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L.Münchow

PRECRITICAL INCREASE OF PARTICLE COLLISION RATES

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1. INTRODUCTION

From a VUU simulation of $C_a + C_a$ collisions the authors [1] concluded that for $E_{lab} > 60$ MeV/A after times t = 80 fm/c "macroscopic" density fluctuations as the precursor of fragmentation set in. These increasing fluctuations reflect the closeness of the system to the region of instability. Really, in these calculations the potential energy per particle as a function of time evolves through a saddle point, and this maximum marks the end of the phase of homogeneous expansion and the beginning of the clusterization process. Depending on the excitation energy the density corresponding to this saddle point is $0.001 < \rho/\rho_0 < 0.60$. Additionally, from the fully thermalized part of the total energy typical temperatures in the region between 3-5 MeV have been estimated. In a thermodynamical sense the system is then close to the spinodal of the P-ρ phase diagram for which the derivative gets zero $(\partial P/\partial\rho) \to 0$. From the thermodynamical relation [2] $(\partial P/\partial\rho)_T^{-1} \sim \sim \int \langle \rho(\vec{r})\, \rho(0) - \rho^2 \rangle d\vec{r}$ strong density fluctuations are expected in the precritical region and just this is the result of the simulation [1].

Now naturally the question arises: What kind of information may be sensible to these fluctuations? Rather to give a general answer we draw our attention to possible dynamical consequences of strong density fluctuations, which to our knowledge have not been discussed formerly. In order to study density fluctuations one has to go beyond the one-particle level, described by the first order density matrix $\rho_1(11')$ or semiclassically by the Wigner function $f(\vec{p}; \vec{R}, T)$. The general structure of coupled equations for first to third order density matrices was derived in ref. [3]. Starting from this hierarchy recently we have shown [4] that the coupling of single-particle motion to density fluctuations may be included into the collision integral of generalized VUU equations. As we will show, the part of the collision integral in the VUU description, which is responsible to density fluctuations, as well as the corresponding part of the s.p. potential energy get prominent in the precritical region and lead to noticeable dynamical consequences.

2. ONE-PARTICLE MOTION COUPLED TO DENSITY FLUCTUATIONS

To describe reactions we apply nonequilibrium statistical mechanics [5,6,7]. In this approach, the one-particle Green function consists of two parts

$$-iG^{<}(\vec{x}_1, t_1; \vec{x}_2, t_2) = \langle \psi^{+}(\vec{x}_2, t_2)\, \psi(\vec{x}_1, t_1) \rangle ,$$

$$+iG^{>}(\vec{x}_1, t_1; \vec{x}_2, t_2) = \langle \psi(\vec{x}_1, t_1)\, \psi^{+}(\vec{x}_2, t_2) \rangle$$

and depends on two space-time points $\vec{x}_1, t_1; \vec{x}_2, t_2$. Introducing microscopic and macroscopic variables $\vec{r} = \vec{x}_1 - \vec{x}_2$, $t = t_1 - t_2$, $\vec{R} = (\vec{x}_1 + \vec{x}_2)/2$, $T = (t_1 + t_2)/2$, the Fourier transform of the Green function of a hole

$$G^{<}(\vec{p}, \omega; \vec{R}, T) = \int d\vec{r}\, dt \exp(-i\omega t + i\vec{p}\vec{r})\, G(\vec{r}, t; \vec{R}, T)$$

with

$$G^{<}(\vec{r}, t; \vec{R}, T) = G^{<}(\vec{x}_1; t_1; \vec{x}_2, t_2)$$

is directly connected weth the Wigner function

$$f(\vec{p}; \vec{R}, T) = -i \int G^{<}(\vec{p}, \omega; \vec{R}, T)\, d\omega/2\pi .$$

