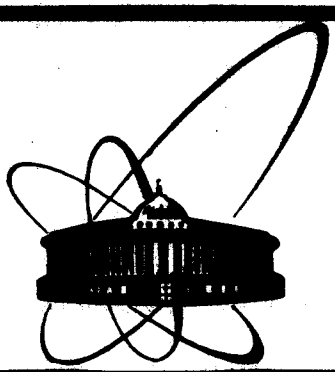

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E7-86-701

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NUCLEAR MOLECULAR DYNAMICS APPROACH TO THE NUCLEUS-NUCLEUS POTENTIALS

1986

1. INTRODUCTION

The relative motion of interacting low-energy heavy ions which imposes limits on cross sections of various processes is described well by the classical equations of motion with friction, namely

$$\frac{d}{dt}(\mu \dot{r}) - \mu r \dot{\phi}^2 + \frac{dV}{dr} + K_r \dot{r} = 0, \qquad \frac{d}{dt}(\mu r^2 \dot{\phi}) + K_\phi r^2 \dot{\phi} = 0. \tag{1}$$

Here, μ is the reduced mass, r and ϕ are the relative coordinates, V is the total potential (including the Coulomb part), and K_r and K_ϕ describe the friction. Usually, they are taken in the form [1]

$$K_r = K_r^0 (V\ V_N)^2, \qquad K_\phi = K_\phi^0 (V\ V_N)^2. \tag{2}$$

In eqs.(2), V_N is the nuclear part of the interaction. As is obviously seen, the friction vanishes at the minimum of the nuclear potential or because of other reasons (reaching the sticking limit) even sooner [2], so that the existence of a (local) minimum in the interaction potential is a necessary condition for the existence of a double nuclear system.

In reality, however, the situation is not so simple as just sketched mainly because the interation potential is a function of time. Whereas the initial stage of a reaction is governed by adiabatic potential (often taken in the proximity form [3]), some adiabatic (e.g., Woods-saxon) potential is supposed thereafter; the typical time which separates these two regions is of the order of 10^{-21} s [4]. What is referred to as the pre-equilibrium emission by an experimentalist, comes out solely from the former stage of the reaction [5].

2. NUCLEAR MOLECULAR DYNAMICS

In the present paper, we construct the nucleus-nucleus interacting potentials within the so-called nuclear molecular dynamics approach [6-8]. Here, the nucleons are treated as mass points whose behaviour is fully described by the Hamiltonian

$$H = \frac{1}{2m} \sum_i p_i^2 + \sum_{i<j} V_{ij}, \tag{3}$$

(m is the nucleon mass). Therefrom

$$\dot{\vec{r}}_i = \vec{\nabla}_{p_i} H , \quad \dot{\vec{p}}_i = - \vec{\nabla}_{r_i} H. \qquad (4)$$

A large variety of effective nucleon-nucleon potentials V_{ij} can be found in literature [6-14]. Also the well-known potentials like M3Y [15-18] or the Paris potential [19,18] can be tried to use. However, the nuclear dynamics is justified well only at intermediate and/or high energies whereas the trajectory-with-friction concept of eq.(1) is applied mainly at low energies (say, about 10 MeV/A). Rather recently, Aichelin and Stöcker suggested the use of the quantum nuclear molecular dynamics [20]. For the potential study, anyway, a simpler approach by Anagnostatos and co-workers [14] aimed at investigating the properties of the effective nucleon-nucleon potential at low energies rather carefully seems to give reasonable results within an affordable computer time. Similarly, the present work lies fully within the classical nuclear molecular dynamics.

3. EFFECTIVE NUCLEON-NUCLEON POTENTIALS

The potentials reported in literature [6-19] can be divided into two groups. One of them makes no difference between the neutrons and the protons whereas the latter differs. Of course, the Coulomb potentials have to be added also to the potentials of the first group.

