объединенный
институт
ядерных
исследований
дубна

E7-80-719

Yu.A.Lazarev, Yu.Ts.Oganessian,
V.I.Kuznetsov

FISSION PROBABILITIES AND BARRIERS OF NUCLEI WITH $Z \gtrsim 80$ AND $N \lesssim 126$ FAR OFF BETA-STABILITY. BETA-DELAYED FISSION AS A TOOL FOR THEIR EXPERIMENTAL DETERMINATION.

Paper presented at the International Symposium on the Synthesis and Properties of New Elements (Dubna, 23-27 September 1980).

1980

1. INTRODUCTION

During the last 10-15 years considerable progress has been made in the experimental and theoretical investigation of fission barriers - a fundamental feature of the process which determines the stability of the heaviest atomic nuclei and the boundaries of the Mendeleev Periodic Table. The exploration of this feature is very important in solving the problem of the existence of the predicted superheavy elements (SHE) of $Z \gtrsim 110$ and $N = 184$ in the vicinity of the classical (liquid-drop) limit of nuclear stability against fission. Fig.1 shows a part of the chart of isotopes with $Z \geq 80$ including the nuclides for which experimental data on fission barriers are available (see, e.g., refs.[1-10] and references therein). From this figure it follows that most of experimental data cover the region of nuclei with $90 \lesssim Z \lesssim 99$ and $140 \lesssim N \lesssim 155$

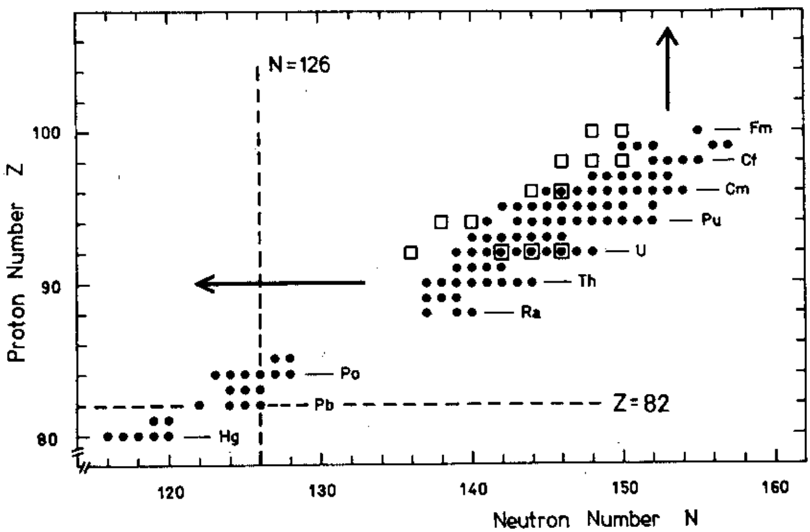

Fig.1. Part of the chart of nuclides with $Z \geq 80$. The nuclei for which there exist experimental data on fission barriers are indicated by squares and points (□ - data obtained from βDF studies[11-17], ● - data obtained by other methods [1-10]).

adjacent to the valley of β-stability. The most detailed and systematic information on fission barriers has been obtained in direct reactions of the type (^{3}He,df), (^{3}He,tf), etc., in the reactions (n,f) and (γ,f), as well as in the studies of spontaneously fissioning isomers. Britt was right to note in ref. [1] that the extensive possibilities of these methods are presently exhausted; they made it possible to obtain information for practically all the nuclei which can really be investigated by these methods.

For a number of neutron-deficient nuclei adjacent to the investigated region, the fission barriers have been recently determined by using β-delayed fission [11-17]. The probability of this process discovered at Dubna by Flerov and his colleagues (refs. [11,12]) is a sensitive function of the fission barrier parameters.

Beta-delayed fission (βDF) substantially complements the variety of traditional methods used in experiments to study the structure of the potential energy surfaces associated with fission. This is a new and promising method, some important aspects of which, however, still need considerable elaboration. Therefore, it is quite natural that the potential possibilities of βDF are much wider than the concrete results so far obtained by this method.

In the present paper, on the basis of βDF we propose and substantiate a method for the experimental determination of the fission probabilities and barrier heights, and the properties of fission fragments of the nuclei having the Z or N values close to the magic ones, and, at the same time, lying considerably, by 15-20 and more neutrons, far from the line of β-stability. We consider the region of nuclei with $Z \geq 80$ and $N \leq 126$*, for which there are no experimental data on fission barriers at all and theoretical predictions are rather uncertain. Moreover, in this region of nuclei, not any information is available about nuclear fission, despite the fact that the ground-state radioactive properties of the majority of nuclides in the region being considered are determined experimentally.

The present paper consists of the following sections:

- The isospin dependence of the fission barrier heights and ground-state nuclear masses.
- Superheavy elements and the fission probabilities of heated nuclei with $N \leq 126$.

---

*Here and below, unless specified otherwise, we mean nuclei very far off the line of β-stability.

- Fission barriers of nuclei lying around ^{208}Pb and the fission barriers of "common" thorium nuclei.
- Beta-delayed fission as a tool for the experimental determination of fission probabilities and barrier heights, and of the properties of fission fragments for nuclei with $Z \gtrsim 80$ and $N \lesssim 126$ far off β-stability.
- General conclusions.

2. THE ISOSPIN DEPENDENCE OF THE FISSION BARRIER HEIGHTS AND GROUND-STATE NUCLEAR MASSES

The modern interpretation of experimental data on the fission barriers of heavy nuclei and on their variations with Z and N is based on the conception that shell effects very strongly influence the deformation energy of the nucleus; according to this conception, nuclear shells only alter rather than disappear at deformation /18,19/. Most of the realistic calculations of the total potential energy of the nucleus, E(q, Z,N) as a function of its shape q and particle number Z,N are performed within the framework of a combined macroscopic-microscopic method developed owing to the investigations of Myers and Swiatecki /20,21/ and especially Strutinsky /18,19,22/. The main idea of this method is that the major part of the total nuclear energy can be calculated macroscopically, for instance, by using the liquid-drop model (LDM) or its generalized versions, whereas the contribution from the effects of the inner structure can be taken into account by the additive introduction of the microscopic corrections - shell and pairing corrections.

In accordance with this basic idea,

$$E(q, Z,N) = \tilde{E}(q, Z, N) + \delta E(q, Z, N),$$

where q is a set of deformation parameters that determine nuclear shape, $\tilde{E}(q,Z,N)$ is the macroscopic part of the total energy responsible for the smooth variations of E, and $\delta E(q,Z,N)$ is the microscopic correction. The latter reproduces the local fluctuations of E and can be calculated using the Strutinsky method /18,19,22/ by means of a given single-particle potential generalized for the case of the deformed shapes of the nucleus. The study of the extremes of the potential energy surface E(q, Z,N) just leads to determination of fission barriers.