Similarly, for particle evolution

$$[1 - f(\vec{p}; \vec{R}, T)] = i \int G^{>}(\vec{p}, \omega; \vec{R}, T) \cdot d\omega/2\pi ,$$

and from the Dyson equation for $G(1,2)$ there follows the generalized Boltzmann equation ($\omega_{\vec{p}} = p^2/2m$)

$$\left[\frac{\partial}{\partial T} + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{R}} + \frac{\partial \operatorname{Re}\Sigma^{+}(\vec{p};\vec{R},T)}{\partial \vec{p}}\frac{\partial}{\partial \vec{R}} + \frac{\partial \operatorname{Re}\Sigma^{+}(\vec{p};\vec{R},T)}{\partial \vec{R}}\frac{\partial}{\partial \vec{p}}\right] f(\vec{p}; \vec{R}, T) =$$
$$= -i\Sigma^{<}(\vec{p},\omega_{\vec{p}};\vec{R},T)[1 - f(\vec{p};\vec{R},T)] - i\Sigma^{>}(\vec{p},\omega_{\vec{p}};\vec{R},T) f(\vec{p};\vec{R},T) , \qquad (1)$$

which represents the basis of our reaction model. The l.h.s. of eq.(1) describes the semiclassical approximation to TDHF dynamics while the r.h.s. is the collision term. It contains two parts of the mass operator $\Sigma^{\gtrless}(\vec{p},\omega;\vec{R},T)$ which are analogous to $G^{\gtrless}(p,\omega;R,T)$. Furthermore, the retarded mass operator is defined by

$$\Sigma^{+}(\vec{x}_1 t_1; \vec{x}_2 t_2) \equiv \Sigma^{+}(12) = \Sigma_{HF}(12) + \theta(t_1 - t_2)[\Sigma^{>}(12) - \Sigma^{<}(12)],$$

where $\Sigma_{HF}(12)$ denotes the HF term. After Fourier transformation then follows the dispersion relation [7]

$$\operatorname{Re}\Sigma^{+}(\vec{p},\omega;R,T) = \Sigma_{HF}(p;R,T) + \mathcal{P}\int \frac{\Gamma(\vec{p},\omega';\vec{R},T)}{\omega-\omega'}\frac{d\omega'}{2\pi} , \qquad (2)$$

where the imaginary part of the mass operator may be represented as

$$\Gamma(\vec{p},\omega;\vec{R},T) = -2\operatorname{Im}\Sigma^{+}(\vec{p},\omega;\vec{R},T) = i(\Sigma^{<}(\vec{p},\omega;\vec{R},T) - \Sigma(\vec{p},\omega;\vec{R},T)) \quad (3)$$

A glance at eq.(1) shows the close connection between the width of the single-particle state Γ and the collision integral.

Next, the mass operator will be evaluated in a T-matrix approximation. As is well known [2,5], just the T-matrix, evaluated in the particle-hole channel, is closely connected with the stability of the system. Namely, the appearance of a pole in the upper half of the imaginary plan signals the set-in of instability. Since our aim is to study the connection between stability and dynamics, it is natural to apply the p-h channel T-matrix in the expression for the mass operator

$$\Sigma(11') = \int d2d2'\, iG(2'2)\, \{T(12;1'2') - T(12;2'1')\}, \qquad (4)$$

where we used the notation $1 \equiv (\vec{x}_1, t_1)$, , $\int d1 = \int d\vec{x}_1 \cdot \int dt_1$ and the time integration is along a closed contour in chronological and antichronological direction [6,7]. After Fourier transformation and time ordering

$$\Sigma^{\gtrless}(\vec{p},\omega;\vec{R},T) = -\int \frac{d\vec{p}_1}{(2\pi)^3}\frac{d\omega}{2\pi} iG^{\lessgtr}(\vec{p}_1,\omega;\vec{R},T) \times$$
$$\times \{ \langle \frac{\vec{p}-\vec{p}_1}{2} | T^{\gtrless}(\omega-\omega_1) | \frac{\vec{p}-\vec{p}_1}{2} \rangle - \qquad (5)$$
$$- \langle \frac{\vec{p}-\vec{p}_1}{2} | T^{\gtrless}(\omega-\omega_1) | \frac{\vec{p}_1-\vec{p}}{2} \rangle \} ;$$

where

$$\langle \frac{\vec{p}_1-\vec{p}_2}{2} | T^{\gtrless}(\omega) | \frac{\vec{p}'_1-\vec{p}'_2}{2} \rangle$$

denotes the Fourier transform of $T^{\gtrless}(12;2'1')$ for $t_1 = t_2$, $t_{1'} = t_{2'}$, with respect to the microscopic variables and

$$\vec{R} = (\vec{x}_1 + \vec{x}_2 + \vec{x}_{1'} + \vec{x}_{2'})/4, \quad T = (t_1 + t_{1'})/2 .$$

The appearance of the T-matrix in eq.(5) reflects the correlations beyond the one-particle level. Mostly the T-matrix is evaluated in the particle-particle scattering channel [5,7]. We additionally include the particle-hole (p-h) channel to take care of scattering of particles on density fluctuations, which correspond to correlated p-h excitations of the TDHF state. Note that in the Landau theory [8] and for the screened Coulomb potential [5] also the p-h channel T-matrix appears in the collision term. Like in the stationary case also for the time dependent T-matrix there exists a spectral decomposition, which is useful to establish the dependence on collective frequencies.