The most frequently used form of the effective nucleon-nucleon potentilas is to be sum of two or three Yukawas with a slight modification of Kitazoe et al.[10] who prefer the attractive term to be of the Woods-Saxon like form. Moreover, ref.[8] introduces one more additional momentum-dependent term which simulates the Pauli principle and is of the form

$$V_{P_{ij}} = \frac{c_1}{r_{ij}^2} \exp\{c_2[(c_3 r_{ij} p_{ij})^4 - 1]\} \delta_{s_i s_j} \delta_{\tau_i \tau_j} . \qquad (5)$$

Therein, $s_i (s_j)$ and $\tau_i (\tau_j)$ are the spin and isospin quantum numbers of the interacting nucleons.

Charge-dependent potentials also frequently use the Yukawa-like form, although a much simpler (finite-range) potential was introduced by Kiselev and Pokrovskyi [11] . To our knowledge, 17 different potential sets usable for our purposes are reported in literature (see, e.g., [21]).

Independently of the potential used, several constraints are put on, as the nuclei are to have reasonable i/ radii, ii/ potential and mean kinetic energies, and iii/ nuclear compressibility. All these quantities were tested thoroughly by the corresponding authors, with the exception of M3Y and the Paris potentials, which have not yet been used within the nuclear molecular dynamics concept. Unfortunately, they both fail completely even with the value of the nuclear radius. A new, rather crude test was used in refs.[12,14,22] ; namely, the values of the longitudinal momentum loss cross section $\sigma^{(1)}(E)$ and of the transverse momentum transfer cross section $\sigma^{(2)}(E)$. Here,

$$\sigma^{(i)}(E) = 2\pi \int_{-1}^{1} \sigma(\theta, E)(1 - \cos^n \theta)\, d(\cos\theta), \qquad (6)$$

and $\sigma(\theta, E)$ is the double-differential nucleon-nucleon cross section given by the corresponding effective nucleon-nucleon potential. At higher energies (say, 50-500 MeV) several potentials do not contradict too much the data; nearly all of them fail with decreasing energy. The only satisfactory therein is that of ref.[14] , whose parameters seem to be very unusual and need much care in their computer treatment. Also the potentials of ref.[11] for two interacting protons give both $\sigma^{(1)}(E)$ and $\sigma^{(2)}(E)$ reasonable, unfortunately, it is not the case of the neutron-proton pair interaction. For comparison, see Fig.1.

4. NUCLEUS-NUCLEUS POTENTIALS

Finally, four potentials are used in our further calculations: the new potential of Anagnostatos et al.[14] together with its older version[12], and the two very simple potentials of ref.[11] . Their parameters are listed in the Table.

Obviously, one has to "prepare" the nuclei themselves as the first step in calculating the interaction of colliding nuclei. In distinction to the isomorphic shell model (see[22] and the former papers of the same authors quoted therein), we preferred the positions and momenta of the individual nucleons within a nucleus to be selected at random. At the very first step, the positions and momenta are generated by a Monte-Carlo procedure filling spheres of given volumes in both the spaces. The relative distances of nucleons must be greater than some minimal value given in advance. The spin and isospin quantum numbers were also assigned randomly to the nucleons. However, as a rule, such a really random sphere filled by nucleons appears to be very hot (i.e., unstable), its energy being by many GeV per nucleon more than needed. A rather sophisticated "cooling procedure" had to be applied therefore. In the case of nucleons without spin and isospin variables (or an interaction independent of them), an introduction of an effective friction force in order to help to create the ground state of a nucleus was suggested[7,8] . Through the same procedure may be applied also to the case of spin- (or isospin-) dependent interaction, it is rather lengthy and time-consuming. Instead, we preferred another way, to our knowledge not mentioned in literature so far: we performed permutations on quantum numbers of individual nucleons and

