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V.K.Lukyanov

METHODS OF THE HIGH ENERGY APPROXIMATION IN STUDYING HEAVY-ION COLLISIONS

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1 Introduction

Peripheral collisions of heavy ions with nuclei at energies $E \gg U$ and $kR \gg 1$ are very sensitive to the parameters of an interaction potential and to the nuclear structure characteristics such as the density distributions, transition matrix elements and others. The developed methods of the high-energy approximation are designed to get the corresponding amplitudes for elastic, inelastic scattering and nucleon transfer reactions in analytic forms to search the mechanisms of the processes. For this purpose, we use the relative-motion quasi-classical wave functions

$$\Psi^{(\pm)} = \exp\left\{ i\vec{k}\vec{r} - \frac{i}{hv}\int_{\pm\infty}^{z} [V(r) \mp iW(r)]\, d\lambda \right\}, \tag{1.1}$$

where $r = \sqrt{b^2 + \lambda^2}$ with b being the impact parameter. The integration over λ runs along the straight line parallel to $\vec{k}$ in asymptotics, and when the deflection of a classical trajectory on $\vec{k}$ is not negligible small the momentum $\vec{\bar{k}} = \vec{k} \mp \vec{q}_c/2$ should be inserted into eq.(1.1) instead of $\vec{k}$ [1], where $q_c = 2k\sin(\theta_c/2)$ and $\theta_c \simeq U(R_t)/E$ with R_t the closest approach radius. In accordance with the DWBA, the matrix elements are taken as follows

$$T = \int d^3r\, \Psi^{(-)*}_{\vec{k}_f} \hat{O}(\vec{r}), \Psi^{(+)}_{\vec{k}_i}, \tag{1.2}$$

where for elastic, inelastic and one-nucleon transfer processes one puts $\hat{O}_{el} = U(r, R)$, $\hat{O}_{inel} \sim U'_R Y^*_{\lambda\mu}(\hat{r})$, $\hat{O}_{tr} \sim u_\lambda(r) Y^*_{\lambda\mu}(\hat{r})$ with the nucleon bound state wave function u_λ. Now, we consider the elastic scattering amplitude for large angles $\theta > 1/kR$, θ_c [2] - [4]

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int d^3r U(r)\, e^{i\vec{q}\vec{r} + i\Phi}, \tag{1.3}$$

where, representing the potential as $U = U_0 f(r)$, we have

$$\Phi = \gamma I(b), \qquad \gamma = -i\frac{U_0}{\hbar v}, \qquad I = 2\int_0^\infty f(\sqrt{b^2 + \lambda^2})\, d\lambda. \tag{1.4}$$

In both the cylindrical and spherical coordinate frames one can simplify eq.(1.3) to the forms appropriate for analytical calculations. Both the results turn out to be rather instructive; therefore, we consider them separately in some detail.

2 Approaches to the large angle scattering

In the cylindrical coordinate frame $oz \uparrow\uparrow |\vec{k}_i$, and we obtain

$$\vec{q}\vec{r} = q_1 b \cos\varphi + q_2 z, \tag{2.1}$$

where $q_1 = q\cos(\theta/2)$ and $q_2 = q\sin(\theta/2)$ are the transverse and longitudinal transfer momenta, respectively; $q = 2k\sin(\theta/2)$ and θ is the scattering angle. Substituting $d^3r = bdbdzd\varphi$ into (1.3) and integrating over the azimuthal angle φ one gets

$$f(\theta) = -\frac{mU_0}{\hbar^2}\int_0^\infty db\; bJ_0(q_1 b)\, e^{i\Phi} F(q_2, b), \tag{2.2}$$

$$F(q_2, b) = \int_{-\infty}^{\infty} dz\; e^{iq_2 z} f(\sqrt{b^2+z^2}), \tag{2.3}$$

where the $F(q_2,b)$-function plays a role of the one-dimensional longitudinal form factor of the distribution function $f(r)$ of a potential at fixed b. For example, following [5], in the case of $f_F = \{1+\exp[(\sqrt{b^2+z^2}-R)/a]\}^{-1}$ for a Woods-Saxon potential we obtain

