends on the normalized dimensions of the solenoid winding a and β (see Fig. 7). For the chosen value of the angle ψ and known value of the angle ϕ_0 we determine the angle $\phi = \phi_0 + \psi$, and, then, for determining the solenoid load line $B^{max}_{\psi}(I)$ (1) we use the coefficient $K_\phi(a,\beta) \equiv K_{\phi_0+\psi}(a,\beta)$ as follows:

$$B^{max}_{\psi}(I) = a_1 F(a,\beta) K_\phi(a,\beta) I/S. \qquad (4)$$

The coefficient $K_\phi(a,\beta)$ depends on winding geometry only, assuming, that the current density is uniform in the whole volume of the winding. The values of the coefficient $K_\phi(a,\beta)$ we have calculated in the range of $a \leq 6$ and $\beta \leq 6$, for the range of the angle $0° \leq \phi \leq 90°$. For $\phi = 0°$ is $K_\phi(a,\beta) \equiv k_m(a,\beta)$. We present the values of the coefficient $K_\phi(a,\beta)$ in graphical form in the a–β plane for $\phi = 30°$, $60°$ and $90°$ – see Figs. 8A,B,C.

To calculate the values of the coefficient $K_\phi(a,\beta)$ we have used an improved (quicker) calculation method of magnetic induction components B_r and B_z [13]. This method is based on the superposition of the contributions of fictive turns (current loops), into which the winding is subdivided.

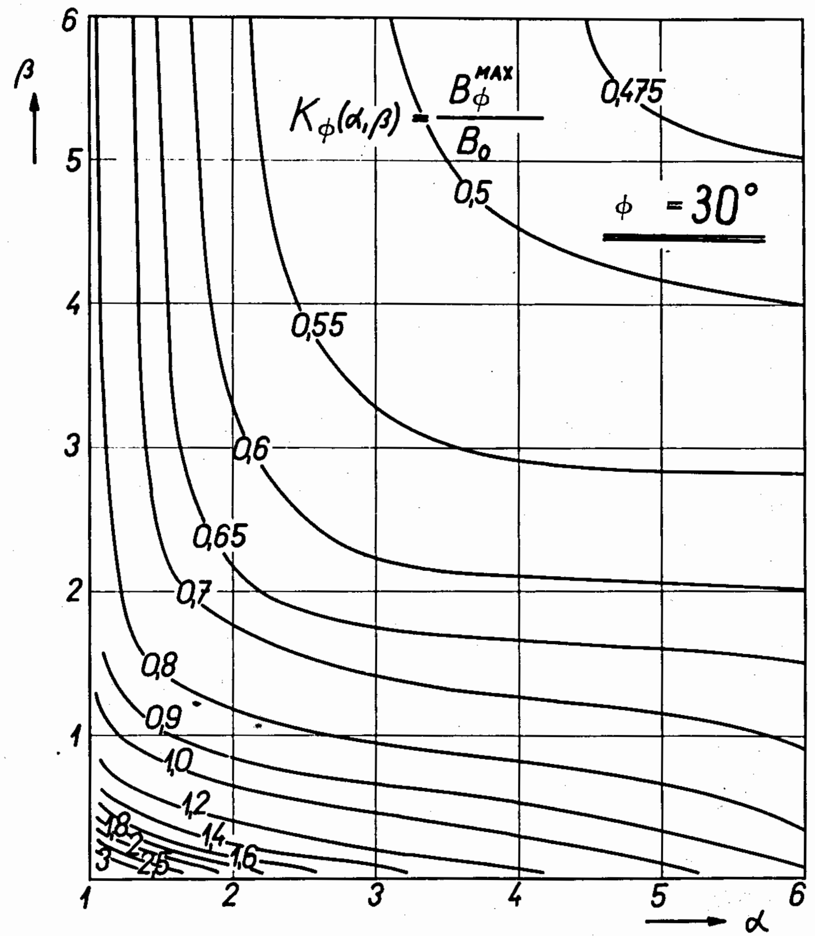

Fig.8A. Function $K_\phi(a,\beta) = B^{max}_\phi/B_0$ in the a–β plane: $\phi = 30°$.

