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A STUDY OF SEMI-CLASSICAL SCATTERING IN A WOODS–SAXON POTENTIAL WITHIN THE HIGH-ENERGY APPROXIMATION

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Чубов Ю.В. и др. E7-97-271
Изучение квазиклассического рассеяния в поле потенциала Вудса–Саксона в высокоэнергетическом приближении

В рамках метода высокоэнергетического приближения исследуется амплитуда упругого ядро-ядерного рассеяния в поле ядерного потенциала Вудса–Саксона. Сравниваются численные расчеты эйконалов и амплитуд «ближнего» и «дальнего» рассеяний с модельными выражениями эйконалов и соответствующими амплитудами, полученными методом перевала. Сделаны выводы об областях применимости моделей в предельных случаях сильного поглощения и рефракции, а также при традиционном выборе параметров взаимодействия. Показано, что во многих случаях модельные эйконалы используются вне области их допустимого применения.

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Chubov Yu.V. et al. E7-97-271
A Study of Semi-Classical Scattering in a Woods–Saxon Potential within the High-Energy Approximation

The elastic scattering amplitudes of heavy ions presented in the form of the high-energy approximation are studied using a Woods–Saxon potential. Numerical calculations for eikonals and for the «near-» and «far-side» amplitudes are compared with those obtained by the saddle point method. Conclusions are made on applicability of models when parameters of potentials are selected in accordance with the cases of strong absorption, refraction and the optical scattering. It is shown that in many cases the models are utilized beyond the scope of their suitability.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

Calculations of the heavy ion elastic scattering at intermediate energies is very sensitive to parameters of an interaction potential. The goal of this paper is to get an analytical expression for the elastic scattering amplitude within the high energy approximation (HEA) [1], [2], [3] which gives a possibility to search for a mechanism of the process depending on the potential included and the dynamical parameters as well. One of the problems of the HEA is that it needs calculating quickly oscillating integrals. Usually, one can avoid these difficulties by extending this sort of integrals into a complex plane. In addition, there exists one more problem since the eikonal phases of an integrand are unknown in an analytic form for a Woods – Saxon potential typical of nuclear physics. To resolve it, in many papers approximate expressions have been suggested instead of a Woods – Saxon potential (see, e.g., refs. [4], [5]) which allow calculations of the eikonal phases in an analytic form. Also, the phases were approximated in explicit forms from the beginnings. For example, the Glauber high energy approach was successfully applied in [6] to the pA – elastic scattering with the eikonal phase calculated by using the residue at only one pole of the Fermi function nearest to the real axis in the first quadrant of a complex plane of impact parameters. In all these cases, approximations are made on the real axis, but one should remember that after transition to the complex plane the behavior of original and approximate functions may be very different.

In Sec.2, we extend the approximate expression for a phase integral from [6] to the fourth quadrant and we also study its applicability in the case of heavy ion scattering. In particular, the saddle point trajectories on the complex plane are compared when the approximate (from [6]) and the exact (from [7]) expressions are used for the nuclear eikonal phases for a Woods - Saxon potential. Sec.3 summarizes the results of our calculations for scattering in the cases of nuclear refraction, optical model scattering and nuclear diffraction.

2 Scattering amplitude and saddle points

We consider the elastic nucleus-nucleus scattering at energies $E \gg |V|$ and $kR \gg 1$ and use the amplitude obtained in [8] for large scattering angles $\theta > |V|/E$, $\theta > (1/kR)$ which covers a wide region of θ where experimental data usually exist:

$$f(\theta) = -\frac{m}{2\pi\hbar^2}[t_{(+)}(q) - t_{(-)}(q)]. \tag{2.1}$$

Here, $q = 2k\sin(\theta/2)$ is momentum transfer and

$$t_{(\pm)} = -\frac{2\pi i}{q}\int_0^\infty r dr U(r)\exp\{i[\pm qr - \frac{1}{\hbar v}\int_{-\infty}^{\infty} U(\sqrt{\rho^2+\lambda^2})d\lambda]\}. \tag{2.2}$$