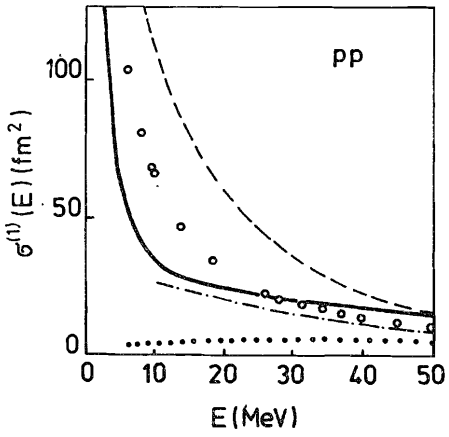

Fig.1a/. Longitudinal momentum loss cross section $\sigma^{(1)}(E)$ for the interacting pp *pair. Full line is for the charge-dependent potential of ref.* /14/, *dotted one for the charge-independent one* /12/, *dashed and dashed-dotted lines for the* P_1 *and* P_2 *potentials of ref.* /11/. *The data together with the calculations of potentials* /12/ *and* /14/ *are taken from* /14/ *and the references quoted therein.*

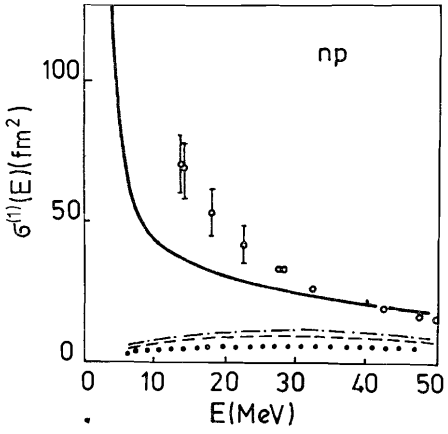

Fig.1b/. The same as in 1a/, but for the np *pair.*

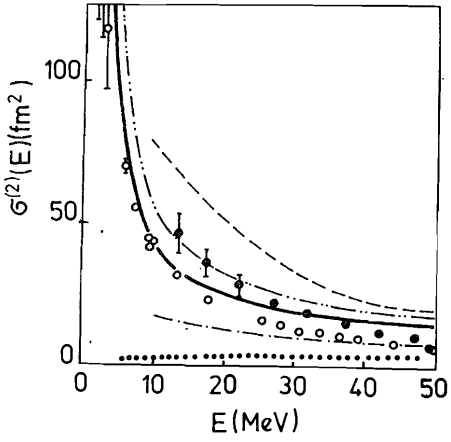

Fig.1c/. Transverse momentum transfer cross section $\sigma^{(2)}(E)$ for the interacting pp *pair (notation as in 1a), and for the* np *pair. There, data are drawn as full points, charge-dependent potential of ref.* /14/ *by a dashed-double dotted line. The calculations of* np *pair based on potentials of* /11/ *is indistinguishable from that of* /12/ *in the scale used.*

followed the total energy of the system. In this way we reduced the energy to several tens of MeV per nucleon, and only thereafter applied the cooling procedure of refs. /7,8/ . The ground state obtained appears to be rather stable for a long time interval with reasonable properties. As an illustration, the ground state for the ^{40}Ca nucleus with the charge-dependent poten-

Table

Parameters of the potentials used.

Potential of refs. /12/ and /14/ is given in the form

$$V(r) = V_R \frac{\exp(-\mu_R r)}{r} - V_A \frac{\exp(-\mu_A r)}{r} .$$

Below, the parameters are listed.

Potential	V_R (MeV fm)	μ_R (fm^{-1})	V_A (MeV fm)	μ_A (fm^{-1})
charge-indep.	7.36775×10^{16}	31.8538	187.0	1.3538
pp	2.4483×10^{22}	29.8017	258.85	1.5017
np	2.8999×10^{22}	28.1016	263.98	1.5016
nn	3.3943×10^{22}	26.6773	156.17	1.2273

Potentials of ref. /11/ are of the form

$$V(r) = a(r - r_o)^2 + b \qquad \text{for } r \le r_o$$

$$\min\{a'(r - r_o)^2 + b,\ 0\} \qquad \text{for } r > r_o .$$

Below, the parameters are given in notation $V_{2S_3+1,\,2T_3+1}$.