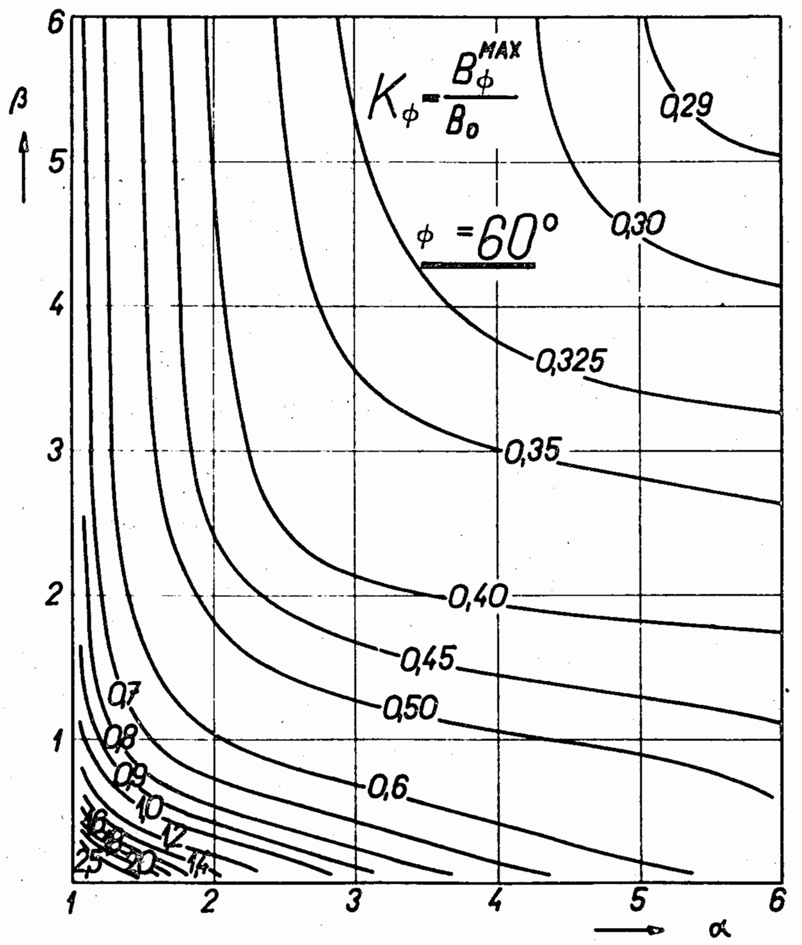

Fig.8B. Function $K_\phi(\alpha,\beta) = B_\phi^{max}/B_0$ in the α–β plane: $\phi = 60°$.