With the main idea unchanged, there exist a large number of the versions of the macroscopic-microscopic method which differ in the following respects: (i) the choice of the way of parametrization of the nuclear surface shape, (ii) the choice of a concrete model for calculation of the smooth part of energy $\tilde{E}$ (the LDM [23-29,20,21,22], the droplet model (DM)[30-35] taking into account the effects due to the finite size of nuclei, such as the dependence of the surface tension of the nuclear droplet on the radius of its surface curvature, the compressibility of nuclear matter, etc., the single-Yukawa model of Krappe and Nix [36] taking into account the finite range of nuclear forces and the diffuseness of the nuclear surface, and its modified version[37], and others), and (iii) the choice of the type of the single-particle potential used to calculate the microscopic correction δE (a modified harmonic-oscillator potential, the generalized Woods-Saxon potential, the folded Yukawa potential, etc.); for details, see, e.g., reviews [22,38,39]. In addition to the more or less important features that characterize different versions of the method, differences can be associated with the choice of numerical values of the parameters incorporated in the calculation of $\tilde{E}$ or δE. For instance, the parameter k_s that determines the changes in the surface energy of the spherical nucleus, $E_s^{(0)} = a_s(1-k_s I^2)A^{2/3}$, as a function of neutron excess $I=(N-Z)/A$, assumes the value 1.78 in the LDM, as given by Myers and Swiatecki[20,21] and the value 2.84 in the version of Pauli and Ledergerber[40]; Krappe and Nix[36] find it to be equal to 4.0, while it is equal to 3.0 in the modified model of Krappe et al.* [37]. At the same time, the constant k_s explicitly enters into the expression for the fissility parameter

$$x = E_c^{(0)}/(2E_s^{(0)}) = (Z^2/A)/[(2a_s/a_c)(1-k_s I^2)],$$

where $a_c = (3e^2)/(5r_0)$ and $E_c^{(0)} = a_c(Z^2/A^{1/3})$, r_0 is the nuclear radius constant and, consequently, it enters into the final expression for the macroscopic fission barrier height $\tilde{B}_f$.

Despite this, different versions of the macroscopic-microscopic method lead to similar results in the region of nuclei on which experimental data are available, i.e., in the valley of β-stability, and it is essentially difficult to give a preference to one of them. In general, a satisfactory description has been obtained, which reproduces the fission barrier heights within the accuracy of about 1-2 MeV[1] and, possibly,

---

*When the present manuscript has been prepared for publication, there appeared a paper by Møller and Nix[109], which contains more precise values of model parameters[37], in particular $k_s = 2.3$.

some specific features of the potential energy surface, which manifest themselves in experiments. On the other hand, the uncertainty involved in the present set of "experimental" values of the fission barrier heights obtained in different types of experiment is significantly smaller than this value. Following Britt [1], it is, on the average, about 0.3 MeV and is due to the inaccuracy of measurement and to systematic errors which occur during data analysis inevitably using certain model representations. However, it is far from being clear whether the discrepancy between experiment and theory is accounted for by the calculational procedure used for the microscopic correction, or vice versa, by the macroscopic component, or by both, or, finally, by the rather phenomenological character of the approach as a whole. If we go farther from the line of β-stability, the situation appears to be much less satisfactory, since here the predictions of different versions of the macroscopic-microscopic method differ both quantitatively and qualitatively. First of all, this concerns the macroscopic part of the deformation energy which is as important as the shell one is.

For instance, the LDM version of Myers and Swiatecki [21], the droplet model [34,35] and the Krappe-Nix model [36] yield quite different, and even opposite trends of the change in the macroscopic fission barrier heights as a function of $I=(N-Z)/A$ at $Z=$const, thus leading to significant quantitative differences at large distances from the line of β-stability. In Fig.2 this is illustrated for thorium isotopes. One can see that in the range N=114-144 the uncertainty is characterized by a factor of >2.

For nuclei lying in the vicinity of lead - from ^{173}Lu to ^{213}At - a similar consideration has recently been carried out by Schrøder [41], who has arrived at the conclusion that the LDM with $k_s=1.78$ [21] best reproduces the experimental data on the isospin dependence of the macroscopic fission barrier heights, in particular, for Pb isotopes with A=204-208. Substantially different conclusions have been drawn concerning the isospin dependence of $\tilde{B}_f$ for the same region of nuclei exactly, e.g., in refs. [6,42]. We would like to add that the macroscopic-microscopic calculations of Möller [43] performed for the actinide region using four different models [21,34,36,37] for the macroscopic part of the deformation energy and one and the same (modified-oscillator) single-particle potential to calculate the shell correction, have led the author to the conclusion that the best agreement with experimental data on fission barriers B_f, in particular, for uranium isotopes with N=142-148, is achieved by using the DM [34] (in the DM the effec-

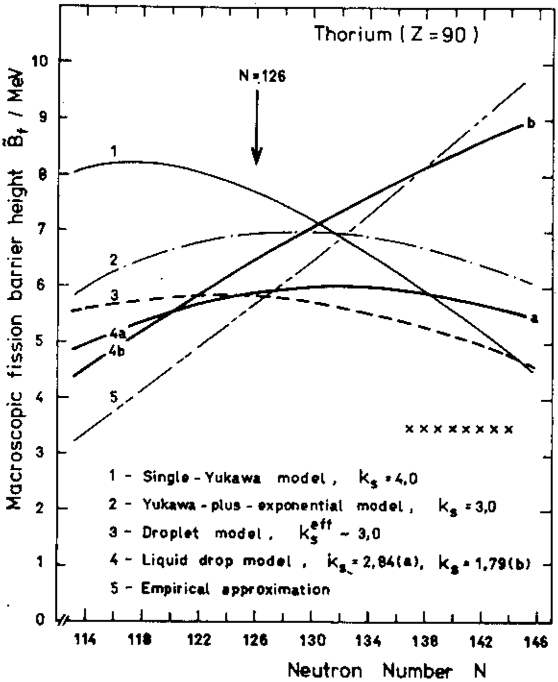

Fig.2. Isospin dependence of the macroscopic fission barrier heights $\tilde{B}_f$ for Th isotopes, calculated in terms of the LDM and its modifications (1 - the model of Krappe and Nix [36], 2 - a modified version [37] of the Krappe and Nix model; 3 - the DM of Myers [35]; 4a and 4b - the LDM using the parameters of Pauli and Ledergerber [40] and of Myers and Swiatecki [20], respectively; 5 - an empirical approximation [82], $\tilde{B}_f = 12.5 - 2.7(33.5 - Z^2/A)^{2/3}$ MeV, obtained by fitting an analytical expression to the results of extraction of shell corrections to ground-state nuclear masses from the experimental values of fission barrier heights). The Th isotopes for which there are experimental data on fission barriers are indicated by (x).

tive k_s for U isotopes is close to 3.0 [44]). In summary, it should be noted that these conclusions as a whole (made on the basis of comparing calculations with experimental data which are available only in a narrow range of N at each value of Z=const, $\Delta N \lesssim 10$) are essentially contradictory and do not allow one to make any choice.

In this connection experimental studies of the fission barriers of heavy nuclei with $Z \gtrsim 80$ and $N \lesssim 126$ are of great interest, in the first place, for the clarification of the isospin dependence of the macroscopic fission barrier heights $\tilde{B}_f$ and of the surface energy of spherical nuclei, $E_s^{(0)}$, in general. The obtaining of experimental data on nuclei with $Z \gtrsim 80$ and $N \lesssim 126$ would make it possible to widen the N range sharply up to $\Delta N \approx$ 25-30, for which the isospin dependence of $\tilde{B}_f$ and $E_s^{(0)}$ can be checked, and to increase the certainty of conclusions about its character. It should be emphasized that such investigations are important for deriving more precise nuclear mass formulae since they incorporate parameters such as k_s and $(2a_s)/a_c$ which are determined very poorly from the experimental ground-state masses of nuclei alone, see, e.g.,[20,38]. It is natural that the information on the fission barriers of nuclei far from the line of β-stability, on the character of their isospin dependence, and the more precise parameters of the mass formulae are of great importance for the calculation of formation of nuclides in astrophysical events and in similar artificial processes.