Introducing the relative momentum $\vec{k}$

$$\vec{p}\,'_1 = \vec{p}_1 + \vec{k}, \quad \vec{p}\,'_2 = \vec{p}_2 - \vec{k}$$

and performing a ladder sum over p-h bubbles, similar to the p-p case [5,7], one ends up with the integral equation

$$\left\langle \frac{\vec{p}_1-\vec{p}_2}{2} \right| T(\omega;\vec{R},T) \left| \frac{\vec{p}_1-\vec{p}_2+2\vec{k}}{2} \right\rangle \equiv$$

$$\equiv T(\vec{p}_1,\vec{p}_2,k;\omega,\vec{R},T) = V(\vec{p}_1-\vec{p}_2) + \tag{6}$$

$$+ \int \frac{f(\vec{p}\,'_1+\vec{k};\vec{R},T) - f(\vec{p}\,'_1;\vec{R},T)}{\epsilon(p_{1'}+k) - \epsilon(p_{1'}) - \omega} V(\vec{p}\,'_1-\vec{p}_2)\, T(\vec{p}\,'_1,\vec{p}_2,\vec{k},\omega) \frac{d\vec{p}}{(2\pi)^3},$$

$$\epsilon(p) = p^2/2m + \mathrm{Re}\,\Sigma^+(p,\epsilon(p)),$$

if the width $\Gamma \ll \mathrm{Re}\,\Sigma^+$. This equation represents the time dependent and semiclassical generalization of the standard RPA equation for the T-matrix. Note the difference how this RPA comes out: Usually, RPA is a small amplitude, linearized approximation to the TDHF theory. Here, however, we assume the full large amplitude TDHF motion is the underlying mean field, and time depending p-h type correlations beyond TDHF correlations are treated. A path integral formulation for these excitations has been given formerly [9].

From eq.(6) one may obtain the spectral decomposition introducing the RPA wave equations

$$\{\omega_\nu - [\epsilon(\vec{p}+\vec{k}) - \epsilon(\vec{p})]\}\phi_\nu(\vec{p},\vec{k}) =$$

$$= \int \frac{d\vec{p}\,'}{(2\pi)^3} v(\vec{p}-\vec{p}\,') \cdot \{f(\vec{p}\,'+\vec{k};\vec{R},T) - f(\vec{p}\,';\vec{R},T)\}\phi_\nu(\vec{p}\,',\vec{k}),$$

and the completeness relation

$$\sum_\nu \phi_\nu(\vec{p},\vec{k})\, \phi^*_\nu(\vec{p}\,',\vec{k})\, [f(\vec{p}+\vec{k};\vec{R},T) - f(\vec{p};\vec{R},T)] = \delta_{\vec{p}\vec{p}\,'}.$$

Really, then

$$T(\vec{p}_1,\vec{p}_2,\vec{k};\omega,\vec{R},T) = \{\epsilon(\vec{p}_1+\vec{k}) - \epsilon(\vec{p}_1) - \omega\} \times$$

$$\times \sum_\nu \frac{X_\nu(\vec{p}_1,\vec{k})\, X^*_\nu(\vec{p}_2,\vec{k})}{\omega - \omega_\nu} \{\epsilon(\vec{p}_2+\vec{k}) - \epsilon(\vec{p}_2) - \omega_\nu\} - \tag{7}$$

$$- \sum_\nu \frac{Y_\nu(\vec{p}_1,\vec{k})\, Y^*_\nu(\vec{p}_2,\vec{k})}{\omega + \omega_\nu} \{\epsilon(\vec{p}_2+\vec{k}) - \epsilon(\vec{p}_2) + \omega_\nu\},$$

where

$$X_\nu(\vec{p},\vec{k}) = f(\vec{p}+\vec{k};\vec{R},T)(1 - f(\vec{p};\vec{R},T))\,\phi_\nu(\vec{p},\vec{k}),$$

$$Y_\nu(\vec{p},\vec{k}) = (1 - f(\vec{p}+\vec{k};\vec{R},T))\, f(\vec{p};\vec{R},T)\,\phi_\nu(\vec{p},\vec{k}).$$