Inserting a Woods Saxon potential

$$U(r) = (V_0 + iW_0)f_N(r), \qquad f_N(r) = \frac{1}{1+\exp\frac{r-R}{a}}, \tag{2.3}$$

we find

$$t_{(\pm)}(\gamma) = 2\pi\frac{\hbar v}{q}\gamma J^{(\pm)}(\gamma), \qquad \gamma = -i\frac{V_0 + iW_0}{\hbar v}, \tag{2.4}$$

where

$$J^{(\pm)} = \int_0^\infty r\, dr\, f_N(r)\, e^{g_{(\pm)}(r,\gamma)}. \tag{2.5}$$

$$g_{(\pm)}(r,\gamma) = \pm iqr + \gamma I(r). \tag{2.6}$$

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$$I(r) = 2\int_0^\infty f_N(\sqrt{\rho^2+\lambda^2})\, d\lambda, \qquad \rho = r\cos\frac{\theta}{2}. \tag{2.7}$$

In [6], the last integral was approximated by a one residue in the first quadrant of the complex ρ- plane. We have generalized this approach and obtained the following expression [9]:

$$\tilde{I}(\rho) \simeq 2R - \frac{2\pi i a r^\pm}{\sqrt{(r^\pm)^2-\rho^2}}, \qquad \left(Im\ \sqrt{(r^\pm)^2-\rho^2} \geq 0\right), \tag{2.8}$$

where $r^\pm = R \pm i\pi a$ are the first poles of the Fermi function in the I and IV quadrants, respectively. The explicit formula for (2. 7) was obtained in [7]:

$$I(\rho) = 2R - 2\pi i a \sum_{p=1}\left(\frac{r_p^+}{\lambda_p^{(+)}} + \frac{r_p^-}{\lambda_p^{(-)}}\right), \qquad Im\ \lambda_p^{(\pm)} \geq 0, \tag{2.9}$$

where $\lambda_p^{(\pm)} = \sqrt{(r_p^\pm)^2 - \rho^2}$, $r_p^\pm = R \pm i\pi a(2p-1)$ with p=1,2,3... For real ρ we have $\lambda_p^{(-)} = -\lambda_p^{(+)}$.

It is clear that for large q the integrals (2. 5) oscillate very quickly. We evaluate them by using the saddle point method (SPM). The saddle points are solutions of the equation:

$$g'_{(\pm)}(r,\gamma) = \pm iq + \gamma I'(r) = 0, \tag{2.10}$$

where the signs "+" and "−" correspond to the so-called near- and farside amplitudes, respectively. The standard SPM expression for integrals of the type of (2. 5) is given by

$$J^{(\pm)}(r_s) = -r_s f_N(r_s)\, e^{g_{(\pm)}(r_s)}\sqrt{-2\pi/g''_{(\pm)}(r_s)}, \tag{2.11}$$

where $r_s = r_s^{(\pm)}$ depends on the transfer momentum q, and s - is the number of solution to (2. 10). We assume that the main contribution to (2. 5) comes from the saddle points around the poles $r_1^\pm = R \pm i\pi a$. It is clear that at large q, saddle points $r_s^{(\pm)}$ are displayed close to the poles $r_p^\pm = R \pm i\pi a(2p-1)$, where p=1,2,3...

When solving eq.(2. 10) numerically, a very effective continuous analog of the Newton method [10] was used. Also it was investigated that for practical cases one can use only 13 terms in the sum (2. 9) for the eikonal phase I (2. 7). If one takes the approximation

(2. 8) for $\tilde{I}$, he roots of equation (2. 10) for saddle points. In fact, substituting (2. 8) into (2. 6) and (2. 10) one obtains

$$g_{(\pm)}(r) = \pm iqr + 2R\gamma + i\frac{\bar{\alpha}r^\pm}{\lambda^{(\pm)}}, \qquad Im\ \lambda^{(\pm)} \geq 0, \tag{2.12}$$

$$g'_{(\pm)}(r) = \cos\frac{\theta}{2}\{\pm i\bar{q} + i\frac{\bar{\alpha}r^\pm\rho}{\lambda^{(\pm)3}}\} = 0. \tag{2.13}$$

