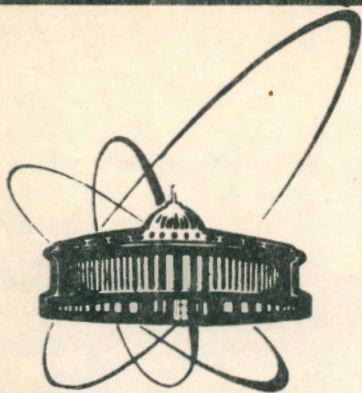


90-327



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L. Münchow

DETERMINATION OF THE NUCLEAR DENSITY
CORRELATION FUNCTION IN HEAVY ION
REACTIONS

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In the theory of Fermi liquids the density correlation function $F(\vec{x}_1 - \vec{x}_2) = \langle \rho(\vec{x}_1) \rho(\vec{x}_2) - \rho^2 \rangle$ and respectively, the form factor

$$S(\vec{x}_1 - \vec{x}_2) = \langle \psi^\dagger(\vec{x}_1) \psi(\vec{x}_1) \psi^\dagger(\vec{x}_2) \psi(\vec{x}_2) \rangle \rho^{-2}$$

play an essential role ^{/1,2/}.

Especially, the strong density fluctuations which occur as the precursor of a liquid-gas phase transition may be expressed by the relation $\rho / (\beta (\partial P / \partial \rho))_\beta = \int F(\vec{r}) d\vec{r}$ ^{/3/} which establishes a connection between macroscopical thermodynamical properties like the pressure P and density ρ on the one hand, and intrinsic characteristics of the correlated system, on the other hand. Furthermore, a direct measurement of the density correlation function in liquids is possible by utilizing the relation between the cross-section for light or particle scattering and the form factor ^{/2/}.

We want to note here that in heavy ion reactions, by using a VUU analysis, it is possible to extract the nuclear density correlation function.

In this case such a procedure may also be useful if the system is close to the spinodal region $(\partial P / \partial \rho)_\beta \rightarrow 0$, where instability sets in (for a recent discussion see, e.g., ^{/4/}).

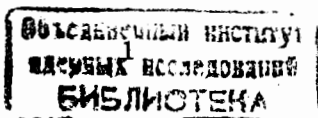
A standard description of HI reactions at intermediate energies starts from the VUU equation, which in a quantum statistical model ^{/5/} may be represented as

$$\left(\frac{\partial}{\partial T} + \frac{\vec{p}}{m} \frac{\partial}{\partial \vec{R}} + \frac{\partial \text{Re} \Sigma^+(\vec{p}; \vec{R}, T)}{\partial \vec{p}} \frac{\partial}{\partial \vec{R}} + \frac{\partial \text{Re} \Sigma^+(\vec{p}; \vec{R}, T)}{\partial \vec{R}} \cdot \frac{\partial}{\partial \vec{p}} \right) f(\vec{p}; \vec{R}, T) =$$

$$= -i \Sigma^<(\vec{p}, \omega_{\vec{p}}; \vec{R}, T) (1 - f(\vec{p}; \vec{R}, T)) - i \Sigma^>(\vec{p}, \omega_{\vec{p}}; \vec{R}, T) f(\vec{p}; \vec{R}, T),$$

$$\omega_{\vec{p}} = \frac{p^2}{2m}.$$

The l.h.s. describes the quasiclassical time-dependent mean field evolution for the Wigner function $f(\vec{p}; \vec{R}, T)$, while the r.h.s. contains the usual gain minus loss structure. In this expression the mass operator is used, where by definition



$$-i \Sigma^<(1,1') = \langle j^+(1') j(1) \rangle,$$

$$i \Sigma^>(1,1') = \langle j(1) j(1') \rangle,$$

$$j(1) = j(\vec{x}_1, t_1) = \quad (1)$$

$$= \int d\vec{x}_2 V(\vec{x}_1 - \vec{x}_2) \psi^+(\vec{x}_2, t_1) \psi(\vec{x}_2, t_1) \psi(1),$$

and the index "irred" means that one eliminates those diagrams for which a cut between the end points passes only through a single particle line. Furthermore, the mean field term in the VUU equation is expressed by the relation

$$\Sigma^+(1,2) = \Sigma_{HF}(1,2) + \theta(t_1 - t_2) (\Sigma^>(1,2) - \Sigma^<(1,2)),$$

where $\Sigma_{HF}(1,2)$ is the HF potential. Finally let us write the general relations ^{5/} for the width of the single particle state

$$\Gamma(1,2) = -2 \text{Im} \Sigma^+(1,2) = i(\Sigma^>(1,2) - \Sigma^<(1,2)), \quad (2)$$

$$\text{Re} \Sigma^+(\vec{p}, \omega; \vec{R}, T) = \Sigma_{HF}(\vec{p}; \vec{R}, T) + \int \frac{d\omega'}{2\pi} \frac{\Gamma(\vec{p}; \vec{R}, T)}{\omega - \omega'}.$$

These formulae are very useful to establish the connection between the Green function description and the density correlation function, as we shall see.