$$F_F(q_2, b) = 2R - 2\pi i a \sum_{p=1}^{\infty}\left(e^{iq_2 b}\frac{r_p^+}{\lambda_p^{(+)}} + e^{-iq_2 b}\frac{r_p^-}{\lambda_p^{(-)}}\right), \tag{2.4}$$

$$I_F(b) = 2R - 2\pi i a \sum_{p=1}^{\infty}\left(\frac{r_p^+}{\lambda_p^{(+)}} + \frac{r_p^-}{\lambda_p^{(-)}}\right), \tag{2.5}$$

where $r_p^\pm = R \pm i\pi a(2p-1)$ and $\lambda_p^{(\pm)} = \sqrt{(r_p^\pm)^2 - b^2}$. Using the sums (2.4), (2.5) one should obey the condition

$$Im\; \lambda_p^{(\pm)} \geq 0, \tag{2.6}$$

which gives for real b the relation $\lambda_p^{(+)} = -\lambda_p^{(-)*}$, so that $F_F(0,0) = I_F(0) = 2R$. In the case of $F = const$, eq.(2.2) is expressed through the same integral which determines the Glauber small angle scattering amplitude [6], [7]

$$f_G(\theta) = -ik\int_0^\infty db\; bJ_0(kb\theta)\, e^{i\Phi}, \qquad \theta < \sqrt{2/kR}. \tag{2.7}$$

Thus, one can see that the amplitude (2.2) extends the Glauber formula to the region of the large scattering angles. Indeed, e.g., for scattering of ^{16}O on ^{60}Ni at $E = 94\; MeV$/nucleon and $|V_0| = 50\; MeV$ one obtains that the Glauber amplitude (2.7) may be used at $\theta < \sqrt{2/kR} \simeq 5°$. At the same time the amplitude (2.2) is applied at $\theta > |V_0|/E \simeq 2°$, the region which partly covers the space of the "small" angles. For analytical evaluations of (2.2) it is fruitful to make use of the asymptotic form of the Bessel function to get

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\left[t_{(+)}(q) - t_{(-)}(q)\right], \tag{2.8}$$

$$t_{(\pm)} = -(1+i)\sqrt{\frac{\pi}{q_1}}U_0 \bar{J}^{(\pm)}, \tag{2.9}$$

$$\bar{J}^{(\pm)} = \int_0^\infty db\sqrt{b}\; e^{g_{(\pm)}(b,\gamma)} F(q_2, b), \tag{2.10}$$

$$g_{(\pm)}(b,\gamma) = \pm iqb + \gamma I(b). \tag{2.11}$$

Here, the typical nearside $t_{(+)}$ and farside $t_{(-)}$ terms characterize the behavior of the amplitude $f(\theta)$ at relatively small and large angles of scattering, respectively.

In the following, we obtain an expression for the scattering amplitude that is most convenient for further applications of the saddle point method (SPM) and of the so-called pole method as well, both giving the final result in explicit forms. For this purpose we represent (1.3) in the spherical coordinate frame where $oz \uparrow\uparrow \vec{q}$, $ox \uparrow\uparrow (\vec{k}_i + \vec{k}_f)/2$ and $d^3r = -r^2 dr d\mu d\varphi$, $\vec{q}\vec{r} = qr\mu$. Thus, we have

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int_0^\infty r^2 dr d\varphi \int_{-1}^{1} d\mu\; U(r)\; e^{iqr\mu + i\Phi}. \tag{2.12}$$

