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D.Kabat

SUPERCONDUCTING SOLENOID CONSISTING OF CYLINDRICAL SECTIONS

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* On leave from Electrotechnical Institute, Slovak Academy of Sciences, Bratislava, CSSR.

1. Introduction

The critical current (I_c) of a superconducting cylindrical solenoid of rectangular cross section and constant current density is determined by the maximum induction of the magnetic field B^{max}. In these areas of the winding, where the induction $B < B^{max}$, , the superconductor is incompletely loaded. The degree of loading the superconductor, I/I_c, depends on the magnetic field distribution in the winding and on the superconductor $I_c - B$ characteristic.

It is well-known /1-4/ that high value of induction in the centre of the cylindrical solenoid with certain geometry can be attained by dividing the winding into sections of various average current density. The winding can be sectioned in the radial or axial directions. Various average current densities can be achieved as follows:

1. All sections are wound by superconductors of constant cross sections, and they are supplied separately from different supplies.

2. The sections are wound by superconductors of various cross sections, then they are connected in series and supplied with equal current from one supply.

In the first case due to inductive couplings between the sections there grows the possibility of quenching the solenoid to the normal state earlier than all the sections are loaded to the maximum.

As the current approaches its critical value, the danger of quenching becomes higher. In this case we need many supplies, and each section needs its current leads. So, the evaporation of liquid helium is more intensive.

It is practically impossible to attain precisely the values of currents from different supplies, needed to achieve a homogeneous field $\Delta B/B_0 \approx 10^{-5}$. In order to achieve high time magnetic field stability, it is necessary to connect the winding by a superconducting current switch. In the second case it is sufficient to connect the winding by one current switch.

The division of the winding into cylindrical sections (by the second method), to obtain homogeneous and time stable magnetic field, is investigated. The rules for optimal sectioning of the winding are drawn up.

2. The Initial Assumptions

We introduce the following assumptions:

A. The critical current density in the superconductor depends on the magnetic induction at $T = 4.2$ K; in the assumed area of the field the characteristic $j_c - B$ is written as

$$j_c = j_{c0} - D_j \cdot B \,, \tag{1}$$

where

$$D_j = \left| \frac{\partial j_c}{\partial B} \right|_{B_x} = \mathrm{tg}\,\alpha . \tag{1a}$$

B. The maximum value of induction in the winding is equal to the value in the centre of the solenoid: $k_m(\alpha,\beta) = B^{max}/B_0 = 1$. This assumption is correct for relatively long windings. For example, for $\beta = 2b/2a_1 > 1.4$ (fig. 1) the coefficient $k_m(\alpha,\beta) < 1\%$ /2-4/ .

C. The maximum value of induction in the winding is at points $r = a_{1n}$, $Z = 0$. This assumption is correct, for example, for $\alpha = 2a_2/2a_1 > 1.05$ and $\beta = 2b/2a_1 > 0.9$ or $\alpha > 1.1$ and $\beta > 0.3$, respectively (see ref. /6/ ,p.4196, fig. 4).

D. The solenoid will operate without the degradation effect. This permits us to determine critical currents of the sections by intersecting the load line of the sections, B^{max}/I, and the characteristic of the superconductors, $I_c - B$. .

E. There are no ferromagnetic circuits and the surrounding conditions are linear so we can apply the superposition method.

3. Induction in the Centre of the Superconducting Solenoid Consisting of Cylindrical Sections

We divide the winding of the solenoid of certain geometry into N coaxial cylindrical sections so that the outside dimensions a_1, a_2, b may remain the same (see fig.1).