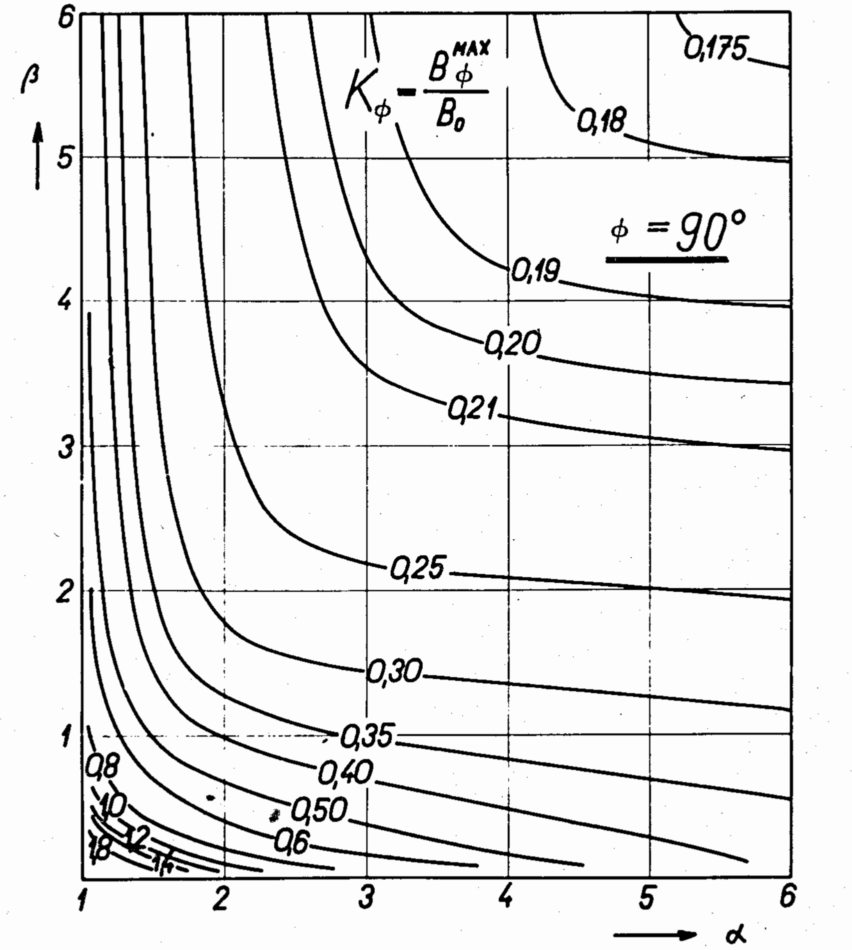

Fig.8C. Function $K_\phi(\alpha,\beta) = B_\phi^{max}/B_0$ in the α–β plane: $\phi = 90°$.

V. CRITICAL CURRENT OF A SUPERCONDUCTING SOLENOID, IF THE $I_c - B|_\psi$ CHARACTERISTICS CAN BE EXPRESSED ANALYTICALLY

In some cases the $I_c - B|_\psi$ characteristics can be truthfully expressed analytically as follows:

a) For a certain angle ψ_M the $I_c - B|_{\psi_M}$ dependence near the assumed value of the magnetic induction B_N can be linearized (see Fig. 9A):

$$I_{c\psi_M}(B) = I_{c\psi_M 0} - D_{\psi_M} B, \qquad (5)$$

where

$$D_{\psi_M} = \left| \frac{\partial I_{c\psi_M}(B)}{\partial B} \right| \equiv \frac{I_{c\psi_M 0}}{B_{c2N}}. \qquad (6)$$

b) For a certain magnetic induction B_N the $I_c - \psi|_{B_N}$ dependence can be expressed elliptically (see Fig. 9B):

$$I_{cB_N}(\psi) = \frac{I_{c\|B_N}}{\sqrt{\sin^2\psi + k_N^2\cos^2\psi}}, \qquad (7)$$

where

$$k_N = \frac{I_{c\|B_N}}{I_{c\perp B_N}}. \qquad (8)$$

c) Using (5) and (7) one obtains:

$$I_c(\psi, B) = \frac{I_{c\|0}(B_{c2N} - B)}{B_{c2N}\sqrt{\sin^2\psi + k^2\cos^2\psi}}. \qquad (9)$$

At the same time we assume, that $k_N = k \neq f(B)$. To determine the critical current of a solenoid for the angle ψ one has to substitute B