Here $\Phi(b)$ depends on $b = b_i = \sqrt{r^2 - z_i^2}$ where $z_i = \vec{k}_i\vec{r}_i = r\left(\alpha\mu + \sqrt{1-\alpha^2}\sqrt{1-\mu^2}\cos\varphi\right)$ with $\alpha = \sin(\theta/2)$ and θ the scattering angle. Bearing in mind that the exponent $iqr\mu$ is large, we integrate in (2.12) over μ by parts retaining only the lowest order term in $1/qr \simeq 1/qR \ll 1$. Then, the dependence of Φ on φ at $\mu = \pm 1$ is released, and (2.12) is transformed into (2.8) with the following near- and farside terms:

$$t_{(\pm)} = -i\frac{2\pi U_0}{q} J^{(\pm)}, \tag{2.13}$$

$$J^{(\pm)} = \int_0^\infty dr\; r\; e^{g_{(\pm)}(r,\gamma)}\left[1+\exp\frac{r-R}{b}\right]^{-1}, \tag{2.14}$$

where $g_{(\pm)}$ is determined by (2.11) with b replaced by $r = b\cos(\theta/2)$. Note that if one appends the Coulomb potential in a whole interaction, additional terms appear in $t_{(\pm)}$.

Formally, two expressions for the amplitudes, first done by eqs. (2.8) - (2.11) and second by (2.8), (2.13), (2.14) must give close results at large angles. Indeed, we have made the comparison of the cross sections when both the integrals (2.10) and (2.14) have been calculated numerically for the same optical potential corresponding to scattering of ^{16}O on ^{60}Ni at E=94 MeV/nucleon [8]. And it is confirmed that in the region of large angles up to 15° (where the cross sections fastly falloff but can be yet measured) two curves occur to be in remarkably good agreement. At small angles the former approach (2.8) - (2.11) is thought to be more precise because of its correspondence to the Glauber formula applied when $\theta < \sqrt{2/kR}$.

3 The SP-methods for trajectories close to poles

To search the mechanism of scattering and to avoid long numerical calculations, the quickly oscillating integrals of the type (2.10) and (2.14) can be evaluated as usually by the standard SPM. However, in our case the specific problem can appear when on the complex plane the saddle points occur to be near peculiarities of the functions F, $g^{(\pm)}$ or

f_F, the regions where integrands change very fastly. Then, when applying the analytical methods of calculations, one needs explicit expressions for the eikonal phase Φ and

$$I_F(b) = 2\int_0^\infty d\lambda \left[1 + \exp\frac{\sqrt{b^2+\lambda^2} - R}{a}\right]^{-1}. \tag{3.1}$$

Formally, eq. (2.5) is the result of integration of (3.1), but it has a form of the infinite sum of residues at poles of $f_F(b,\lambda)$ on the complex λ-plane. However, it is desirable to obtain for I_F a simpler closed form which can then be used, e.g., for applications of the SPM or of other methods. To this aim, in many papers the integrand of (3.1) has been replaced, for example, by the step function [1], [9], [10], the trapezoidal function [11], or it has been adjusted by a set of the Gaussians [12], and others. However, in such approximations one can be very careful when doing extensions to the complex b- or r-planes. Indeed, in the regions near the poles at $R \pm i\pi a(2p-1)$, $p = 1,2,3...$, the integrand I_F changes very fastly. Then, it is easily seen that the main contribution to the oscillating integrals (2.10) and (2.14) comes from the regions near two poles with $p = 1$. This is a reason why for studying the pA-scattering at intermediate energies, in ref.[13] in the total sum (2.5) only the first terms are taken in account, i.e.,

$$\Phi = \gamma \tilde{I}^{(\pm)}, \qquad \tilde{I}^{(\pm)} = 2R - 2\pi i a\frac{r^\pm}{\lambda^{(\pm)}}, \qquad Im\ \lambda^{(\pm)} \geq 0, \tag{3.2}$$

where $\lambda^{(\pm)} = \sqrt{(r^\pm)^2 - b^2}$, $r^\pm = R \pm i\pi a$. This approach has been used by the authors of [13] (ADL) for the SPM calculations of the Glauber amplitude (2.7) for small angles. It seems to be interesting to test the phase (3.2) also for heavy ions, when $kR \gg 1$ and to calculate the large angle scattering amplitudes (2.8), (2.13), (2.14). In general, the saddle points $r_s \equiv r_s^\pm(q)$ are the solutions of the following equation:

$$g'_{(\pm)} = \pm iq + \gamma I'(r) = 0. \tag{3.3}$$

These points $r_s^\pm(q)$ fill the complex trajectories which behave as a function of the momentum transfer q or the scattering angle θ where s is the number of the solution. Then, applying the SPM we find the standard result

$$J^{(\pm)}(r_s) = -r_s f_F(r_s) e^{g_{(\pm)}(r_s)}\sqrt{-2\pi/g''_{(\pm)}(r_s)}. \tag{3.4}$$