Since RPA eigenfunctions depend on collective frequencies as $\omega_\nu^{-1/2}$, the T-matrix and by eq.(5) also the mass operator obtains the dependence ω_ν^{-1} showing the enhancement of these quantities for soft, precritical modes. Note that the same dependence on frequency appears in a fermion-boson coupling description, in which each vertex contains the factor $\omega_\nu^{-1/2}$ [6].

Using the relation [5]

$$T(t_1,t_2) = V + \theta(t_1-t_2)\, T^>(t_1,t_2) + \theta(t_2-t_1)\, T^<(t_1,t_2),$$

we obtain from (7) the equation for the $\left\langle \frac{p_1-p_2}{2} \right| T^{\gtrless}(\omega) \left| \frac{p_1-p_2}{2} \right\rangle$ matrices appearing in formula (5)

$$T^>(\vec{p}_1,\vec{p}_2,\vec{k};\omega,\vec{R},T) = i\pi\{\epsilon(\vec{p}_1+\vec{k}) - \epsilon(\vec{p}_1) - \omega\} \times$$

$$\times \sum_\nu \delta(\omega_\nu - \omega) \cdot X_\nu(\vec{p}_1,\vec{k})\, X^*_\nu(\vec{p}_2,\vec{k}) \times$$

$$\times [\epsilon(\vec{p}_2 + \vec{k}) - \epsilon(\vec{p}_2) - \omega_\nu], \qquad (8)$$

and a similar relation for $T^<(\vec{p}_1, \vec{p}_2, \vec{k}; \omega, \vec{R}, T)$.

From the structure of equation (6) we conclude that for k = 0 the contribution to the T-matrix, which is due to p-h correlations, disappears. As a consequence, in equation (5) only the exchange term remains. Furthermore, similar to the case of the p-p channel T matrix /5,7/ applying a generalized optical theorem one may obtain from formula (5)

$$\Sigma^<(\vec{p}, \epsilon(p); \vec{R}, T) = \frac{i}{2} \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}'}{(2\pi)^3} \frac{d\vec{p}'_1}{(2\pi)^3} \cdot (2\pi)^4 \times$$

$$\times \delta(p + \vec{p}_1 - \vec{p}' - \vec{p}'_1)\, \delta(\epsilon(\vec{p}) + \epsilon(\vec{p}_1) - \epsilon(\vec{p}') - \epsilon(\vec{p}'_1)) \} \times \qquad (9)$$

$$\times \left| < \frac{\vec{p} - \vec{p}_1}{2} | T(\epsilon(\vec{p}) - \epsilon(\vec{p}'); \vec{R}, T) | \frac{\vec{p}_1 - \vec{p}}{2} > \right|^2 \times$$

$$\times (1 - f(\vec{p}_1; \vec{R}, T)) \cdot f(\vec{p}', \vec{R}, T) \cdot f(\vec{p}'_1, \vec{R}, T),$$

and a similar expression for $\Sigma^>(\vec{p}, \epsilon(\vec{p}); \vec{R}, T)$.

Let us denote now that the width of the single-particle state, given by the relation (3), sets the rate at which equilibrium is reached /7/. Corresponding, $\tau = h/\Gamma$ has the meaning of the relaxation time. Furthermore, for the delute system close to the spinodal regime we have $f(\vec{p}; \vec{R}, T) \ll 1$ and from eq.(1) therefore $\Sigma^< \ll \Sigma^>$ and hence $\Gamma \sim i\Sigma^>$. From this consideration, together with eq.(8), we conclude that close to the critical point we expect a strong decrease in relaxation time to equilibrium, and from equation (2) also an increase in the shift of the self-consistent potential. Formerly /10/, we have analyzed the similar problem of coupling the single-particle motion to an oscillator and found the same behaviour of the relaxation time near the point of instability.