3. SUPERHEAVY ELEMENTS AND THE FISSION PROBABILITIES OF HEATED NUCLEI WITH $N \leq 126$

Another important problem is to extrapolate the available knowledge about the ground-state masses and fission barriers of nuclei to the region of the largest Z values; the existence of SHE's is essentially determined by the fact whether superheavy nuclei have a fission barrier preventing their spontaneous deformation and prompt decay to two or more fragments. Concrete calculations of fission barriers for SHE's are carried out using the same macroscopic-microscopic method and the main field where it is tested is the comparatively narrow in Z and N region of actinides.

In the region of SHE's the shell correction of the deformation energy plays a crucial role although the macroscopic part remains also important. For instance, Möller[45] noted that the spontaneous-fission half-life of $^{298}114$ decreases by a factor of more than 10^6 if the calculation of the macroscopic energy is performed using the DM[34] rather than the LDM[21].

In the region of superheavy nuclei, where the fission barrier heights may reach 9-14 MeV, predictions have been made by different groups of authors (see, e.g., refs.[22,38,46-51]) and, in general, they are rather reliable in the sense that they do not depend qualitatively on the choice of the concrete version of the macroscopic-microscopic method. However, in the next approximation the contours of the stability island, which, according to different calculations, can considerably differ (cf., e.g.,[50] and [51]) become important. The landscape of the island of stability is especially important in terms of experiments to synthesize SHE's. In practice, the only means of producing SHE's are heavy ion collisions, of which the reactions induced by ^{48}Ca seem to us to be the most promising ones. The latter, however, do now allow one to produce directly the double magic nucleus with N=184, but lead to $N \lesssim 178$ isotopes. In addition, if we take into account the α-decay, the centre of the stability island is displaced in Z from 114 to ≈ 110. Therefore, it is important to know at least qualitatively to what extent the fission barrier heights decrease as one moves away from the doubly magic nucleus. A unique possibility of learning something about this without extending to the region of unknown elements is offered by studies of the fission barriers of nuclei lying in N and Z round the closed nucleon shells of N=126 or/and Z=82.

Finally, the fission barrier is one of the main factors which determine the fission probability $G_f = \Gamma_f/\Gamma_{tot}$ or the probability of neutron emission $G_n = \Gamma_n/\Gamma_{tot}$ from a heated nucleus; here Γ_n and Γ_f are the partial widths of the decay of an excited nucleus by fission and neutron emission, respectively; and Γ_{tot} is the total decay width. Since, irrespective of the type of reaction chosen for synthesis, we always deal with more or less excited and rotating nuclei, the fission barrier heights, through a ratio like (Γ_n/Γ_f)*, substantially influence the ground-state formation probability for a given superheavy product.

Despite this, the important question as to how the closed nucleon shell influences the magnitude of the (HI, xn) reaction cross section, or the survival probability for the heavy and superheavy nuclei formed in other processes involving heavy ions, remains rather unclear.

In the region of highly fissionable nuclei ($G_f \sim 1$, $\Gamma_n \ll \Gamma_f$), the magnitude of the cross section of the (HI, xn) reaction is almost entirely determined by the factor $G_{xn} = \prod_{i=1}^{x} (\Gamma_n/\Gamma_f)_i$. The Γ_n/Γ_f value in turn is a strong and composite function of many variables such as the fission barrier, the excitation energy E* (see, e.g., [52,53]), angular momentum ℓ, and others. Therefore, the study of G_n or Γ_n/Γ_f as a function of all variables is or primary interest to clarify the original causes of the changes that arise in the cross sections of the xn -reactions and in the probabilities of formation of final products in other heavy-ion induced processes used to produce new nuclei as we vary the Z and A of the nucleus to be synthesized, the target-projectile combination, the bombarding energy and, consequently, to predict isotopic yields in experiments aimed at the synthesis of new elements.

On the other hand, the experimental determination, in different types of reactions, of the energy dependence of the fission cross section $\sigma_f(E^*)$ or the fission probability $G_f(E^*)$ (in the particular case of Γ_f/Γ_n) is a classical method

---

*In considering mainly highly fissionable nuclei in this paper, here and below we often "replace" Γ_{tot} by Γ_f only for the sake of visualizability; of course, in the general case $\Gamma_{tot} = \Gamma_f + \sum_m \Gamma_m$, where m is the index of the decay mode.

for obtaining quantitative information about fission barriers, the foundations of which have been laid by Bohr and Wheeler [23]; since then various aspects of the method were elaborated and unified repeatedly. Therefore, it is, in principle, clear that in the region of highly fissionable nuclei, $[1 - G_f] \approx G_n \ll 1$, complete-fusion reactions of the type (HI, xn) can also serve as a source of information about fission barriers if, from experimental studies of these reactions, one can extract the energy dependence of $G_n(E^*)$ or, at least, the values of $(\Gamma_n/\Gamma_f)_i$ for each i. As is known, the main parameters used in the adjustment and fitting of the theoretical expression for $G_f(E^*)$ (or $G_n(E^*)$) obtained using an adequate (statistical, as a rule) model of the process and of the experimental dependence of G_f on excitation energy are the fission barrier B_f and the ratio a_f/a_n, where a_f is the level density parameter for the fissioning nucleus at the saddle point, and a_n is the level density parameter of the residual nucleus (after evaporation of one neutron) in the main minimum of the potential energy surface. It is natural that none of the concrete models used in such a procedure takes into account (and, in contrast to the rigorous theory, it is uncapable of doing so) the total variety of possible physical effects. In addition, despite the presence of a number of elaborate versions of the statistical model (see, e.g., [54-57, 6, 52, 53] and references therein), rather rough versions are often used in practice. Therefore, it should be especially emphasized that the parameters found from the best-fit procedure, in particular, the B_f values, should ever be considered to be "effective" to a greater or lesser extent.

However, it is extremely difficult, if only possible at all, to solve the reverse problem and extract reliable values of $(\Gamma_n/\Gamma_f)_i$ for each stage i of the neutron cascade separately, on the basis of only the experimental data on the excitation functions of the xn-reactions induced by heavy ions, $\sigma_{xn}(E^*)$, because all the variables on which Γ_n/Γ_f depends, change at each stage of the cascade and are in no case completely independent. For evident reasons, it is especially complicated to extract $(\Gamma_n/\Gamma_f)_i$ for the final stages of the cascade, if the analysis is performed on the basis of the heavy-ion reactions for which the minimum excitation energy of the compound nucleus corresponds to $\langle i \rangle = 4-5$. On the other hand, just at the last stages the ratio Γ_n/Γ_f is the most sensitive to the shell structure of the nucleus as a whole and, in particular, to the fission barrier height of the cold nucleus.

For lack of something better, from the experimental values of σ_{xn}^{max} one often extracts (see, e.g., [58-61]) some effective quantities $\langle\Gamma_n/\Gamma_f\rangle = (G_{xn})^{1/x}$, the geometric mean of the x values of $(\Gamma_n/\Gamma_f)_i$. This procedure gives nothing but a different notion of the totality of the experimental σ_{xn}^{max} which is more convenient for systematization and extrapolations. However, this does not change the situation substantially. Without denying the evident practical usefulness of this procedure, we stress that, in principle, the relationship of $\langle\Gamma_n/\Gamma_f\rangle$ with the fission barrier height of the final nucleus remains to be as complicated and conditioned as that of the initial quantity σ_{xn}^{max} proper.