In a particular case of the ADL-approach when eq.(3.2) takes place and the impact parameters $b = r\cos(\theta/2)$ are suggested close to the poles, i.e. $b = r^\pm + \delta$ with $|\delta| \ll |r^\pm|$, one can obtain approximate expressions for $g(r_s)$ and $g''(r_s)$ [13] – [16]:

$$g_{(\pm)}(r) = \pm iqr + 2R\gamma + i\frac{\bar{\alpha} r^\pm}{\lambda^{(\pm)}}, \quad g''_{(\pm)}(r) = \cos^2\frac{\theta}{2}\,\frac{\bar{\alpha} r^\pm}{\lambda^{(\pm)5}}\left[\left(r^\pm\right)^2 + 2\left(b_s^{(\pm)}\right)^2\right], \tag{3.5}$$

where $b_s^{(\pm)} = r^\pm + \delta_s^{(\pm)}$, $\delta_s^{(\pm)} = -(|\lambda^{(\pm)}|^2/2|r^\pm|)\exp[i\beta_\delta^{(\pm)}]$ and $\bar{\alpha} = -2\pi a\gamma$.

Fig.1 demonstrates the cross sections calculated in [15], [16] for scattering of ^{17}O on ^{90}Zr at $E = 1430\ MeV$ with $R = 7.05\ fm$, $a = 0.5\ fm$. The solid lines are numerical

calculations with the help of eqs.(2.8), (2.13), (2.14); the dashed curves are the SPM evaluations of (2.14), and in both cases the exact $g(r)$ (2.5) was used; the stars correspond to the ADL-approach (3.2), (3.5). The cases considered are: (a) refraction $V_0 = -50\ MeV$, $W_0 = 0$, (b) optical model $V_0 = -50\ MeV$, $W_0 = -25\ MeV$, (c) diffraction $V_0 = -1\ MeV$, $W_0 = -50\ MeV$. One can conclude that the ADL-approach is not well working for heavy ion scattering, and one needs to make use of exact expressions for the eikonal phases (2.5), (2.6) (note that in the given example it was enough to use 13 terms in the total sum for I (2.5)). One should mention that at comparably small angles of scattering the peculiarities of the eikonal phases (2.5) at $b_p = r_p\cos^{-1}(\theta/2) = R \pm i\pi a(2p-1)$, $p = 1,2,\ldots$ occur to be close to the poles $r_p = R \pm i\pi a(2p-1)$ of the integrand Fermi function $f_F(r)$. In this case a standard SPM must be modified. To this purpose, we represent $f_F(r)$ near the poles by the leading terms of the series

$$f(r) = \frac{1}{1 + \exp\frac{r-R}{a}} \simeq -\frac{a}{r - r_p} + \tilde{f}, \tag{3.6}$$

where $\tilde{f}$ is a nonpole correction. Then, including only the pole term in (3.6) at $p = 1$ we obtain

$$J^{(\pm)}(r_s) = ar_s\, e^{g_{(\pm)}(r_s)}\int_{-\infty}^{\infty}\frac{du}{u + \tilde{\delta}^{(\pm)}}\ \exp\left[-\left(-\frac{g''_{(\pm)}(r_s)}{2}\right)\right]u^2, \tag{3.7}$$

where $\tilde{\delta}_s^{(\pm)} = r_s - r^\pm$. We put $t = u\sqrt{-g''_{(\pm)}(r_s)/2}$, $z^\pm = -\tilde{\delta}_s^{(\pm)}\sqrt{-g''_{(\pm)}(r_s)/2}$ and get [14]

$$J^{(\pm)}(r_s) = -ar_s\, e^{g(r_s)}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{z^\pm - t}dt = i\pi ar_s\, e^{g(r_s)}\omega(z^\pm), \tag{3.8}$$

$$\omega(z) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{z - t}dt, \qquad Im\ z > 0. \tag{3.9}$$

Here ω is the error functions determined, e.g., in [17]. We have checked that in a particular case of the previous example the results of evaluations of J by (3.4) and (3.8) are in rather good coincidence.