3. APPLICATION TO THE LANDAU THEORY

Let us illustrate this result applying the Landau theory for the description of instability of nuclear matter /11/. Thorough variational calculations for cold and hot nuclear matter /12/ have shown that for densities $\rho < 0.1$ fm^{-3} the sound velocity $s = \sqrt{(\frac{\partial P}{\partial \rho})_T \cdot \frac{1}{m}}$ gets imaginary. From $s(\rho) = v_F \sqrt{(1+F_0)/3}$

one may introduce then an effective F_0-parameter and assume that the mean field evolution has brought the system close to the point $F_0 = -1$. Let us remember that in the Landau theory the parameter F_0 is connected with the zero range approximation of the interaction potential

$$F_0 = V_0 \cdot m p_F / (\pi^2 h^3),$$

where p_F is the Fermi momentum and $V(p - p') = \sum_\ell V_\ell P_\ell (\cos \hat{\vec{p}\vec{p}'}) \sim V_0$ is assumed. In a homogeneous, low-temperature approximation eq.(6) has the solution

$$T = \frac{V_0}{1 + F_0 \{1 - \frac{x}{2} \ln \frac{x+1}{x-1}\}}; \quad x = \frac{\omega}{v_F \cdot k} \qquad (10)$$

and the frequencies of collective density fluctuations are given by

$$\omega_c^2 = \frac{1}{3}(1 + F_0) v_F^2 k^2.$$

Expanding the denominator in eq.(9) around this solution, we get

$$T = -\frac{V_0 v_F^2 k^2}{6\omega_c} \left\{ \frac{1}{\omega - \omega_c + i\delta} - \frac{1}{\omega + \omega_c - i\delta} \right\}$$

which has just the structure of equation (7) and

$$T^> = i \frac{V_0 v_F^2 k^2}{3\omega_c} \delta(\omega - \omega_c).$$

Let us return now to eq.(9). For an isotropic T-matrix this expression has been evaluated in /8/. Introducing like in this paper the angle ϕ between the momentum vectors p and p' we get from eq.(10)

$$T(\vec{p}, \vec{p}', \vec{p}', \vec{p}; \epsilon(\vec{p}) - \epsilon(\vec{p}')) = \frac{V_0}{\frac{1 + F_0}{3} - x^2}$$

with

$$x = \frac{[\epsilon(\vec{p}) - \epsilon(\vec{p}')]}{(|\vec{p} - \vec{p}'|) v_F} = \sin\phi$$

and for $F_0 \to -1$ the transition probability $w \sim |T|^2$ in eq.(9) is strongly forward peaked into the region $\phi \sim 0$, leading after ϕ-integration to the appearance of a $(1+F_0)^{-2}$ factor in the collision integral as compared with the isotropical case. In this way, the model predicts strong increase of collisions near the point of instability due to density fluctuations.

4. SUMMARY

Generally, the increase of the mass operator caused by the coupling of the single-particle motion to soft precritical density fluctuations may be understood as a manifestation of a dynamical anharmonicity effect. In the case of nuclear spectroscopy, it is well-known that the corresponding coupling effect to soft quadrupole modes, e.g., leads to considerable shifts of s.p. energies [13]. Like in the spectroscopical situation for a quantitatively improved description one has to include higher order graphs to take care of nonlinear mode-mode coupling effects. Additionally we remark, that if the width is comparable with particle energies the dynamics may not be decribed with the Boltzmann equation, as has been underlined formerly [7]. The qualitative prediction of increased equilibration near the critical point as well as the increasing role of meanfield effects may be very well of experimental significance for the dynamics of heavy ion collisions.

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Мюнхов Л. E7-90-326

Предкритичный прирост интеграла столкновения

Интеграл столкновений в квантовой кинетике определяется мнимой частью массового оператора. Пользуясь этим соотношением мы докажем, что связь одночастичного движения с предкритичными флуктуациями плотности вызывает сильный прирост интеграла столкновений вблизи точки фазовой неустойчивости.

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Münchow L. E7-90-326

Precritical Increase of Particle Collision Rates

In quantum kinetics the collision integral follows from the imaginary part of the mass operator. Using this connection we demonstrate that the coupling of single particle motion to precritical density fluctuations causes a strong increase of the collision integral near the point of phase instability.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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