Ideologically, the influence of the fission barrier height B_f or the value of $(B_f - B_n)$, where B_n is the neutron binding energy, on the ratio Γ_n/Γ_f is a relatively simpler problem. It is apparent that the change in the fission barrier height leads to sharp variations in the probability of fission: a distinct and clear correlation between the experimental values of B_f^{max}* and $\lg(\Gamma_n/\Gamma_f)$ as a function of Z and A has been observed[5] for actinides in the region of excitation energies of $\lesssim 10$ MeV. As the excitation energy and angular momentum increase, both the "macroscopic" properties and nuclear shell structure and, consequently, the amplitude and shape of the fission barrier, change leading to immediate changes in Γ_n/Γ_f. In particular, the problem of the correlation between B_f and Γ_n/Γ_f or Γ_f at different excitation energies was considered in detail in refs.[62-64,5]. For instance, in ref.[64], by comparing the experimental data on Γ_n/Γ_f obtained in the reactions (n,f) and (HI, xn), and by using ^{250}Cf as an example, the authors have clearly shown that strongly heated nuclei ($E^* \gtrsim 45$ MeV) have a fission barrier height of about 2 MeV, which is close to the expected LDM-value of B_f, but is nearly thrice smaller than the B_f^{max} of the cold nucleus ^{250}Cf. Thus, at low excitation energies $E^* \lesssim 10-15$ MeV the energy dependence of Γ_n/Γ_f is associated with

*Here and below, we term the quantity B_f^{max} as the fission barrier amplitude and find it as the maximum value of deformation energy (measured with respect to the ground-state energy) along the path (in the multi-dimensional space of deformations) of the real evolution of the nucleus from the main energy minimum to the scission point. In the case of a one-dimensional double-humped curve that is often used to describe the deformation dependence of the nuclear energy, B_f^{max} is the height of the larger hump, $B_f^{max} = \max\{B_f^{(A)}, B_f^{(B)}\}$.

the structure of the fission barrier of the cold nucleus, whereas at high excitation energies $E^* \geq 50$ MeV it is related with a certain "asymptotic" value of the fission barrier /62,63/, which is in height close to the LDM-value for a cold nucleus but not necessarily equal to it because of the possible changes of $\tilde{B}_f$ due to the thermal expansion of a highly excited nucleus and to other effects (see, e.g., /65-67/ and references therein). Even qualitatively, very little is known about the character of the change in the barrier structure at intermediate values of E^*. At $\ell \neq 0$, this complicated picture of the evolution of the fission barrier with increasing E^* is superimposed by the angular momentum dependence of the barrier /68-71,55,56/. This dependence strongly influences the amplitude and the shape of both the macroscopic part of the barrier and the microscopic correction, and there are no adequate grounds to consider the character of the ℓ dependences of both components to be identical since they are completely different in nature. We would also like to stress than changes in the shape of the fission barrier (and symmetry effects /72,1/), which accompany the change in its amplitude as the shell structure is destroyed with increasing E^* and ℓ, can lead to significant changes in the ratio of the level density parameters a_n/a_f in the main minimum of potential energy and at the saddle point. This ratio regulates the energy dependence of Γ_n/Γ_f, and naturally depends on the positions of the corresponding stationary points of the potential energy surface in the space of deformations.

Thus, in considering the analysis of cross sections of complete-fusion reactions followed by neutron evaporation as a method for obtaining information on the fission barriers of cold nuclei, we would like to point out quite positively that in the case of adhering to the approach under discussion, an analysis of experimental data on the energy dependence of Γ_n/Γ_f or $(\Gamma_n/\Gamma_f)_i$ for each i, obtained in the (HI, xn) reactions at minimum x values, $x \leq 2$, can offer the most valuable source of information of this kind. As has been shown in a long run of experiments performed at Dubna /73-80/, such reactions occur in bombardments of targets made of the isotopes of Pb and neighbouring elements with heavy ions of $A_I \gtrsim 40$, such as ^{40}Ar, ^{48}Ca, ^{50}Ti, ^{54}Cr, and others. Recently, similar reactions, in particular, $^{208}Pb(^{50}Ti, n)^{257}Ku$, have been observed also by Armbruster and his colleagues /81/ at the UNILAC in Darmstadt. These reactions lead to the isotopes of elements from Fm, via Ku, to the heavier ones, and their analysis is of great interest in terms of information on fission barriers in the transfermium region, in which it is absolutely absent.

Moreover, at present it is difficult to indicate any alternative possibilities of obtaining information about fission barriers of nuclei with $Z \geq 102$, which is very useful in solving the problem of the synthesis of heavy and superheavy elements lying around the predicted closed shells with Z=114 and N=184. However, one can try to produce similar reactions in the region of the $Z \geq 80$ and $N \leq 126$ nuclei adjacent to the known closed shells. The above proposed approach, of course, needs a substantial development in many respects before it can be used for the determination (or estimation) of the fission barrier heights for cold nuclei with $\ell = 0$. In our view, this development is possible although the question as to what extent this approach will be informative remains open a priori. These investigations should be carried out separately.

The analysis becomes much more complicated if the fission barrier information is extracted from experimental data on the cross sections of those (HI, xn) reactions, for which the compound nucleus excitation energy E^*_{min} corresponds to $x \geq 4$. In this case one can determine from experiment only the quantities $\langle\Gamma_n/\Gamma_f\rangle$ averaged over a wide range of excitation energies, ≈25 MeV or more. To illustrate the difficulties arising in the analysis of $\langle\Gamma_n/\Gamma_f\rangle$, we shall consider the conclusions made in a recent paper /58/, in which these quantities were extracted for a number of Ar-induced complete-fusion reactions leading to evaporation residues with $84 \leq Z \leq 91$ and $N \leq 127$. In particular, the values of $\langle\Gamma_n/\Gamma_f\rangle$ were determined (and reduced to $\ell = 0$ by using the rotating LDM /69/) for the Th isotopes with N=122-126 produced in the reactions $^{176,...,180}Hf(^{40}Ar, 4n)$. These values lie almost on a straight line in the range of (N+1)=123-127 depending on N (on a semilogarithmic scale) and show no pronounced variation across the N=126 shell, which was expected by the authors of ref. /58/. From this fact the following two far-reaching conclusions have been made: (i) the influence of the ground-state shell effect on the fission probability is rather weak around N=126; if this is a general effect, then production of SHE by fusion reactions will be extremely difficult; (ii) the fission barrier heights $\tilde{B}_f$ of very neutron-deficient isotopes near Th are smaller than those predicted by the Myers DM /35/ and the DM does not reproduce their isospin dependence properly.

In our opinion, the analysis method used in ref. /58/ is uncapable of providing the acceptable level of reliability of the conclusions made irrespective of the fact whether the conclusions themselves correctly characterize the situation under discussion. In fact, in analyzing complex (since most averaged) quantities such as $\langle\Gamma_n/\Gamma_f\rangle$ at x=4 one cannot predict a priori with any degree of certainty how they should behave across the shell. It is far from being obvious that these effective values should undergo a pronounced rupture or another kind of irregularity. The closed shell, if it has an effect on the quantities being considered at all, is very likely to manifest itself in such a way that all the set of the values of $<\Gamma_n/\Gamma_f>$ in the vicinity of N=126 will turn to be somewhat overrated. The question, however, arises as to in comparison with what and to what extent it may be overrated.

