ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E7-81-653

R.Dymarz, K.V.Shitikova

IS THERE A GIANT MONOPOLE RESONANCE IN ^{12}C?

Submitted to "Zeitschrift für Physik"

1981

1. INTRODUCTION

During the recent years the existence of the giant monopole resonances (GMR) in heavy nuclei has experimentally been established in various reactions[1-9]. For not very heavy nuclei ($A \leq 65$) the results of various experiments have been rather contradictory. In some experiments with these nuclei 0^+ states were found at the energies that followed the $80/A^{-1/3}$ rule established for the GMR energies in heavy nuclei, and which exhaust a great amount of energy weighted sum rules (EWSR)[1,2,5]. In other experiments the monopole states were not found at the expected energies[6], or the amount of the EWSR exhausted by such states was only few percent[10,11].

In this paper we deal with the ^{12}C nucleus for which a 0^+ state at 20.3 MeV was recently found in inelastic scattering of ^3He ions[11] the width of this state being 1.1 MeV. A few years ago in the framework of the hyperspherical functions method the existence of 0^+ states in very light nuclei ($A \leq 16$) has been predicted[12]. These states would exhaust a great amount of the EWSR and they would have very small widths. In this paper we intend to show that using the transition densities as calculated in the hyperspherical functions method we are able to explain the measured 0^+ state in ^{12}C as being a collective one, in contradiction to the original interpretation[11] that this state exhausts only 2.6% of the monopole EWSR.

In Sect.2 we briefly present the hyperspherical functions method and compile the results of previous calculations of properties of the 0^+ state in ^{12}C. In Sect.3 we describe the way of constructing the transition potential and that of analysis of inelastic scattering. In Sect.4 we discuss the approximations and uncertainties of the analysis.

2. PREDICTION OF THE PROPERTIES OF MONOPOLE GIANT RESONANCE IN LIGHT NUCLEI IN THE HYPERSPHERICAL FUNCTIONS METHOD

The monopole or "breathing mode" oscillations in nuclei correspond to variations in the nuclear matter density i.e., they characterize nuclear matter compressibility. The hyperspherical functions method provides a convenient basis for a microscopic description of such vibrations[12]. The point of this method is that a collective variable is being introduced which can be directly associated with the mean-square nuclear radius: $\rho^2 = A\langle r^2\rangle$, i.e., with the mean nuclear density. The excitations in this variable correspond to the monopole vibrations of the nucleus as a whole; the density being then a dynamical variable.

In the method of hyperspherical functions the wave function of the nucleus is thought to be an expansion over standard hyperspherical harmonic polynomials

$$\Psi(1,2,\dots,A) = \rho^{-(3A-4)/2}\sum_{K\gamma} \chi_{K\gamma}(\rho)\, Y_{K\gamma}(\theta_i), \tag{1}$$

where θ_i are the hyperspherical angles, and

$\int \chi^2_{K\gamma}(\rho)d\rho = 1.$

The Hamiltonian has the form

$$H = -\frac{h^2}{2m}\frac{1}{\rho^{3A-4}}\frac{\partial}{\partial\rho}\left(\rho^{3A-4}\frac{\partial}{\partial\rho}\right) - \frac{h^2}{2m}\frac{\Delta_\theta}{\rho^2} + V(\rho). \tag{2}$$

The hyperspherical harmonics are eigenfunctions of the angular part of the Laplacian

$$\Delta_{\Omega_n} Y_{K\gamma}(\theta_i) = -K(K+n-2)Y_{K\gamma}(\theta_i). \tag{3}$$

The K is an analogue of the angular momentum at $n=3$ and it is called the global angular momentum. The subscript γ denotes all the quantum numbers necessary to enumerate various degenerated states of Eq.(3).

The system of equations for radial eigenfunctions $\chi(\rho)$ and eigenvalues E can be written as follows:

$$\left\{\frac{d^2}{d\rho^2} - \frac{L_K(L_K+1)}{\rho^2} - \frac{2m}{h^2}\left(E + W^{K\gamma}_{K\gamma}(\rho)\right)\right\}\chi_{K\gamma}(\rho) = \tag{4}$$

$$= \frac{2m}{h^2}\sum_{K'\gamma' \neq K\gamma} W^{K'\gamma'}_{K\gamma}(\rho)\chi_{K'\gamma'}(\rho),$$

where m is nucleon mass $L_K = K + ½(3A-6)$ and $W^{K'\gamma'}_{K\gamma}(\rho)$ are the matrix elements of the potential energy of the nucleon-nucleon effective interaction

$V = \sum_{i<j}^{A} V(r_{ij});\quad V(r_{ij}) = f(r_{ij})W_{\sigma\tau}$,

σ and τ denote spin and isospin, respectively.