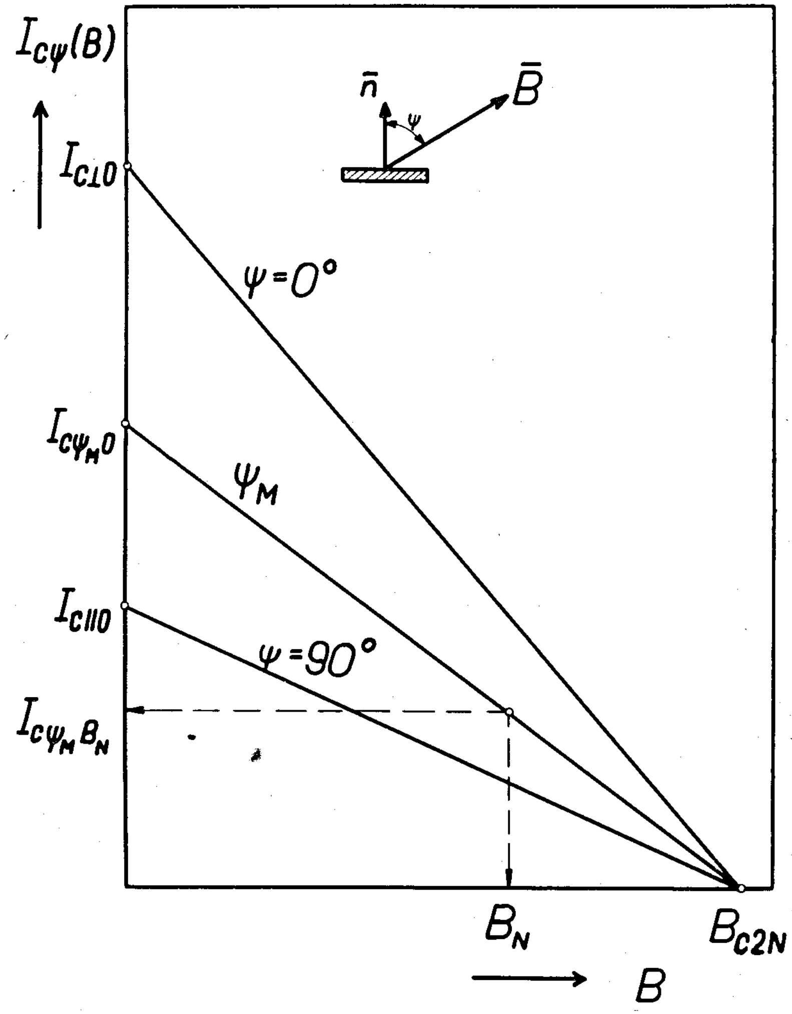

Fig. 9A. Definition of parameters of the $I_c - B|_\psi$ superconductor characteristics if it is expressed analytically: $I_c - B|_{\psi_M}$ characteristics.

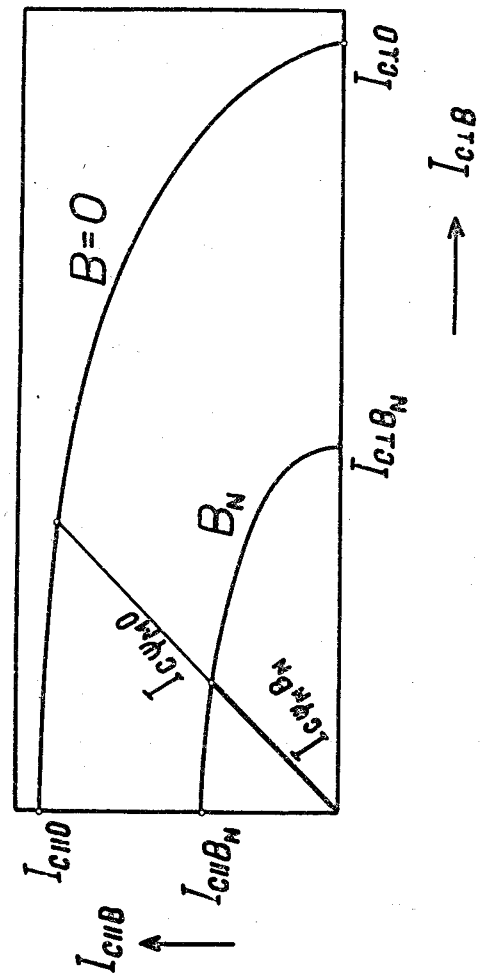

Fig.9B. Definition of parameters of the $I_c - B|_\psi$ superconductor characteristics if it is expressed analytically: $I_c - \psi|_{B_N}$ characteristics.

in the term (9) by the value $B_{c\psi}^{max}$ (see(4)). In such a way one obtains:

$$I_{c\psi} = \frac{I_{c\perp 0}k}{\sqrt{\sin^2\psi + k^2\cos^2\psi}}\left[1 - \frac{I_{c\perp 0}K_0(\alpha,\beta)K_\phi(\alpha,\beta)}{B_{c2N} + I_{c\perp 0}K_0(\alpha,\beta)k_m(\alpha,\beta)}\right], \quad (10)$$

where $K_0(\alpha,\beta) = a_1F(\alpha,\beta)/S$ and the angle $\phi = \phi_0 + \psi$ (see Fig. 1).

The values of the critical current $I_{c\psi}$ can be calculated for different values of the angle ψ as a parameter (10). The solenoid critical current I_{cM} is determined by the minimum values of $I_{c\psi}$; $I_{cM} = I_{c\psi}^{min}$.