In order to answer the question what the N=126 shell effect lies in, one should completely "switch off" the spherical shell in this region (this, however, is not necessarily equivalent to the full exclusion of any shell structure) and determine the σ_{xn} and $<\Gamma_n/\Gamma_f>$ values for this set of pseudonuclei and for exactly identical reactions. Unfortunately this can be done only theoretically. However, one can hardly rely upon a calculation performed for the region of nuclei in which, taking into account all the presently available knowledge about fission barriers and nuclear masses, even the value of the macroscopic fission barrier height $\tilde{B}_f$ can be estimated with an accuracy not better than a factor of 2(!). In addition, it is almost evident that the given value of the product $\prod_{i=1}^{x}(\Gamma_n/\Gamma_f)_i$ can be obtained by using quite different ad hoc sets of 4 separate $(\Gamma_n/\Gamma_f)_i$ values, i.e., entirely different assumptions concerning the functions and parameters used in the analysis [58], which are rather numerous in this case. Generally speaking, the number of combinations of several factors, which is required to obtain the set value of the product is infinite. If we fix the value of each factor, i.e., of each $(\Gamma_n/\Gamma_f)_i$ value, even then at $i=x=4$, by mere permutation, one can equally well satisfy say twenty four ($x!=4!=24$) different assumptions about the energy dependence of Γ_n/Γ_f, among which at least two will be exactly opposite ones. This means that the $<\Gamma_n/\Gamma_f>$ value, in general, is absolutely insensitive to the character of the energy dependence $G_n(E^*)$ (see also [82]) and, consequently, the analysis method based on $<\Gamma_n/\Gamma_f>$ is ambiguous. As to the dependence $G_n(E^*)$ itself, e.g., in ref. [52,53], for actinide nuclei, it has been shown to be strong, rather complex and, in particular, nonmonotonic in the E^* range of 6-35 MeV.

A drastic decrease in ambiguity can be achieved in the case where experimental excitation functions for neutron evaporation residues are employed to extract and analyse the $(\Gamma_n/\Gamma_f)_i$ values for each stage of the neutron cascade, as has been done, e.g., in ref.[83] for direct reactions (^{7}Li, αxn) ,or directly the energy dependence $G_n(E^*)$, which has been found, e.g., in refs. /52,53/ for reactions (α, xn) at x=1-4. In principle, similar analysis can be carried out also for (HI, xn) reactions leading to very neutron-deficient nuclei lying in Z or N around the $Z \approx 82$ or $N \approx 126$ closed nucleon shells. It is evident that this analysis would require all the presently available experimental data on $\sigma_{xn}(E^*)$ in this region of nuclei, including those obtained in refs. /84-86/ for reactions induced by ^{12}C, ^{16}O, ^{20}Ne, ^{22}Ne, ^{24}Mg, ^{31}P and other data. In particular, this would allow one to improve the procedure of separating the ℓ-dependence of Γ_n/Γ_f and check it to a certain extent. It is also possible that new systematic measurements would be required to obtain "complete" sets of the $(\Gamma_n/\Gamma_f)_i$ values ("complete" in terms of the extraction of $(\Gamma_n/\Gamma_f)_i$ values or $G_n(E^*)$ by comparing cross sections for the reactions couples [xn - (x-1)n] leading to the same final products). Finally, in the region of the $Z > 80$ and $N < 126$ nuclei it would be of particular interest to consider experimental data on $\sigma_{xn}(E^*)$ for fusion reactions involving the emission of few neutrons, x = 1 and 2. Neither of the possibilities described by us above is used or even mentioned in ref. /58/. The scope of the present paper does not allow us to investigate these possibilities either. This should be done in a separate paper.

In summary, we do not incline to think that the authors of ref. /58/ have proven the absence of the N=126 shell effect on the fission probabilities for nuclei lying in this region and on the cross sections of (HI, xn) reactions, and, consequently, comment (i) on the behaviour of these quantities near $N = 184$ is unjustified. The conclusion (ii) of the authors of ref. /58/ about the magnitude and isospin dependence of the fission barriers of the $N < 126$ nuclei should also be considered in the context of the above discussion. In view of the very complex character of the effective values of $\langle\Gamma_n/\Gamma_f\rangle$, on the one hand, and of strong changes in $(B_f - B_n)$ with E^* variations in the cascade, on the other, the result of the direct comparison made in ref. /58/ of the $(\tilde{B}_f - B_n)$ values calculated using the DM /35/ with the experimental values of $\langle\Gamma_n/\Gamma_f\rangle$ for nuclei with identical $\langle\Gamma_n/\Gamma_f\rangle = \text{const.}$, but with different values of $I = (N-Z)/A$, cannot be interpreted unambiguously. Even if we neglect all the possible ways of the influence of the fission barrier structure on Γ_n/Γ_f others than the direct one following from the schematic relation

$$\Gamma_n/\Gamma_f \simeq \text{const} \cdot \exp[(B_f - B_n)/T]$$

one should rather compare the given $\langle\Gamma_n/\Gamma_f\rangle = \text{const}$ to some averaged effective value of $\langle(B_f - B_n)\rangle = \frac{1}{x}\sum_{i=1}^{x}(B_f^{(i)} - B_n^{(i)})$.

In the case of the reaction Hf(^{40}Ar, xn) with $x=4$, the averaging occurs just in that excitation energy range in which the shell structure is destroyed and the fission barrier of the cold nucleus evolves in the direction of an "asymptotic" barrier. Therefore, in terms of information about the fission barriers of heated nuclei, which are commonly assumed to be close to the LDM barriers in height and shape, an analysis of experimental data on $\sigma_{xn}(E^*)$ for the reactions (HI, xn) with $x>5$ can lead to more definite conclusions.

As to the fission barriers of cold nuclei, it is a too indirect method to judge their heights on the basis of the effective values of $\langle\Gamma_n/\Gamma_f\rangle$ determined at ≥ 45 MeV initial excitation energy of the rotating compound nuclei, particularly, for such exotic, in nucleon composition, nuclei as the $N \leq 126$ isotopes of thorium. We note that the fission barrier is a superposition of the macroscopic and microscopic components and the existing notions of each of them for this region of nuclei are rather uncertain. Moreover, the result of the superposition may be different, and its prediction is still more complicated. For instance, if the macroscopic fission barrier heights $\tilde{B}_f$ for Th isotopes with $N<126$ decrease strongly, the isospin dependence of the total fission heights may not show the pronounced peak at $N=126$ at all. Of course, this would in no case imply that the $N=126$ shell has no effect on the fission barrier heights.

We now shall make a number of comments on the procedure of extracting $\langle\Gamma_n/\Gamma_f\rangle$ from experimental data on $\sigma_{xn}(E^*)$. For this purpose one typically uses /58-61/ the relation

$$\sigma_{xn}(E^*) = \sigma_{fus}\, P_{xn}(E^*) \prod_{i=1}^{x} \bar{G}_n^{(i)}\ , \tag{3.1}$$

where σ_{fus} is the complete-fusion cross section, $\bar{G}_n^{(i)} = [\Gamma_n/(\Gamma_n+\Gamma_f)]_i$, and $P_{xn}(E^*)$ is the emission probability for exactly x neutrons from the compound nucleus having an initial excitation energy E^*, if other modes of its decay are forbidden; naturally all quantities entering into (3.1) depend on angular momentum. However, expression (3.1) implies the assumption that after the emission of x neutrons the nucleus reaches its ground state without difficulty, and other processes, in particular, fission, do not compete with γ-ray emission, i.e., the radiative width $G_\gamma \equiv \Gamma_\gamma/\Gamma_{tot} = 1$. For highly fissionable neutron-deficient nuclei formed in HI-reactions, this is generally invalid (see also ref. [87]). In fact, one should write

$$\sigma_{xn}(E^*) = \sigma_{fus} P_{xn}(E^*)[1 - \bar{G}_f(E^*_{res})] \prod_{i=1}^{x} \bar{G}_n^{(i)} , \qquad (3.2)$$

where $[1 - \bar{G}_f(E^*_{res})] = \bar{G}_\gamma(E^*_{res})$ if neutron emission is forbidden, and E^*_{res} is the residual excitation energy that has not been carried off by the cascade of x neutrons; in addition to thermal energy, E^*_{res} contains a certain amount of rotational energy.