Having found the effective interaction matrix element $W^{K'\gamma'}_{K\gamma}(\rho)$, we insert it into Eq.(4) to find its eigenvalues E and eigenfunctions $\chi_{K\gamma}(\rho)$. Subsequently Eq.(4) is firstly being solved

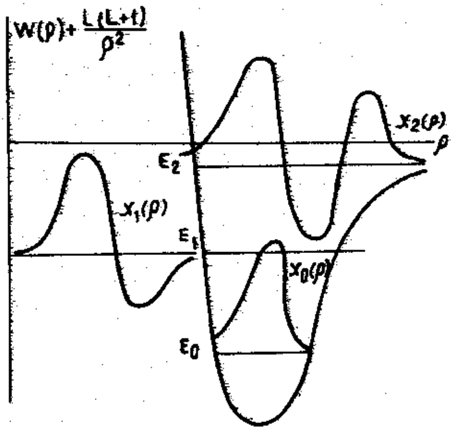

Fig.1. The radial wave functions for the ground and the two monopole excited states in an effective potential.

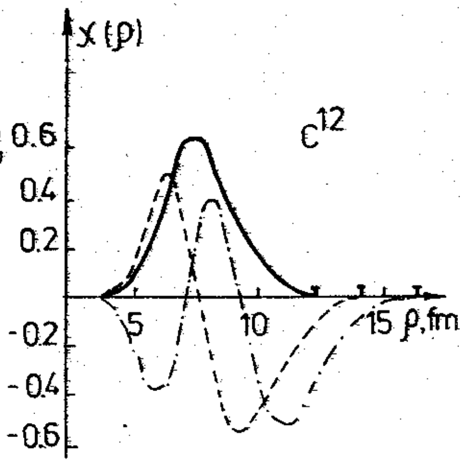

Fig.2. Three wave functions: for ground, first and second excited monopole states in ^{12}C nucleus.

for the ground state and then for the first, second, etc., monopole excited states of the nucleus as a whole. Thus the solutions for the ground and the monopole excited states are found in a consistent way. The radial wave functions for the ground and the two monopole states in an effective $W(\rho) + L(L+1)/\rho^2$ potential are schematically shown in Fig.1. It can be seen that the wave functions of the excited states have some nodes while those of the ground state have none. What is more interesting, the wave functions for the excited states are pushed out towards larger radii. This is illustrated more precisely in Fig.2, where three wave functions: for ground, first and second excited monopole states in ^{12}C nucleus are shown. It is then evident that the increase in the nuclear size in the excited states is automatically accounted for in the hyperspherical functions method.

Now, we briefly review the properties of the GMR in ^{12}C nucleus obtained in the framework of the hyperspherical functions method.

i) The Excitation Energy

The excitation energy of the GMR was found directly as a difference between first and second eigenvalues of Eq.(4). In Ref.[13] the energies of the GMR in ^{12}C were calculated using various nucleon-nucleon effective interactions. It appeared there that the E_n value calculated[13] with the Brink-Bocker B1 potential[14] coincides with the excitation energy of the 0^+ state measured recently in inelastic ^3He scattering[11]:

ii) <u>The Monopole Energy Weighted Sum Rules (EWSR)</u>

In Ref.[15] the EWSR were estimated according to the formula

$$\Sigma(E_n - E_0)|M_{n0}|^2 = \frac{h^2 A}{2m} \langle 0 | r^2 | 0 \rangle , \tag{5}$$

where

$$M_{n0} = \langle n | \tfrac{1}{2} \sum_{i=1}^{A} r_i^2 | 0 \rangle$$

$|0\rangle$ and $|n\rangle$ are the wave functions of the ground and excited states, respectively. n enumerates all the 0^+ states, m is the nucleon mass and A is the mass number of the nucleus.