Potential	a (MeV fm^{-2})	a (MeV fm^{-2})	r_o (fm)	b (MeV)
P_1 $V_{11} = V_{13}$	168.75	12.5.	1.6	-8.
V_{31}	203.125	46.875	1.6	-30.
V_{33}	97.65625	0	2.4	0.
P_2 $V_{11} = V_{13}$	675.	5.555...	1.2	-8.
V_{31}	812.5	20.833...	1.2	-30.
V_{33}	150.	0.	1.8	0.

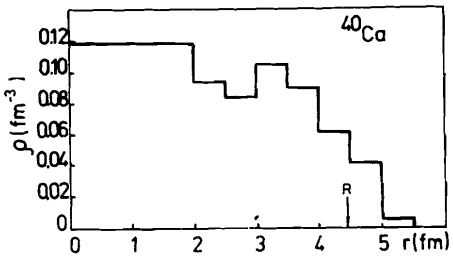

Fig.2. An example of the density distribution generated within the charge-dependent version of potential /14/ for the nucleus ^{40}Ca. The fluctuations seen below 3.5 fm are of purely statistical nature, and also because of low number of nucleons in the central region of nucleus only one value has been assigned to the density below 2 fm.

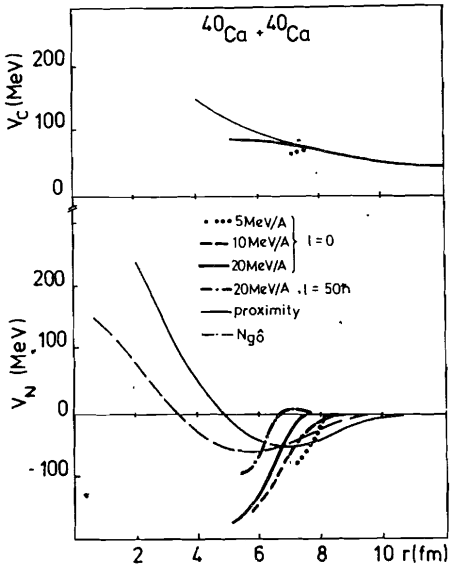

Fig.3. The nuclear part of the interacting potential (lower part). The heavy lines are the potentials calculated using the P_1 potential of ref./11/ at four different initial energies and orbital momenta. For comparison, also the sudden potentials of proximity /3/ and Ngô and Ngô /23/ are drawn as thin lines. The upper part of the picture brings the Coulomb part of the potential for 5 and 20 MeV/A (thick lines), together with a point-charge Coulomb, drawn for a comparison (thin line).

tial of ref./11/ had the potential energy - 12.3 MeV per nucleon, the kinetic energy of 4.3 MeV per nucleon, and the root mean square radius of 3.68 fm. Its density distribution is shown in Fig.2. To save the computer time, a random rotation was applied to the nucleus as a whole yielding relative initial configurations.

The calculations of the nucleus-nucleus potential were straight-forward thereafter: given two nuclei at a sufficiently large distance (so that the nuclear interaction could be neglected), an initial relative velocity was assigned to them, and their time development was followed. Fig.3 brings the dependence of the resulting potential for ^{40}Ca+^{40}Ca at the relative energies of 5, 10, and 20 MeV/A for head-on collisions, and of 20 MeV/A also for $\ell = 50\hbar$. We see that the effective nuclear potential radius varies with the incident energy, the less is the impact