Here $\lambda^{(\pm)} = \sqrt{(r^\pm)^2-\rho^2}$, $\bar{q} = q\cos^{-1}\theta/2$, and

$$\bar{\alpha} = -2\pi a\gamma = \frac{2\pi a}{\hbar v}(|W_0| - i|V_0|) = |\alpha|\ e^{i\beta_\alpha}, \tag{2.14}$$

$$\beta_\alpha = 2\pi - \arcsin\frac{1}{\sqrt{1+(W_0/V_0)^2}}, \tag{2.15}$$

$$r^\pm = R \pm i\pi a = |r^\pm|\ \exp[i\beta_r^{(\pm)}], \tag{2.16}$$

$$\beta_r^{(+)} = \arcsin\frac{\pi a}{\sqrt{\pi^2a^2+R^2}} \simeq \frac{\pi a}{R}, \qquad \beta_r^{(-)} = 2\pi - \beta_r^{(+)}. \tag{2.17}$$

At large momentum transfer the saddle point will be near the poles $r^\pm$; therefore, the solutions of (2. 13) may be represented as

$$r^{(\pm)} = \rho^{(\pm)}\cos^{-1}\frac{\theta}{2}, \qquad \rho^{(\pm)} = r^\pm + \delta^{(\pm)}, \qquad |\delta^{(\pm)}| \ll |r^\pm|. \tag{2.18}$$

Then, one can rewrite (2. 13) in the form:

$$\bar{q} = \mp\frac{\bar{\alpha}r^\pm(r^\pm+\delta^{(\pm)})}{\lambda^{(\pm)3}} \simeq \mp\frac{\bar{\alpha}(r^\pm)^2}{\lambda^{(\pm)3}}, \tag{2.19}$$

where

$$\lambda_n^{(\pm)} = |\lambda|\ \exp[i\beta_\lambda^{(\pm)}], \qquad |\lambda| = \left[\frac{|\alpha||r^\pm|^2}{\bar{q}}\right]^{\frac{1}{3}}, \tag{2.20}$$

$$\beta_\lambda^{(\pm)} = \frac{\pi}{3}(2n + \frac{1}{2} \pm \frac{1}{2}) + \frac{1}{3}\beta_\alpha + \frac{2}{3}\beta_r^{(\pm)}. \tag{2.21}$$

Here, n=0, 1, 2 are the numbers of roots of (2. 19), and they are selected to satisfy the condition $Im\ \lambda_n^{(\pm)} \geq 0$. In the I quadrant, one gets

$$\sin\beta_\lambda^{(+)} = \sin\left[\frac{\pi}{3}(2n+1) + \frac{1}{3}\beta_\alpha + \frac{2}{3}\beta_r^{(+)}\right] \geq 0, \tag{2.22}$$

and for solutions in IV, one has

$$\sin\beta_\lambda^{(-)} = \sin\left[\frac{\pi}{3}2n + \frac{1}{3}\beta_\alpha + \frac{4}{3}\pi - \frac{2}{3}\beta_r^{(+)}\right] \geq 0. \tag{2.23}$$

Now, since $|\delta^{(\pm)}| \ll |r^{\pm}|$ we find:

$$\lambda_n^{(\pm)} = \sqrt{(r^{\pm}+\rho)(r^{\pm}-\rho)} \simeq \sqrt{-2r^{\pm}\delta_n^{(\pm)}}, \tag{2.24}$$

and then

$$\delta_n^{(\pm)} = -\frac{1}{2}\frac{|\lambda|^2}{|r^{\pm}|}\,\exp[i\beta_\delta^{(\pm)}]. \tag{2.25}$$