Therefore the values of $\langle\Gamma_n/\Gamma_f\rangle$ obtained on the basis of (3.1) and (3.2) can noticeably differ and these differences will be the larger the smaller the x values. The use of the more correct expression (3.2) instead of (3.1) will lead to an increase in $\langle\Gamma_n/\Gamma_f\rangle$. In addition, owing to the direct and strong relation with the fission barrier height of almost cold (however rotating) nuclei, the factor $[1 - \bar{G}_f(E^*_{res})]$ strongly depends on N and Z. For evident reasons, for nuclei having N and/or Z close to the magic ones, it is natural to expect local and relatively sharper changes in the value of $[1-\bar{G}_f(E^*_{res})]$ and, consequently, in $\langle\Gamma_n/\Gamma_f\rangle$ extracted on the basis of (3.2). Thus, it is very likely that if we take into account $\bar{G}_f(E^*_{res}) \neq 0$ for nuclei having N around 126, the increase in absolute values will be accompanied by the slowing down of the rate of changes in $\langle\Gamma_n/\Gamma_f\rangle = f(N+1)$, especially if the isospin dependence of the fission barrier height has indeed a pronounced "peak" at N=126. This effect, however, will be smoothed if $\langle\Gamma_n/\Gamma_f\rangle$ are extracted on the basis of (3.1).

The above consideration shows that the existing systematics [58-61] of the effective $\langle\Gamma_n/\Gamma_f\rangle$ values for strongly fissionable nuclei should be redetermined according to (3.2); it can be expected that as a result of this, the N dependences of $\lg\langle\Gamma_n/\Gamma_f\rangle$, typically characterized for each Z by straight lines with almost identical slopes, will become less regular - separate points will more scatter with respect to these lines. It is undoubtful that such a redetermination will enhance the reliability of σ_{xn} extrapolations.

Thus, the authors of ref. [58] have performed a pioneering study of a question that is important in terms of the synthesis of new elements. Their method, however, appears to be rather sketchy in character because it is oversimplified without necessity in its main aspects. Therefore, the conclusions made in ref. [58] remain disputable until direct experimental data on fission barriers and on fission probabilities $G_f(E^*)$ at low excitation energies are obtained in the nuclear region under discussion.

The βDF offers these possibilities. As soon as the fission barriers of cold nuclei with $Z \geq 80$ and $N \leq 126$ and their fission probabilities at $E^* \leq 10$ MeV are determined, a thorough analysis of all the available experimental data on the excitation functions of the (HI, xn) reactions and other data in this nuclear region will allow one to draw unambiguous conclusions about the character, strength and concrete ways of the manifestation of the closed N=126 neutron shell effect on the value of Γ_n/Γ_f and on the cross sections of xn-reactions. This will make it possible to see to what extent this situation with fission barriers and the reactions cross sections may be similar to that predicted for the hypothetical region of enhanced stability around N=184. Only after all this has been done, more or less grounded forecasts can be made regarding the synthesis and properties of SHE's.

4. THE FISSION BARRIERS OF NUCLEI LYING AROUND ^{208}Pb AND THE FISSION BARRIERS OF "COMMON" THORIUM NUCLEI

Prior to discussing the possibilities offered by the studies of βDF of nuclei with $Z > 80$ and $N \leq 126$, it is natural to consider the fission barrier experimental data obtained in adjacent fields.

For the time being, a certain amount of experimental data have been accumulated for the region around the double magic ^{208}Pb. These data have been obtained mainly from studies of the energy dependence of the fission cross sections by using electrons, protons, and α-particles (see refs./2-4,6-10/ and references therein). A characteristic fragment of these data is presented in fig.3. From this figure one can clearly see that the Z=82 and N=126 closed shells have a strong effect on the fission barrier heights: for $^{204-208}Pb$, the fission barrier heights reach 24-28 MeV, which is by 10-12 MeV larger than those predicted by the LDM or its modified versions. An almost equal shell effect follows from the comparison of the experimental ground-state masses of nuclei with those calculated using the LDM. This forms the basis for the conclusion /2/ that near ^{208}Pb the microscopic part of the fission barrier heights is mainly due to a strong decrease in the ground-state mass of nuclei lying around Z = 82 and N=126. The saddle-point shell and pairing effects seem to be small though there is no certainty in details. Apparently, the value of these effects is of the order of the experimental inaccuracy, i.e., about 1-1.5 MeV. From fig.3 it follows that as one moves from the doubly magic ^{208}Pb, the fission barrier heights decrease sig-

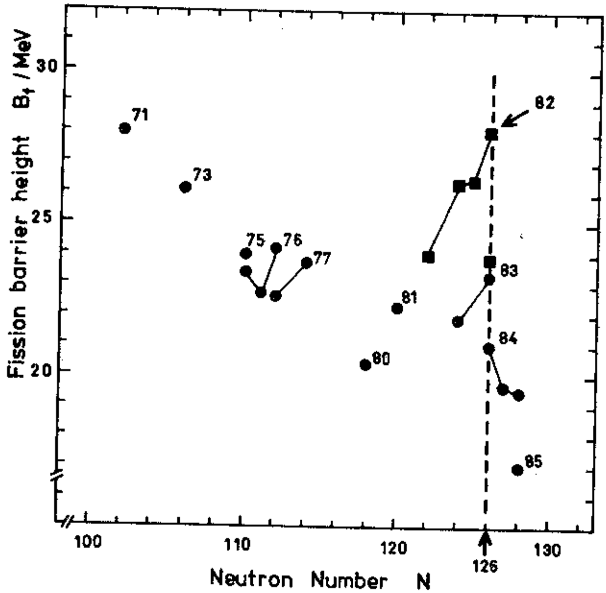

Fig.3. Experimental data [2,7-9] on the fission barrier heights B_f of preactinide nuclei with Z and N close to the doubly magic nucleus ^{208}Pb; the results of refs. [3,4,6] give a similar picture of B_f changes as a function of Z and N in this region.

nificantly: for instance, the transition from A=208 to A=204 leads, for Pb isotopes, to a barrier decrease by about 4.5±2.5 MeV due to both the decrease in the macroscopic part (~2.5 MeV following Schrøder [41]), and the decrease in the shell correction; the latter can be estimated to be ~1-2 MeV/ΔN=2 in the immediate vicinity of N=126.

Thus, the presently available experimental data, on the whole, characterize rather definitely the changes in the fission barrier heights in the region of ^{208}Pb, which can be summarized in the following way.

(i) Near ^{208}Pb, shell effects in fission barriers manifest themselves strikingly; however, the macroscopic part is the main contributor (≈60%) to the fission barrier amplitude.

(ii) The closed proton and neutron shells decrease mainly the ground-state energy of the nuclei; they have a joint effect, the strict separation of which is complicated, although some attempts at doing so have been made[88]. The effects of both shells on the fission barrier height are approximately equal in strength; at any rate, one can hardly draw other/or more definite conclusions on the basis of existing data.

(iii) In the region of ^{208}Pb, fission barriers are determined only for the nuclei lying in the close proximity of the line of β-stability.