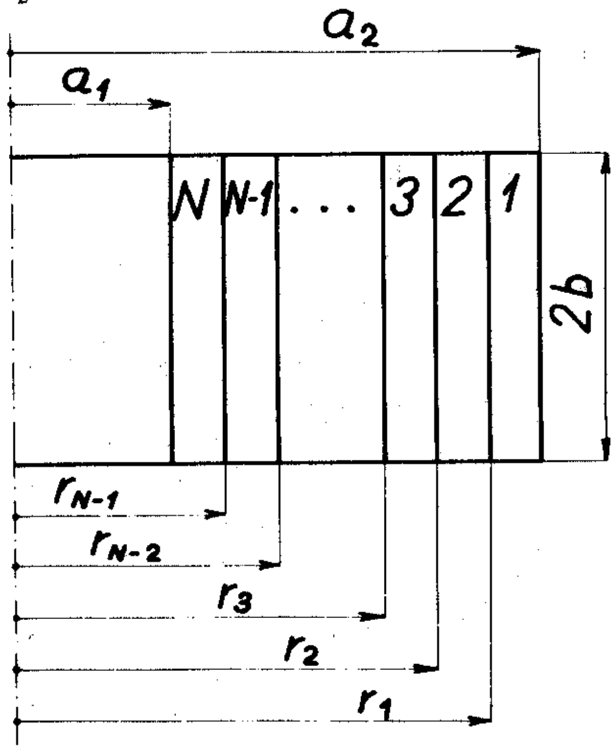

Fig. 1. Axially symmetric cylindrical solenoid with the rectangular cross section of the winding ($2a_1$, $2a_2$, $2b$) divided into N coaxial cylindrical sections.

The sections are denoted by the index N = 1 up to N, starting from the outside one. As the symbol k_{jn}, we designate

$$k_{jn} = a_{1n} F(\alpha_n, \beta_n), \quad (2)$$

$F(\alpha, \beta)$ is the form factor of the winding

$$F(\alpha,\beta) = \mu_0 \beta \ln \frac{\alpha + \sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}} . \quad (2a)$$

It is possible to show that

$$a_1 F(\alpha,\beta) = \sum_{n=1}^{N} [a_{1n} F(\alpha_n, \beta_n)], \qquad (3)$$

i.e., $K_j = \sum_{n=1}^{N} k_{jn}$. (3a)

The value of k_j is designated as K_j for the undivided winding of the solenoid (a_1, a_2, ,b).

The contributions of the sections to the induction in the centre of the solenoid, if each section has the critical average current density, are denoted by $\Delta B_{0cn} = k_{jn} \cdot j_{cn}$. We assume that each section has the same j_c-B characteristics. Starting from (1), we obtain the critical average current densities in the sections

$$\begin{aligned} j_{c1} &= j_{c0} \frac{1}{M_1}, \\ j_{c2} &= j_{c1} \frac{1}{M_2} = j_{c0} \frac{1}{M_1 \cdot M_2}, \\ &\vdots \\ j_{cN} &= j_{nN-1} \frac{1}{M_N} = j_{c0} \frac{1}{M_1 \cdot M_2 \cdots M_N}, \end{aligned} \qquad (4)$$

where

$$M_n = 1 + D_j k_{jn} . \qquad (4a)$$

The induction in the centre can be calculated by the equation

$$B_{0cN} = \sum_{n=1}^{N} \Delta B_{0cn} = \sum_{n=1}^{N} k_{jn} j_{cn} . \qquad (5)$$

Starting from (4), we obtain

$$B_{0cN} = \sum_{n=1}^{N} \frac{k_{jn}\, j_{c0}}{\prod_{n=1}^{N} M_n} = j_{c0} \frac{\sum_{n=1}^{N} [k_{jn} \prod_{x=n+1}^{N} M_x]}{\prod_{n=1}^{N} M_n}, \quad (5a)$$

for $n = N$ the $\prod_{x=n+1}^{N} M_x = 1$ is valid.

We determine the values of k_{ij} so that for a certain number of sections, N, the induction in the centre may become maximum. The values of k_{jn} can be determined using the method of gradual division of the winding. The first step is to divide into two sections; N = 2. From (5a) we get the condition for B_{0c2}^{max} in the form $k_{j1} = k_{j2} = K_j/2$. Each of the two sections is divided again into two parts and so on. In such a way it is possible to show that the maximum induction in the centre of the solenoid, divided into N sections, is achieved on condition that

$$k_{j1} = k_{j2} = k_{j3} = \dots = k_{jn} = \dots = k_{jN} = K_j/N. \quad (6)$$

Starting from (6), we obtain for the induction in the centre

$$B_{0cN}^{max} = \frac{k_j \cdot j_{c0}}{M^N} \sum_{n=1}^{N} M^{n-1} = \frac{j_{c0}}{D_j}(1 - \frac{1}{M^N}), \quad (7)$$

for $N \to \infty$

$$B_{0c\infty}^{max} = \frac{j_{c0}}{D_j} [1 - \frac{1}{\exp(D_j K_j)}]. \quad (8)$$

Taking the relative increase of the induction in the centre $(B_{0c\infty}^{max} - B_{0c})/B_{0c}$, for 100%, we get for N=3 the relative increase $(B_{0c3}^{max} - B_{0c})/B_{0c}$=60%. Taking into account this fact and construction simplicity, the winding is usually divided into 3 sections.