Here

$$\beta_\delta^{(+)} = \frac{2}{3}\pi(2n+1) + \frac{2}{3}\beta_\alpha + \frac{1}{3}\beta_r^{(+)}, \tag{2.26}$$

$$\beta_\delta^{(-)} = \frac{2}{3}\pi(2n+1) + \frac{2}{3}\beta_\alpha - \frac{1}{3}\beta_r^{(+)}. \tag{2.27}$$

It has been shown in [9] that for the nucleus-nucleus collisions when $R = R_1 + R_2 \gg \pi a$ we have the following approximate values:

$$\beta_\lambda^{(+)} = \begin{cases} \frac{2}{3}\pi n + \pi & \text{for} \quad |W_0| \gg |V_0|, \\ \frac{\pi}{3}(2n+1) + \frac{\pi}{2} & \text{for} \quad |W_0| \ll |V_0|. \end{cases} \tag{2.28}$$

$$\beta_\lambda^{(-)} = \begin{cases} \frac{2}{3}\pi n & \text{for} \quad |W_0| \gg |V_0|, \\ \frac{2}{3}\pi(n+2) + \frac{\pi}{2} & \text{for} \quad |W_0| \ll |V_0|. \end{cases} \tag{2.29}$$

Thus, using eqs. (2. 28), (2. 29) and bearing in mind that $0 \leq \beta_\lambda^{(+)} \leq \frac{\pi}{2}$ and $\frac{3}{2}\pi \leq \beta_\lambda^{(-)} \leq 2\pi$, one can select the root number n among the values n=0, 1, 2.

3 Results and conclusion

First, we have studied applicability of the approach [6] in the case of the heavy ion scattering when the approximate expression (2. 8) is used for the eikonal phase with a Woods – Saxon potential. An attractive feature of this approach is that in both the cases of scattering at small and large angles θ one can obtain an explicit formula for the amplitudes by using the SPM of calculations. Below, as an example we consider elastic scattering of two nuclei with atomic numbers 17 and 90 by the complex Woods – Saxon potential with the geometrical parameters R=7.05 fm and a=0.5 fm. The parameters V_0 and W_0 vary. The kinetic energy in the c.m. system is E=1435 MeV.

In Fig.1. we show the results of calculations for the complex trajectories of saddle points.

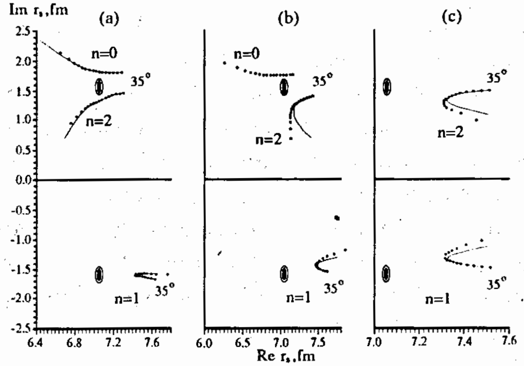

Figure 1: *The behavior of the saddle points on the complex r plane with different parameters of the nuclear potential. Scattering angle interval is from 5° up to 35°. Solid lines are exact numerical solutions, stars correspond to approximate calculations, the circled black spots present the poles. (a) V_0= -50 MeV, W_0=0; (b) V_0= -50 MeV, W_0= -25 MeV; (c) V_0= -1 MeV, W_0= -50 MeV.*

Trajectories in Fig.1(a) correspond to the refractive scattering when the absorption parameter W_0=0. The two big black spots represent the poles $r^{\pm} = R \pm i\pi a$ nearest to the real axis for a Woods – Saxon potential in the integrand function. One can see that in this case the solutions (stars) of the approximate saddle point equation (2. 19) coincide with numerical calculations of the exact equation (2. 10) (solid lines). Then, it is seen that in the I quadrant there exist two trajectories ($n = 0, 2$), while in the IV quadrant there is only one root $n = 1$. When absorption is included ($W_0 \neq 0$), a distinct behavior is seen of numerical and approximate solutions. For strong absorption (Fig.1(c)) only one root exists in the I quadrant and one root is in the IVth one. This property is a consequence of (2. 26), (2. 27) because of β_α=0.