The βDF permits an attempt to determine the fission barrier amplitude B_f^{max} for Pb isotopes (and neighbouring elements) far from the N=126 shell by 20 and more neutrons, such as ^{188}Pb or ^{186}Pb. The DM[35] predicts for ^{186}Pb and ^{208}Pb equal values of the macroscopic part of the fission barrier $\tilde{B}_f \approx 13$ MeV; at the same time, for ^{186}Pb, it gives a decrease in B_f^{max} by 14-15 MeV compared with ^{208}Pb. The LDM with $k_s=1.78$[20,21] which, according to[41], best reproduces the value and the isospin dependence of $\tilde{B}_f$ for the A=208-204 Pb isotopes, predicts a 6-7 MeV decrease in $\tilde{B}_f$ and a decrease in B_f^{max} to ≈ 11 MeV as one goes from ^{208}Pb to ^{186}Pb. In addition to clarifying the isospin dependence of $\tilde{B}_f$, it would be very important, at Z=82, to experimentally establish whether there acts and to what consequences leads the closed spherical proton shell in very proton-rich nuclei. For the above reasons, an attempt to observe the βDF of Bi isotopes with N=106-108 is of extremely great interest and, undoubtfully, it is worth making even if the detection probability for fission fragments from these nuclei may seem to be a priori vanishingly small (see also Section 5 below).

On the other hand, by preserving the magic N=126 one can move by 8-10 protons to the region of elements of Z>82. For nuclei of $Z \gtrsim 85$ and $N \lesssim 126$, we should expect a dramatic change in the situation outlined above.

First of all, because of a strong decrease in Z^2/A, the macroscopic fission barrier heights should apparently decrease to a value of about 5 MeV, i.e., by a factor of about 2-3 compared with the region of ^{208}Pb. The shell correction in the ground-state masses of nuclei with $Z \geq 85$ and $N \lesssim 126$ is also about (-5) MeV[35,89]. So, the sharp change in the correlation between the values of the macroscopic and microscopic components indicates the possibility of a substantial change not only in the height, but also in the shape of the fission barriers. It is very difficult to predict the net result of the two contributions to the total deformation energy in a completely new (in terms of fission information) region of nuclei. However, it would not be surprising if the amplitude and shape of the fission barriers of the nuclei being considered be determined mainly by the microscopic correction and its deformation dependence.

Secondly, it is natural to assume that in the case of the Z>88 nuclei the strong effect of the spherical Z=82 shell vanishes entirely, and the N=126 shell effect, if it exists at all, should manifest itself in a "pure" form in an experiment. Finally, the isotopes of Ra, Th, U of N<126 are also very neutron-deficient. This fact is a distinguishing feature of the nuclear region proposed for investigation, compared with that around ^{208}Pb and the already investigated region of the Z>85 nuclei, in which fission barriers have been determined experimentally only for N≥137 (see fig.1).

Now, by using Th isotopes as an example, we shall consider in brief the main features of the experimental data and theoretical predictions concerning the structure of the fission barriers of "common" (N≥137) isotopes lying in the range of Ra to U, assuming that for elements adjacent to Th the situation is similar qualitatively (see, e.g., refs. /1,10/). As shown in fig.4, for thorium, fission barriers have been measured for seven isotopes with A ranging from 228 to 234 /1,10/. The height of the largest external peak in the double-humped fission barrier for all these nuclei lies between 6.0 and 6.5 MeV, and macroscopic-microscopic calculations /39,40,43,91,92/ lead to approximately the same values. For the first ("internal") peak, these same calculations show that its height should be by 2-3 MeV smaller than the second one. In contrast to the theory, in experiments /1/ aimed at measuring the energy dependence of the fission probabilities in direct reactions resonance phenomena are clearly observed, which are indicative of the fact that the fission barrier has really two maxima of nearly the same height and which are separated by a minimum 2 MeV in depth. This "contradiction" known as the "Ra-Th anomaly" seems to be explained /92/ by the fact that the two maxima of nearly the same height that cause the resonance structure in fission cross sections are conditioned by the development of a third minimum in the dependence of potential energy upon deformation, which lies in the region of the outer (mass-asymmetric) saddle point.

So, the shape of the fission barriers of heavy thorium isotopes is fairly complex - it is virtually a three-humped curve. The second (external) peak in the barrier, usually characteristic of actinides, is very broad in this case and is divided into two peaks about 6.0-6.5 MeV in height with a comparatively shallow (≲ 2 MeV) well between them, whereas the first (internal) peak lies in height 2-3 MeV below and, therefore, does not show itself in experiment.

In contrast to the region of ^{208}Pb, where, in the main, shell effects only strongly decrease the ground-state energy

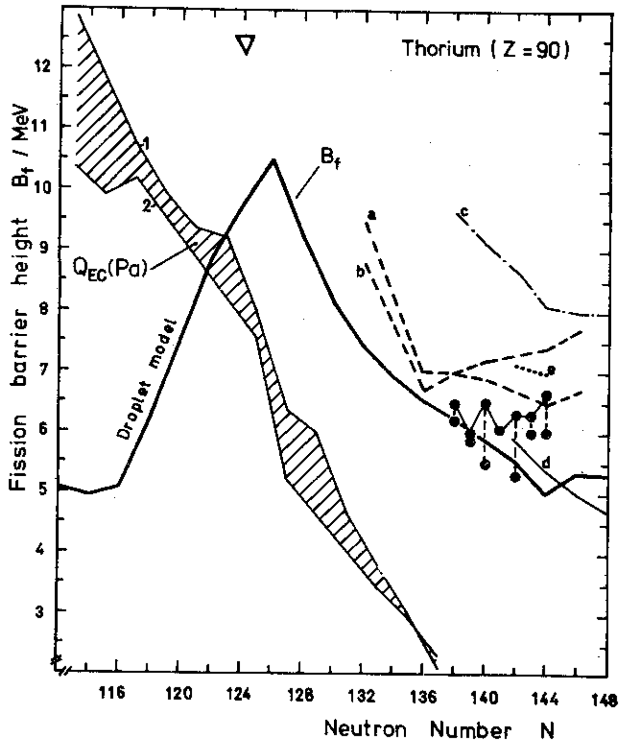

Fig.4. Experimental (•) [1] and theoretical values of fission barrier heights for Th isotopes. Two experimental points(:) for each N correspond to the heights of two separate peaks, into which the external (mass-asymmetric) hump in the fission barriers of Th isotopes is divided (see text). The solid line is the predictions of the DM of Myers[35], curves a,b,c, d, and e and point indicated as (∇) are the results of the macroscopic-microscopic calculations of B_f^{max} (curves a and b are the results of Møller (ref.[91]), for the two variants of parameters G=const, k_s = 2.53 and G ∞ S, k_s = 1.78, respectively; curves c,d, and e are the results of Møller[43], Møller and Nix[92] and Pauli and Ledergerber[40], respectively, point(∇) are predictions of Aberg et al.[70] for ^{214}Th). The shaded area is the Q_{EC} values (MeV) for Pa isotopes: curve 1 is predictions of the semiempirical systematics of Viola et al.[90]; curve 2 is predictions of Myers[35]; both curves are drawn through the points for Pa isotopes with even A.

of the nucleus, the complex structure of the fission barriers of "common" Th isotopes serves as an excellent example of pronounced shell effects at very large deformations. For Th isotopes with $N \leq 126$, both of these features may manifest themselves simultaneously agaist the background of the macroscopic part of the deformation energy. The behaviour of the latter, at such a large distance from the line of β-stability, remains to be most problematic (see fig.2). There are practically no realistic calculations for fission barriers in the nuclear region of interest to us. Although Myers and Swiatecki [20,21] and Myers [34,35] make some concrete predictions (e.g., the DM curve in fig.4), these predictions rest in a sense on rather a sketchy basis - they use the phenomenological shell correction [20,21] which is, moreover, damped with increasing deformation θ as $(1-2\theta^2)\exp(-\theta^2)$, i.e., the shell correction to the ground-state energy only and this is unreal for heavy nuclei. As to realistic calculations, we shall mention one of Möller's papers [91] in which fission barriers were determined, in particular, for the $N \geq 132$ isotopes of Ra, Th, and U. This study was performed using the LDM [21] with $k_s = 1.78$ or $k_s = 2.53$ to calculate the macroscopic energy, the modified harmonic-oscillator potential to determine the shell correction, and under the two assumptions concerning the deformation dependence of the pairing strength - $G = \text{const}$ and $G \sim S$, where S is the nuclear surface area. For $Z = 90$, the results obtained by Möller [91] are shown in fig.4. One can see that as N decreases from 138 to 132, the height of the external peak in the calculated fission barrier increases by 1.5-2.5 MeV. Thus, one can state a priori that in many aspects that are important in terms of fission barriers, the region of nuclei $Z \geq 80$ and $N \leq 126$ substantially differs from the "neighbouring" ones and any others, in which experimental data are already available. This is the reason why the experimental studies of fission barriers and of other characteristics of fission in the new region of nuclei can provide an original and valuable material for the critical examination and development of the existing theoretical representations of fission phenomena.