Using the method of successive division of the winding taking into account the condition (6), one can determine "division radii r_n" (see fig. 1):

$$r_n = \frac{V_{1n} V_{2n} - b^2}{2\sqrt{V_{1n} \cdot V_{2n}}}, \qquad (9)$$

where

$$V_{1n} = a_{1n} + \sqrt{a_{1n}^2 + b^2}; \; V_{2n} = a_{2n} + \sqrt{a_{2n}^2 + b^2}, \qquad (9a)$$

Using the values of r_n, we can determine the thicknesses of the sections.

4. Determination of the Optimum Cross Sections of the Superconductors in the Sections

The superconductors must have "optimum" effective cross sections in order to achieve the critical current density in each section. In this case the critical current of the solenoid will be at the same time the critical current of all the sections. The optimum values of the effective cross sections are determined as follows: taking into account (4) and (6), the critical currents in the sections are equal to

$$\begin{aligned} I_{c1} &= j_{c1} A_{ef1} = j_{c0} \frac{1}{M} A_{ef1}, \\ I_{c2} &= j_{c2} A_{ef2} = j_{c0} \frac{1}{M^2} A_{ef2}, \\ &\vdots \\ I_{cN} &= j_{cN} A_{efN} = j_{c0} \frac{1}{M^N} A_{efN}. \end{aligned} \qquad (10)$$

From the value of induction in the centre of the solenoid, B_{0cn}, and the j_c-B characteristic of the superconductors, we can determine the value of j_{cN} (inside section). The value of the critical (nominal with respect to some reserve) current of the solenoid, I_{cm}, can be chosen, for example, from the nominal current of the power supply, which we have, or from the value of inductance of the solenoid. The effective cross section A_{cfn} is determined by the values of j_{cn} and I_{cm}: $A_{enf} = I_{cm}/j_{cn}$. The values of $A_{efn-1} \div A_{ef1}$ can be determined from the equality of the critical currents of all the sections:

$$A_{efn-1} = \frac{1}{M} A_{efn} \quad . \tag{11}$$

5. Critical Current of the Sectioned Solenoid in the Case of Nonoptimum Values of A_{efn} and k_{jn}

The critical currents of the sections for nonoptimum values of A_{efn} (for example, there are no conductors of necessary cross sections) and k_{jn} (for example, the lengths of the pieces of the conductors are not suitable) can be unequal.

As k_{In} we designate:

$$k_{In} = k_{jn}/A_{efn} = a_{In} F(\alpha_n, \beta_n)/A_{efn} \; . \tag{12}$$

The form of the characteristics of the superconductors, $I_{cn}-B$, at $T = 4.2$ K is assumed as:

$$I_{cn} = I_{c0n} - D_{In} B \; , \tag{13}$$

$$D_{In} = \left| \frac{\partial I_{cn}}{\partial B} \right|_{B_x} . \qquad (13a)$$

We determine the critical currents of the sections likewise the values of j_{cn} (4)

$$I_{cn} = \{ I_{c0n} \prod_{i=1}^{n-1} P_i - D_{In} \sum_{i=1}^{n-1} [I_{c0i} k_{Ii} \prod_{x=1}^{i} P_{x-1}] \} / \prod_{i=1}^{n} P_i , \qquad (14)$$

where

$$P_i = 1 + D_{Ii} k_{Ii} . \qquad (14a)$$

If $i = 0$, $P_i = 1$.

If $I_{c1} < I_{c2} < \dots < I_{cn} < \dots < I_{cN}$, $\qquad$ (15)

the critical current of the solenoid, I_{cm}, is determined by the critical current of the outside section: $I_{cm} = I_{c1}$.

If

$$I_{c1} > I_{c2} > \dots > I_{cn} > \dots > I_{cN} , \qquad (16)$$

the critical current I_{cm} is determined by the term (14). If both the conditions (15) and (16) are not valid, the critical current I_{cm} is determined by the minimum value of the critical currents of the sections: $I_{cm} = I_{cn}^{min}$. In this case to determine I_{cm}, it is possible to use the graphic iteration process.

6. Conclusion

Considering that the optimum conditions are achieved when the magnetic field induction is maximum in the centre of the solenoid, we come to the conclusion that the best division is such that the coefficient k_{jn} values are equal. The optimal effective cross sections of the superconductors, cross section of one turn in the winding, are chosen (eq. 11) such that the critical current density in all sections and in the solenoid is achieved at the same time. By dividing the solenoid into cylindrical sections (using the second method), we draw up the equation (7) which determines the induction in centre. The solenoid critical current is determined by the critical current of the section having minimum critical current.

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