The calculations were carried out for nuclei with A = 4,6, 12,16 and it was shown for all these nuclei the first 0^+ state exhausts 80-90% of the monopole EWSR. One may believe that this result is not very sensitive to the kind of the nucleon-nucleon effective potential used.

iii) <u>The Width of the GMR</u>

It is easy to show in the hyperspherical functions method that among the giant resonances of different multipolarity the width of the GMR should be the smallest. It is determined by a particular nature of its radial wave function. In Fig.3 we show three wave functions in their potential wells: the wave

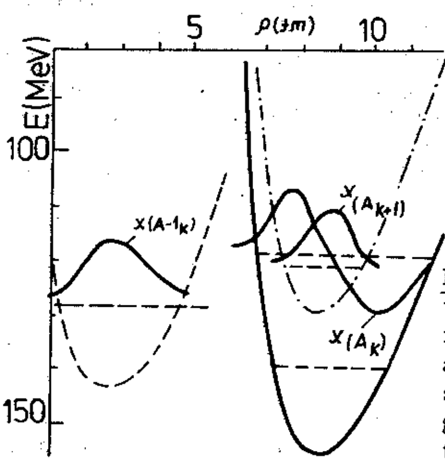

Fig.3. Three wave functions in their potential wells: the wave function of the monopole $\chi(^N A_K)$ and dipole $\chi(^N A_{K+1})$ excited states in A nucleus and the ground state wave function of the A-1 nucleus.

function of the monopole $\chi(^N A_K)$ and dipole $\chi(^N A_{K+1})$ excited states in A nucleus and the ground state wave function of the A-1 nucleus. The widths for decay and spread of the GMR are determined by overlap integrals and that is why all the integrals involving monopole excited state wave function (that have a node) will be small. In Ref. [16] the widths of the GMR in ^{16}O has been estimated to be 1 MeV what is similar to the width of the 0^+ state recently measured in ^{12}C.

3. THE POTENTIALS AND THE METHOD OF ANALYSIS

A. Folding Model

The radial wave functions $\chi_{Ky}(\rho)$ have been used to calculate the matrix elements of the density operator $n_{ij}(r)$ [17]. Folding these densities with the effective nucleon-nucleon interaction we obtain the potentials

$$V^{A_1A_2}_{ij,\ell_k}(r) = \int n_{ij}(r_1)\, n_{\ell k}(r_2)\, \mathcal{V}(r - r_1 - r_2)\, dr_1\, dr_2 . \qquad (6)$$

For the effective nucleon-nucleon potential we have taken spin-, and isospin-independent zero-range two-, and three-body terms of the Skyrme form with parameters as given in Ref. [18]. The potentials $V_{11,11}$, $V_{11,22}$ and $V_{11,12}$ for the ^3He-^{12}C system are shown in Fig.4 (the convention is the following: the first pair of indices corresponds to projectile and the second pair of indices to target). The potentials $V_{11,11}$ and $V_{11,22}$ represent the interaction in elastic and inelastic channels, respectively with ^3He in ground state and ^{12}C in ground state and in monopole excited state in ^{12}C nucleus.

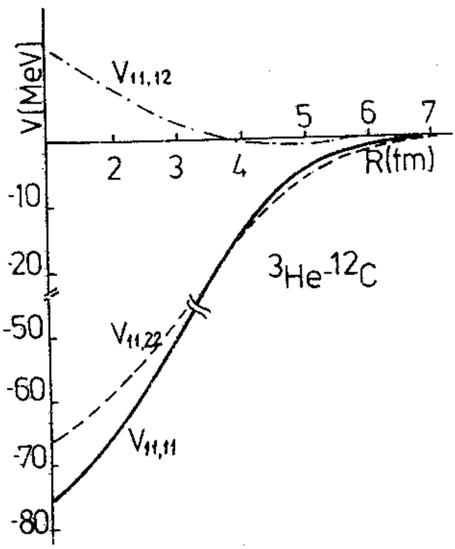

Fig.4. The potentials $V_{11,11}$, $V_{11,22}$ and $V_{11,12}$ or the $^3He^{12}$ system. (The first pair of indices corresponds to projectile and the second pair of indices to target).

B. Elastic Scattering

The potentials calculated according to Eq. (6) are real. The diagonal components (in both the pairs of indices) are used as real parts of the optical model potentials. It has been a common practice to define the imaginary part of optical potential in a Woods-Saxon form and to search parameters by fitting theoretical elastic differential cross section to experimental data. However, we have followed another way supposing that the imaginary potential should have a similar form to the real one

$$\mathcal{V} = V_{ii,\ell\ell}\,(1 + iX_{ii,\ell\ell}). \qquad (7)$$

The scaling factor $X_{ii,\ell\ell}$ in elastic channel was searched upon by fitting the theoretical differential cross sections to experimental data. Unfortunately, we have had at our disposal no experimental data of the ^3He-^{12}C scattering in elastic channel at $T_L(^3He) = 108.5$ MeV where inelastic cross section to 0^+ ($E_x = 20.3$ MeV) state has been measured. We used then for $X_{11,11}$ the same value (0.63) which we has found in our analysis of elastic scattering of ^4He ions on ^{12}C at $T_L(^3He) = 139.0$ MeV, i.e., at the same energy per nucleon as in the former case.