It is obvious that in the scattering amplitude proportional to (2. 11) the exponential term $\exp[g(r_s^{(\pm)})]$ plays a dominant role. Since $g(r_s^{(\pm)}) = \pm iqr_s^{(\pm)} + \gamma I(r_s^{(\pm)})$, the behavior of the eikonal integral I on the complex plane significantly influences the absolute value and the angular dependence of cross sections. Thus, the approximations for eikonal phases made for real

r can have no meaning when these phases are considered on the complex r- plane. As an example, Fig. 2 exhibits the calculations for Im I and Re I with the help of exact (2. 7) and approximate (2. 8) expressions shown by the solid and dashed lines, respectively. The stars are when only 13 terms in the sum (2. 9) are taken into account. It is seen that, in practice, in calculating one may use several terms only. And the main result is that an approximate formula (2. 8) behaves on the complex plane in a very different way (dashed lines) as compared with the exact one (2. 7) (solid lines). Thus, one should be very careful when expanding approximate models of the eikonal integrals into the complex r- plane.

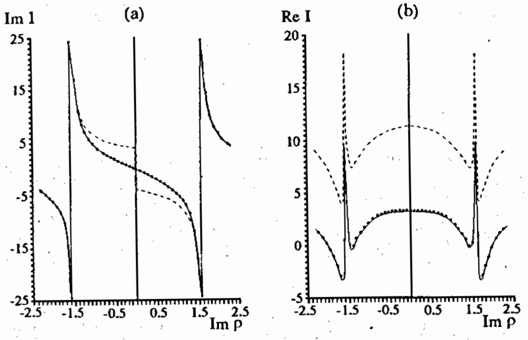

Figure 2: *The behavior of eikonal integral I dependending on the imaginary part of the impact parameter ρ when its real part is equal to R=7.05 fm.*

To see how well the approximate amplitudes work, we compare them with the corresponding exact numerical calculations. Fig.3 shows the cross sections obtained by the SPM (dashed lines) compared with the full numerical calculations (solid lines) for various parameters of a Woods – Saxon nucleus-nucleus potential.

The exact numerical calculations by (2. 5) have used the exact eikonal phase (2. 9). The dashed lines are the SPM calculations with the help of (2. 11) where also the exact phase (2. 9) was used. The stars are calculations with the approximate phase $\tilde{I}$ (2. 8). It is seen that evaluations made by using $\tilde{I}$ are different from others. Their absolute values are in strong dependence on the parameters of the real part of a potential. All

calculations by the SPM, which use the exact phase (2. 9), are in good coincidence with the results of numerical integration up to the angles where the slope of curves of cross sections changes. In this region, one has to elaborate other methods which include contributions from the saddle point trajectories passing near each of the poles $r_p^{\pm} = R \pm i\pi a(2p-1)$, $p = 1, 2, 3 \ldots$.

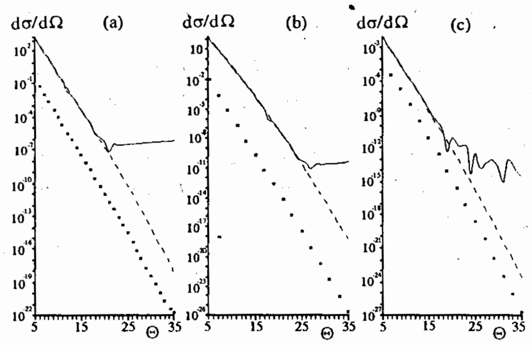

Figure 3: *Eikonal cross sections in (fm^2/st) versus the scattering angles. (a) V_0= -50 MeV, W_0=0; (b) V_0= -50 MeV, W_0= -25 MeV; (c) V_0= -1 MeV, W_0= -50 MeV.*

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