4 The pole method

When studying the integral $J^{(\pm)}$, eq.(2.14), one has temptation to represent them as a sum of residues at the poles $r_p^\pm = R \pm i\pi a(2p-1)$, $p = 1,2\ldots$ of the Fermi function. Indeed, supposing that these poles give the main contribution to the amplitude, we obtain

$$J^{(\pm)} = -2\pi i a\, e^{\pm iqR}\sum_{p=1}^{\infty}\left(r_p^\pm\right)\, e^{-q\pi a(2p-1)}\, e^{iI(r_p^\pm)}, \tag{4.1}$$

and neglecting the contributions of the terms with $p > 1$, which are reduced by a factor of $e^{-2q\pi a} \ll 1$ as compared to every previous term, we get the so-called two pole approximation

$$f(\theta) = -\frac{2i\pi amU_0}{q\hbar^2}\left\{\left(r^+\right)e^{iqR + iI^{(+)}} - \left(r^-\right)e^{-iqR + iI^{(-)}}\right\}e^{-\pi aq}. \tag{4.2}$$

Here, an important role is revealed of the diffuseness parameter a which regulates the exponential slope of the cross sections in the region of the Fraunhofer diffraction of particles. Then, for a qualitative interpretation, one can write at $R \gg \pi a$ the relation $I(r^{\pm}) \simeq I(R)$ and the expression in brackets in (4.2) becomes as follows

$$\Big\{\ldots\Big\} = 2iR\, e^{iI(R)} \Big\{\sin qR + \frac{\pi a}{R}\cot qR\Big\}. \tag{4.3}$$

This result exhibits the role of the radius R which introduces oscillations of amplitudes, the typical phenomena of diffraction processes. Here, the phase function $I(R)$ does not depend on diffuseness a. This kind of behaviour can be obtained in a closed form for a step distribution function $f_\Theta(r) = \Theta(R-r)$ of the potential. In this case, to imitate the dependence of the phase on a, an additional parameter can be introduced (see, e.g., [1], [9]). But a better way to include the a-dependence is to use a more realistic distribution function which, at the same time, gives a possibility to calculate the eikonal phase in an explicit form. We show the results of calculations I for the trapezoidal f-function

$$f_t(r) = \Theta(R_1 - r) + \frac{R_2 - r}{\Delta}\Theta(r - R_1)\Theta(R_2 - r), \tag{4.4}$$

where $\Delta = R_2 - R_1$, $R_2 = R + \Delta/2$. Note that at $\Delta = 4a$ its derivative $f_t'(r = R)$ coincides with that for the Fermi function $f_F'(r = R)$. In this case, the explicit expression for the phase integral takes place [11]

$$I_t(b) = -2\Big\{\Big[\frac{R_2}{2\Delta}\sqrt{R_2^2 - b^2} - \frac{R_1}{2\Delta}\sqrt{R_1^2 - b^2} - \frac{b^2}{2\Delta}\ln\frac{R_2 + \sqrt{R_2^2 - b^2}}{R_1 + \sqrt{R_1^2 - b^2}}\Big]\Theta(R_1 - b) +$$

$$+ \Big[\frac{R_2}{2\Delta}\sqrt{R_2^2 - b^2} - \frac{b^2}{2\Delta}\ln\frac{R_2 + \sqrt{R_2^2 - b^2}}{b}\Big]\Theta(R_1 - b)\Theta(R_2 - b)\Big\}. \tag{4.5}$$

We remind that $b = r\cos(\theta/2)$ and θ is the scattering angle. It was checked numerically in [11] for the elastic scattering of ^6Li on ^{12}C that the approximate expression (4.5) for I_t describes well behaviour of the exact I_F, (3.1) in dependence of the real r. Also, to exemplify scattering, a rather good agreement has been obtained between both the real and imaginary parts of I_F and I_t, respectively, in the region of complex r which gives the main contributions to the considered process. Fig.2 shows the corresponding cross sections by the solid lines for the Fermi distribution function f_F and by the points for the f_t-distribution (both the cross sections are the numerical integrations over real r). The dashed lines are the two-pole approximation [9] with the phase (4.5) for the trapezoidal distribution. It is seen that the two-pole approach with the analytical form (4.5) for the phase works well at comparably large energies and angles of scattering.