In addition to the ^3He-^{12}C and ^4He-^{12}C systems we have analysed the scattering of ^6Li ions ^{12}C at $T_L(^6Li) = 90.0$ MeV. For this case we have obtained $X_{11,11} = 1.0$. The calculated elastic scattering differential cross sections together with experimental data (whenever available) are shown in Fig.5. In spite of the simplicity of the potential used here, with only one free parameter, the agreement with experiment appears to be good.

C. Inelastic Scattering

Experimental data for inelastic scattering to 0^+ ($E_x = 20.3$ MeV) state in ^{12}C [11] were analysed using the coupled channel code CHUCK 2 [19] with two channels (elastic and 0^+ inelastic) explicitly included. In elastic channel we have used the optical model potential described in the previous section. In inelastic channel we have used the potential in the form (7) but with the scaling factor $X_{11,22} \simeq 0.8\,X_{11,11}$. This somewhat weaker absorption in inelastic channel is physically well grounded.

For the interchannel coupling potential $\Delta\mathcal{V}$ it is customary to use the collective model [20]. A deformation parameter $\beta_L(n)$ for the transition to the n-th state is defined from the effective nuclear matrix element [21,22]. The scattering interaction

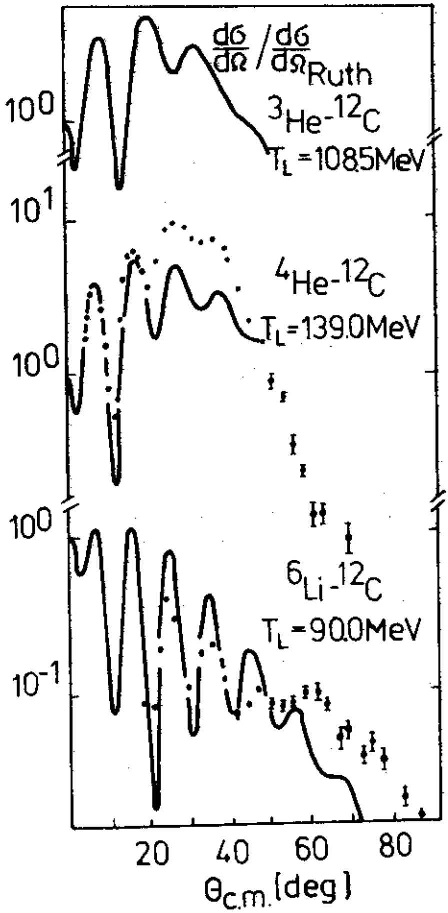

Fig.5. The calculated elastic scattering differential cross sections together with experimental data (points) for $^3He-^{12}C$ at $T_L(^3He)=108.5$ MeV, $^4He-^{12}C$ at $T_L(^4He)=139.0$ MeV /21/, $^6Li-^{12}C$ at $T_L(^6Li) = 90.0$ MeV /24/ systems.

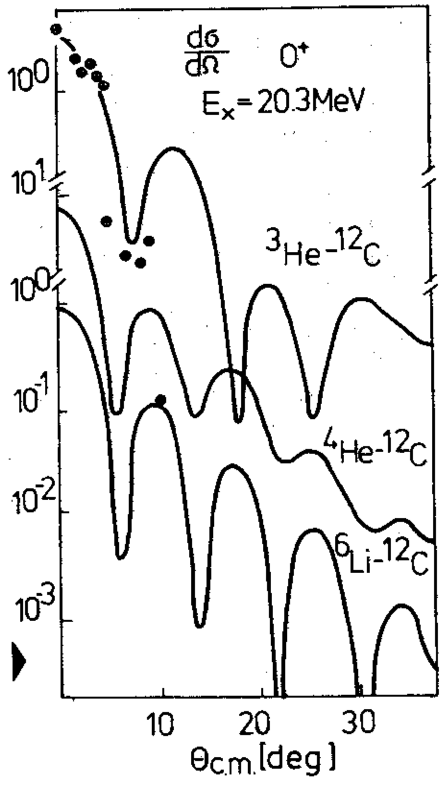

Fig.6. The calculated differential cross sections to 0^+ (E_x = = 20.3 MeV) state in inelastic scattering of ^3He , ^4He , ^6Li on ^{12}C. Points - experimental data /24/.