Starting from the definition of the mass operator (1), performing a Fourier transformation and time ordering and neglecting three-body correlations, we obtain

$$\Sigma^<(\vec{p}, \omega) = \int G^>(\vec{p}', \omega') F^<(\vec{p}' - \vec{p}, \omega' - \omega) (V(\vec{p}' - \vec{p}))^2 d\vec{p}' / (2\pi)^3 d\omega' / (2\pi), \quad (3)$$

with the Green functions calculated from the definitions

$$-iG^<(1,2) = \langle \psi^+(2) \psi(1) \rangle, \quad iG^>(1,2) = \langle \psi(1) \psi^+(2) \rangle,$$

and the density correlation functions

$$F^>(1,1) = \langle \rho(1) \rho(2) - \rho^2 \rangle, \quad F^<(1,2) = \langle \rho(2) \rho(1) - \rho^2 \rangle.$$

Applying in the last equation a completeness relation we have

$$F^>(\vec{q}, \omega) = 1/V \sum_n |\langle 0 | \rho(\vec{q}) | n \rangle|^2 2\pi \delta(\omega - \omega_{n0}),$$

$$|F^<(\vec{q}, \omega) = 1/V \sum_n |\langle 0 | \rho(\vec{q}) | n \rangle|^2 2\pi \delta(\omega + \omega_{n0}). \quad (4)$$

From these relations together with eq.(3) it follows that $\Sigma^>$ describes the scattering on p-h excitations (RPA-type modes) of the time-dependent TDHF-state $|0\rangle$.

The general expression (3) is simplified if the system is close to the spinodal region and if thermodynamical equilibrium is assumed. Near the point of instability a low-lying collective mode gets soft and therefore $\omega_c \rightarrow 0$. Since the corresponding transition probability in eq.(4) is $\langle 0 | \rho | n \rangle \sim \omega_c^{-1/2}$, together with eq.(3) we get

$$\Sigma^>(\vec{p}, \omega) = \int d\vec{p}' / (2\pi)^3 (V(\vec{p}' - \vec{p}))^2 F(\vec{p}' - \vec{p}) G^>(\vec{p}', \omega),$$

with the Fourier transform of the static density correlation function

$$F(\vec{q}) = \int F(\vec{x}_1 - \vec{x}_2, \rho) e^{+iq(\vec{x}_1 - \vec{x}_2)} d(\vec{x}_1 - \vec{x}_2).$$

Integrating over ω , taking account of the relation ^{5/}

$$\int d\omega / (2\pi) (iG^>(\vec{p}, \omega) - iG^<(\vec{p}, \omega)) = 1$$

together with the definition (2) we finally obtain the very instructive relation

$$\int \Gamma / (2\pi) d\omega = \int F(\vec{q}) (V(\vec{q}))^2 d\vec{q} / (2\pi)^3. \quad (5)$$

Formally this expression is a general result, if the contour is chosen to include all energies $= \pm \omega_{n0}$. The formula (5) has been quoted earlier ^{5/} without proof, and an equivalent condition expressed by the collision term has recently been derived ^{6/}.

Let us take in the thermodynamical limit, following ^{3/}, the approximation for the static density correlation function derived from the RPA

$$F(\vec{q}) = \rho / ((T - T_c) / T + T_c / T a q^2), \quad a = r_v^2 / 6, \quad (6)$$

where r_v is a range parameter of the force. By the Fourier transformation it then follows that

$$F(r) = \rho T (a T_c \cdot 4\pi r)^{-1} \exp\{-[(T - T_c) / (a T_c)]^{-1/2} r\},$$

which is just the Ornstein-Zernicke density correlation function near the critical point $T = T_c$. Applying this formula to the relation $\rho/(\beta(\partial P/\partial\rho)\beta) = \int F(\vec{r})d\vec{r}$, one verifies, that for $T \rightarrow T_c$ this expression diverges thus signifying the onset of the liquid-gas phase transition. A related divergent behaviour occurs also in the mass operator, and therefore, in the collision integral and in the mean field. Really, from eqs. (5) and (6) we get

$$\int \Gamma/(2\pi)d\omega - F(\vec{q} = 0) \cdot \int (V(\vec{q}))^2 d\vec{q}/(2\pi)^3 \sim \int F(\vec{r}) d\vec{r} \cdot (V(\vec{q}))^2 d\vec{q}/(2\pi)^3,$$

demonstrating the strong increase of this quantity near the critical point. This result is similar to the well-known critical opalescence phenomenon. We have recently obtained the same behaviour in a Landau-type theory directly calculating the particle-fluctuation contribution to the mass operator⁷⁷.

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Мюнхов Л.

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Определение ядерной корреляционной функции плотности в реакциях с тяжелыми ионами

Реакции с тяжелыми ионами при промежуточных энергиях часто описываются уравнением Власова - Уленбека - Улинга. Пользуясь методами квантовой кинетики, мы вывели полезное соотношение между интегрированной по частотам одночастичной шириной и статической корреляционной функцией. Вблизи точки неустойчивости из этого соотношения следует прирост интеграла столкновения и увеличение скорости релаксации.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Münchow L.

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Determination of the Nuclear Density Correlation Function in Heavy Ion Reactions

Heavy ion reactions at intermediate energies are mostly described by the VUU equation. Using methods of quantum kinetics we derive a useful relation between the frequency integrated width of s.p. states and the static correlation function. Near the point of instability from this relation follows the increase of the collision integral and enhanced equilibration.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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