5 Conclusions

Summarizing the results of investigations of the high energy approximation methods the following conclusions can be made. First, the extended Glauber formula (2.2) is working apparently well at both small and large angles of the heavy ion scattering, however, further

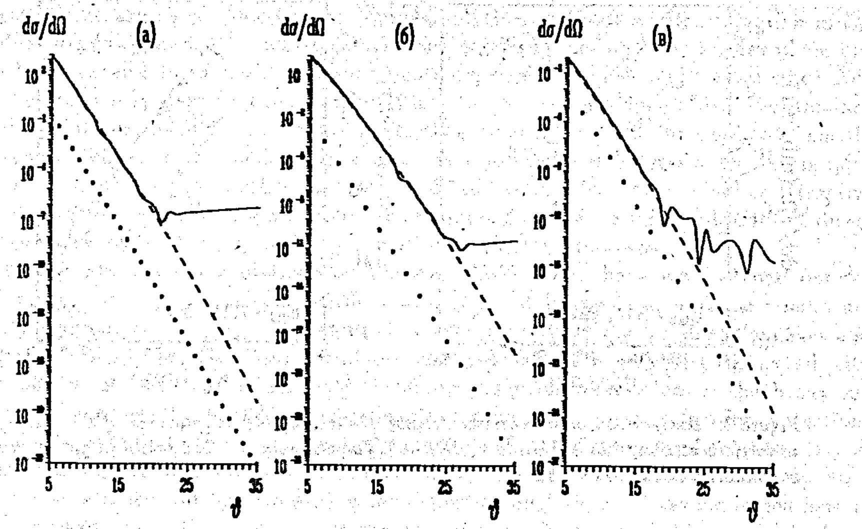

Figure 1: Cross sections of $^{17}O + ^{90}Zr$ at E = 1430 MeV. The solid and dashed lines are numerical and the SPM calculations with the exact phase integral (2.5) for the Fermi distribution f_F and stars – the SPM with the ADL-approach for phases (3.2), (3.5).

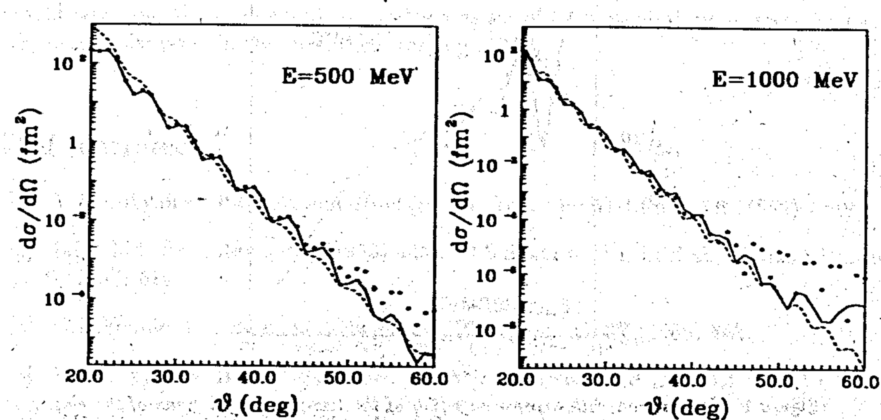

Figure 2: *Cross sections of ^6Li+^{12}C for $V_0 = -28$ MeV, $W_0 = -8$ MeV, $R = 4.68$ fm, a = 0.38 fm. The solid lines and points are the cases corresponding to the numerical integration of (2.14) with the f_F-distribution and its trapezoidal approximation f_t. The dashed curves are the two-pole approach with f_t.*