for L=0 is

$$\langle n|\Delta\mathcal{C}|0\rangle = \frac{\beta_0(n)}{(4\pi)^{1/2}}\,\mathcal{C}_1 .$$

The form factor $\mathcal{C}_1$ is taken to be proportional to transition potential $V_{11,12}$

$$\mathcal{C}_1(r) = V_{11,12}(r)(1 + iX_{11,12}) .$$

The value of $X_{11,12}$ was arbitrarily put equal to 1.

The monopole sum rule (5) now becomes

$$\sum_n (E_n - E_x)\,\beta_0^2(n) = 4\pi \frac{h^2}{2m} \cdot \frac{1}{A\langle r^2\rangle}, \tag{8}$$

$\langle r^2\rangle$ is mean square radius of the nucleus. If the EWSR (8) is exhausted by excitation of a single state, then

$$\beta_0^2 = 4\pi \frac{h^2}{2m} \cdot \frac{1}{E_x A\langle r^2\rangle},$$

where E_x is the excitation energy of the considered state.

Comparing β_0^2 and β_{ex}^2 as extracted from scaling the calculated inelastic differential cross section in order to get the experimental one ($\beta_{ex}^2 = d\sigma^{ex}(\theta)/d\sigma^{th}(\theta)$), we can determine the percentage depletion of the EWSR. In our analysis we have obtained $\beta_{ex}^2/\beta_0^2 = 2.7$. Certainly, this value should not be greater than 1. The latter value would mean that 100% of the EWSR is depleted in the considered state. However, in view of many approximations and uncertainties in our analysis the quoted result seemd to be reasonable. These approximations and uncertainties we discuss in the next section.

In Fig.6 the calculated differential cross section to 0^+ ($E_x = 20.3$ MeV) state in the inelastic scattering of ^{3}He on ^{12}C is compared with experimental data using $\beta_{ex}^2 \simeq 0.5$, the value obtained from analysis. For completness the theoretical cross sections (with the same β_{ex}^2) for two other reactions: ^{4}He – ^{12}C and ^{6}Li – ^{12}C are also shown.

4. DISCUSSION

It has been a common practice to use the deformed potential model to analyse inelastic scattering to low lying excited states and to giant resonances in heavy nuclei. This is a standard model in which the surface of the optical potential is being deformed. A different approach is to use the transition potential calculated from a folding model. The transition density is being inserted into the folding integral (6) in order to get the transition potential. The transition density may be obtained from microscopic nuclear structure calculations, for example, those using the random-phase approximation. In practice, however, in analyses of inelastic scattering to the giant resonances the Tassie or hydrodynamical[24] model density are being used[2]. In this approximation the depletion of the EWSR for giant quadrupole resonance is very similar to that obtained in the deformed potential model. However, these two models give very different results for the GMR: for medium-weight nuclei the folding model yields for the percentage depletion a value which is a few times greater than that given by the deformed potential model. Moreover, this value is often greater than 100%; for not very heavy nuclei a value of about 180% was obtained /2/. In view of these considerations our result for the depletion of the EWSR for ^{12}C should not be surprising: since we have used a fully microscopic approach to the transition potential for very high nucleus, then we should have expected an overestimation of the depletion factor. This overestimation is mainly connected with the weakness of the used transition potential. This weakness has at least two sources. Firstly, we have been using in the folding integral the zero-range effective nucleon-nucleon potential; a finite range elementary potential would considerably strengthen the transition potential. Secondly, for the imaginary part of the transition potential we have chosen the same shape and strength as for the real one.

In deformed potential model the imaginary part of the transition potential is pushed out towards greater radii and from this range of radii there comes a great amount of transmission amplitude.

In conclusion more experimental data and more careful analyses are needed before definite conclusions will be drawn concerning the existence of monopole resonances in light nuclei.

ACKNOWLEDGEMENTS

The authors are indebted to R.Nazmitdinov for help in calculations of the potential and A.Kobosa for reading the manuscript.

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Received by Publishing Department
on October 20 1981.