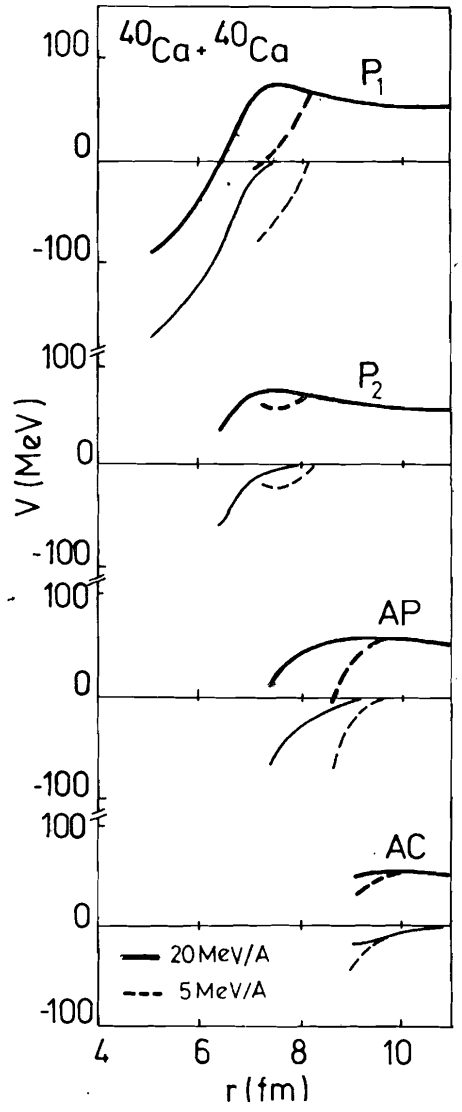

Fig.4. Nuclear parts (thin lines) and the total interaction potential, i.e. nuclear plus Coulomb (heavy lines) for two incident energies (central collisions) and four effective nucleon-nucleon potentials, P_1 and P_2 of /11/, charge-independent (AP) and charge-dependent (AC) potentials of Anagnostatos et al./12,14/.

energy, the more time the interacting nuclei have "to feel" each other and the larger the interaction radius is. For comparison, also two adiabatic potentials are drawn, namely the well-known proximity potential /3/ and the sudden potential of Ngô and Ngô /23/ obtained within the energy density formalism. Of course, both of them are energy independent. The last picture, Fig.4, brings nucleus-nucleus potentials for all four sets of the effective nucleon-nucleon potential, given for the incident energy of 5 and 20 MeV/A ($\ell = 0$). We see that the above conclusion on the change of the potential radius with the incident energy holds for all of them. (Note that the potentials of ref./11/ assume nuclear radii by about 20% smaller than the other two potentials).

5. SUMMARY

We built the nucleus-nucleus interaction potentials for different incident energies within the approach of the classical nuclear molecular dynamics. Comparison with sudden potentials shows a similar behaviour, but with one feature more: the range of a potential obtained within our approach is a decreasing function of the incident energy, independently of the exact form of the effective nucleon-nucleon potential used.

Acknowledgement

The author is grateful to V.D.Toneev for discussions, to C.Ngô for availability of his computer code, and to T.Kosmas for bringing the papers of Anagnostatos et al. to his attention.

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Received by Publishing Department
on October 21, 1986.

Бетак Э. E7-86-701

Ядро-ядерные потенциалы, полученные методом ядерно-молекулярной динамики

На основе нескольких эффективных нуклон-нуклонных потенциалов в рамках ядерно-молекулярной динамики построены динамические ядро-ядерные потенциалы. В качестве примера рассматривается система $^{40}Ca+^{40}Ca$. Показано, что полученные ядро-ядерные потенциалы при заданном нуклон-нуклонном потенциале зависят от энергии столкновения и радиус взаимодействия понижается с ее увеличением.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1986

Běták E. E7-86-701

Nuclear Molecular Dynamics Approach to the Nucleus-Nucleus Potentials

Nuclear molecular dynamics approach with several sets of the effective nucleon-nucleon potentials is used in order to construct dynamical nucleus-nucleus potentials. The method is illustrated on the case of the $^{40}Ca+^{40}Ca$ system. The resulting nucleus-nucleus potential varies with the incident energy of interacting nuclei for a fixed nucleon-nucleon potential; the range of the former one is a decreasing function of energy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986