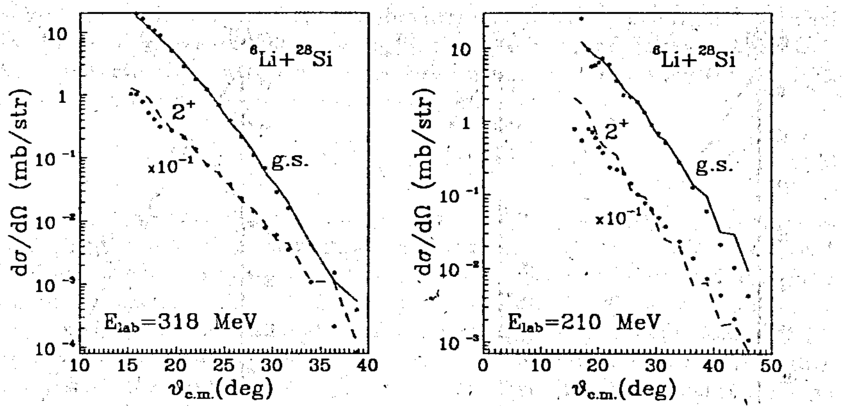

Figure 3: *Comparison with experiments [20] of the two-pole calculations of the elastic and inelastic scattering $^6Li + ^{28}Si$ (g.s., 2^+) at different energies. The modified phases for the step distribution f_Θ are used.*

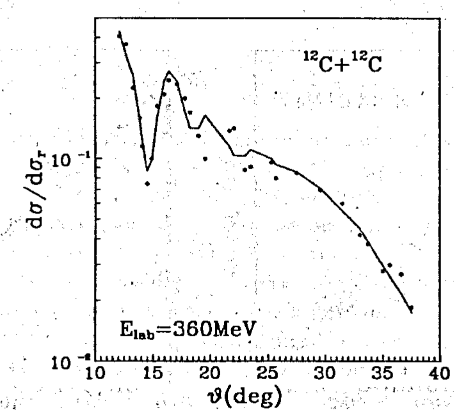

Figure 4: *Comparison with experiments [20] of the two-pole calculations of the elastic scattering $^{12}C + ^{12}C$ with the trapezoidal distribution f_t in the phase integral.*

investigations are to be done for developing analytical methods for its evaluation in the explicit form. Second, in analytic calculations of amplitudes on the complex plane one should be careful when selecting approximate expressions for eikonal phases since they can be sufficiently good for the real r but faulty for the complex one. Third, the standard saddle point method for evaluations of the fastly oscillating integrals for amplitudes should be generalized when the saddle points occur close to the integrand poles on the complex plane. Fourth, the pole method and its simplified two-pole version are rather attractive for applications since they give evident and simple expressions for amplitudes of direct collisions, clever for the physical interpretation of their mechanisms.

In the last years, calculations by the two pole method have been carried out for scattering and for the nucleon transfers with incident heavy ions, and the results are partly summarized in [18]. The method is working well at dozens MeV/nucleon and higher of the ion energy. Also, he allows one to resolve the problems connected with calculations of the recoil effects and the finite range approximation in the heavy ion transfer reactions [19]. In those applications a step function f_Θ was taken for evaluations of the phases Φ that were then modified to include effects of dimness of the boundary of an interaction potential at $r = R$. The effect of distortion of the relative motion trajectory was taken into account by introducing the deflection angle θ_c as it was explained here in the comments to eq.(1.1). As a recent example of applications of this approach, Fig.3 demonstrates calculations [10] of elastic and inelastic scattering of ^6Li on ^{28}Si (g.s., 2^+) at $E_{lab} = 318\ MeV$ and 219 MeV and their comparison with experimental data [20]. The results from [11] for the trapezoidal distribution function in the phase integral is shown in Fig.4 for elastic scattering $^{12}C + ^{12}C$ at $E = 360\ MeV$. The optical model parameters obtained in [10], [11] occur to be close to those in [9] at comparably larger energies. One should note that the problem of the ambiguous set of the potential parameters for heavy ions is still discussed in the literature (see, e.g., [22]).