Using $I_c - B|_\psi$ characteristics of the superconductor with the parameter: $I_{c\perp 0} = 328$ A, $B_{c2N} = 9.5$ T, $k = 0.387$ and $S = 2$ mm^2 (S is the cross section of one turn in the winding) for axisymmetric solenoids with rectangular cross section, uniform current density and the angle $\phi_0 = 0°$ in the $\alpha - \beta$ plane, we have determined the area where the critical current of a solenoid is determined by some of the $I_{c\psi}$ values for the angle $\psi > 0°$. The result is presented in a graphical form - see Fig. 10. For the points (α,β) situated below the curve $I_{c\psi}/I_{c\perp} = 1$ the critical current of a solenoid is determined by the value $I_{c\psi}$ for an angle $\psi > 0°$. For example, for $\alpha = 2.0$ and $\beta = 1.5$ the critical current of the solenoid is determined by the value $I_{c\psi} = 97.2$ A for $\psi \sim 80°$ through the value of $I_{c\perp} = 123.6$ A.

VI. CONCLUSION

We have shown how to calculate the critical current of magnets wound from an aniso-

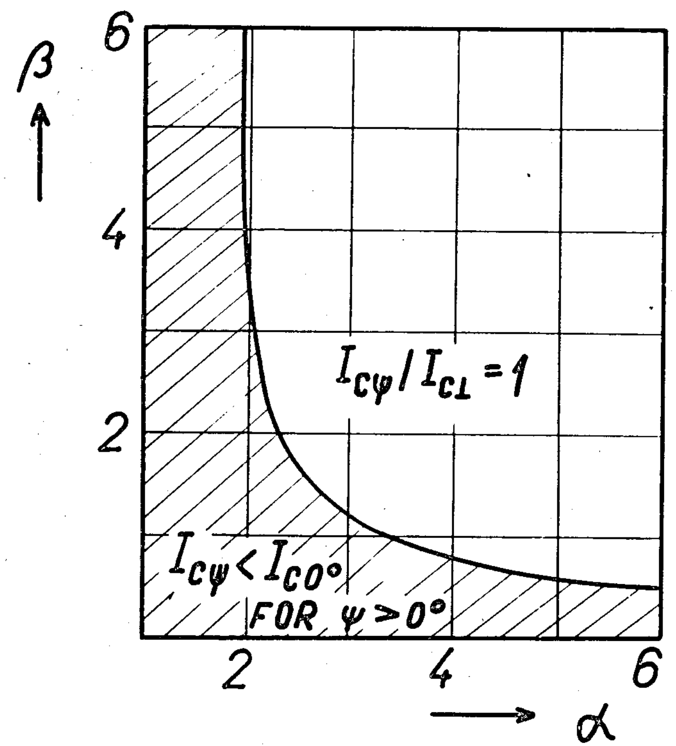

Fig. 10. Using the I_c-$B|_\psi$ characteristics with parameters $I_{c\perp 0}$ = 328 A, k = 0.387, B_{c2N} = 9.5 T and S = 2 mm^2 for axisymmetric solenoids with rectangular cross section and uniform current density the area of $I_{c\phi} < I_{c\perp}$ for $\phi > 0°$ ($\phi_0 = 0°$) is shown.

tropic superconducting tape or ribbons. For axisymmetric solenoids with rectangular cross section and uniform current density, the magnetic induction vector $\bar{B}$ at any point of the winding is perpendicular to the plane

of the tape. In such a case we have calculated the values of the coefficient $K_\phi(\alpha,\beta) = B_\phi^{max}/B_0$. Using $K_\phi(\alpha,\beta)$ and the value of the angle ϕ_0 (see Fig. 1) it is possible for a certain angle ψ (ψ is the angle between $\bar{B}$ and normal $\bar{n}$ to the plane of the tape) to determine the load line of the solenoid. The critical current $I_{c\psi}$ is determined by the point of interaction of the load line of the solenoid for the angle ψ(4) and the corresponding I_c-$B|_\psi$ characteristics of the superconductor. The critical current (regardless of the degradiation) of the solenoid I_{cM} is given by the lowest value of the critical currents $I_{c\psi}$: $I_{cM} = I_{c\psi}^{min}$. If the I -$B|_\psi$ characteristics can be truthfully expressed analytically, we have shown for axisymmetric solenoids with rectangular cross section and uniform current density how to determine its critical current. In this case we add a numerical example.

We have also pointed out the possibility of effecting the critical current of the superconducting magnet by choosing the most convenient type of winding or superconductor technological process.

We also present some results of our experimental investigations, showing typical examples of a critical current anisotropy of tape and/or flattened superconductors used for construction of magnet systems.

All our calculations are based on a uniform current density assumption, regardless of the diamagnetic screening current influence. Their influence can be considerably decreased by using twisted multifilamentary superconsuctors.

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