References

[1] V.K.Lukyanov, Phys.Atomic Nucl.(transl. from russ.Yad.Fiz.), **58** (1995) 1848.

[2] L.I.Schiff, Phys.Rev., **103** (1956) 443.; D.S.Saxon and L.I.Schiff., Nuovo Cimento., 6 (1957) 614.

[3] D.R.Yennie, F.L.Boos, D.C.Ravenholl, Phys.Rev., **B137** (1965) 882.

[4] V.K.Lukyanov, Bull. Russ. Acad. Sc., Physics (russian ed.), **58** n.1 (1994) 6.

[5] J.R.Shepard and E.Rost, Phys.Rev. C, **25** (1982) 2660.

[6] R.J.Glauber, *in* Lectures on Theor. Phys., **1** (Interscience, New York, 1959).

[7] A.G.Sitenko, Ukr.Fis.Zh. (russian ed.), **4** (1957) 152.

[8] P.Roussel-Chamaz et al, Nucl.Phys., **A477** (1988) 345.

[9] V.K.Lukyanov, K.A.Gridnev, A.V.Embulaev, Bull. Russ. Acad. Sc., Physics (russian ed.), **58** n.1 (1994) 23; V.K.Lukyanov, A.V.Embulaev, V.P.Permyakov, JINR Rapid Comm., **4[67]-94** (1994) 12, Dubna.

[10] A.V.Embulaev, V.K.Lukyanov, V.P.Permyakov, Bull. Russ. Acad. Sc., Physics (russian ed.), **59** n.11 (1995) 83.

[11] E.V.Zemlyanaya, V.K.Lukyanov, V.P.Permyakov, Yu.V.Chubov, Bull. Russ. Acad. Sc., Physics (russian ed.), **61**, n.1 (1997) 132.

[12] P.J.Korol, Phys.Rev. C, **11** (1975) 1203.

[13] R.D.Amado, J.P.Dedonder, F.Lenz, Phys.Rev. C, **21** (1980) 647.

[14] V.K. Lukyanov, Bull. Russ. Acad. Sc., Physics (russian ed.), **60**, n.1 (1996) 8.

[15] A.V.Embulaev. E.V.Zemlyanaya, V.K.Lukyanov, V.P.Permyakov, Yu.V.Chubov, Preprint JINR, **P7-97-185** (1997), Dubna.

[16] Yu.Chubov, A.V.Embulaev, V.K.Lukyanov, V.P.Permyakov, E.V.Zemlyanaya, Preprint JINR, **E7-97-271** (1997), Dubna.

[17] M.Abramowitz and I.A.Segun, Handbook of mathematical functions (Dover, New York, 1968).

[18] A.V.Embulaev, S.I.Fedotov, K.A.Gridnev, V.K.Lukyanov, V.P.Permyakov, *in* Low Energy Nucl.Dynamics, Proc. XV Nucl.Phys.Div.Conf., W.S. (1995) 369.

[19] S.I.Fedotov, V.K.Lukyanov, JINR Rapid. Comm., **4[67]-94** (1994) 5; K.A.Gridnev, V.K.Lukyanov, S.I.Fedotov, Bull. Russ. Acad. Sc., Physics (russian ed.), **59**, n.5 (1995) 46; V.K.Lukyanov, S.I.Fedotov, Bull. Russ. Acad. Sc., Physics (russian ed.), **59** n.11 (1995), 74.

[20] A.Nadasen et al, Phys.Rev. C, **39** (1989) 536; A.Nadasen et al, Phys.Rev. C, **47** (1993) 674.

[21] Liguori Neto R. et al, Nucl.Phys. **A560** (1993) 733.

[22] G.Bartnitzky et al., *in* Low Energy Nucl.Dynamics, Proc. XV Nucl.Phys.Div.Conf., W.S. (1995) 392; M.-E.Brandan, *ibid*, 399; Y.Sugiyama, *ibid*, 384.

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