5. BETA-DELAYED FISSION AS A TOOL FOR THE EXPERIMENTAL DETERMINATION OF FISSION PROBABILITIES AND BARRIER HEIGHTS, AND OF THE PROPERTIES OF FISSION FRAGMENTS FOR NUCLEI WITH $Z \geq 80$ AND $N \geq 126$ FAR OFF BETA-STABILITY

The advantages of βDF can be used most fully for those nuclei that in Z or N are close to the magic ones and, at the same time, are substantially (by 15-20 and more neutrons) removed from the line of β-stability. In fact, a mere consideration of the possibilities of βDF studies for nuclei with $Z>80$ and $N<126$ convinces us that the radioactive properties of nuclei in this region are most favourable to carry out such experiments. Indeed, the effect of the $Z=82$ and $N=126$ closed shells reduces the Q_α values and increases the α-decay half-lives to $\sim(10^{-3}-10^{2})$s for a fairly wide range of nuclei. This range of half-lives is especially convenient for experiments aimed at detecting βDF fragments with a maximum sensitivity. At the same time, at these large distances from β-stability the values of the total energy of electron capture (EC), Q_{EC}, reach $\approx$ 7-11 MeV. One can easily see that in this case the α-decay and EC half-lives compare, i.e., EC is a very probable process. On the other hand, the probability (or branching ratio) of the βDF of these nuclei, $P_{\beta-df}$, is quite sensitive to the barrier amplitude since $Q_{EC} \lesssim B_f^{max}$. This picture is rather a typical one and can occur also near other closed shells (in other nuclear regions).

We shall carry out a further consideration by using principally the examples of Ac and Pa isotopes and taking into account the fact that the situation for other elements in the nuclear region under discussion will be qualitatively similar. For Ac and Pa isotopes, it is illustrated in figs.4 and 5. It should be noted that, in principle, the βDF of even-Z isotopes can also be considered in the given region of nuclei.

Thus, if the amplitude of the fission barriers B_f^{max} for, say, Th isotopes does not increase as one goes from $N=138-144$ to $N\lesssim126$, already for ^{216}Pa, a strong βDF effect should be observed. If, vice versa, a noticeable increase in the barrier heights occurs at $N=126$, then the $P_{\beta-df}$ value will be rather small at $N=126$. However, in any case the crossing of the $N=126$ shell and a further advance in the direction of neutron deficit should inevitably be accompanied by a decrease in B_f^{max}. The Q_{EC} value continues to grow and, consequently, at some value of $N<126$ the βDF effect should be "switched on" more or less sharply.

It is noteworthy that experiments to study βDF in the nuclear region under discussion can be performed with a very high sensitivity. For instance, to produce Ac and Pa isotopes of $N\lesssim126$ the following reactions appear to be the most suitable ones:

$$^{197}\mathrm{Au}(^{20,22}\mathrm{Ne},4\text{-}9n)^{208,\ldots,215}\mathrm{Ac}, \quad \text{and}$$

$$^{197}\mathrm{Au}(^{24}\mathrm{Mg},5\text{-}9n)^{212,\ldots,216}\mathrm{Pa},$$

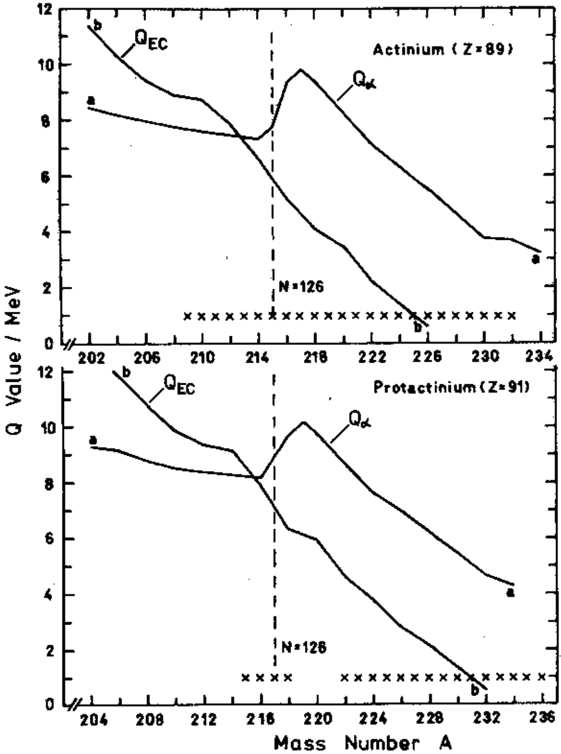

Fig.5. The Q_α (a) and Q_{EC}(b) values for Ac and Pa isotopes taken from the semiempirical systematics of Viola et al.[90] (curves a and b are drawn through points for even-A isotopes). The known Ac and Pa isotopes are indicated by (x).

the cross sections of which, σ_{xn}, are known from refs.[84,93] to be $\approx 5\times10^{-27} - 6\times10^{-30}$ cm^2 for x=4–5. The very neutron-deficient Bi isotopes can be produced[94,95] with large yields, for instance, in the reactions $^{151,153}Eu(^{40}Ca, xn)^{[191(193)-x]}Bi$.

One can indicate rather effective reactions leading to the synthesis of the neutron-deficient isotopes of other elements with $Z \gtrsim 80$. The use of intense heavy ion beams from the U-400 accelerator of the JINR Laboratory of Nuclear Reactions (Dubna) and of the low-background techniques developed by us[78-80] to detect short-lived spontaneously fissioning nuclei in the transfermium region, makes it possible, for several hours of irradiation, to achieve and "display" very small cross sections for formation of βDF fragments, $\sigma_{\beta-df} \sim 10^{-36}$ cm^2. Here $\sigma_{\beta-df} = \sigma_\beta \cdot P_{\beta-df}$, σ_β is the formation cross section for the precursor of βDF fragments, determined directly for the β-decay branch. This means that under favourable (from the point of view of σ_β) conditions βDF fragments can be recorded even if $P_{\beta-df}$ values are as small as $\sim10^{-9}$. In general, βDF fragments can be easily detected for the majority of nuclei in the region considered, if $P_{\beta-df} \gtrsim 10^{-6}$. It is qualitatively clear that at $Q_{EC} \approx 7-11$ MeV the values of $P_{\beta-df} \sim 10^{-6} - 10^{-9}$ "control" rather high fission barriers.

Under the conditions of such a high experimental sensitivity, the absence of the βDF effect over the entire range of nuclei considered (of course, its absence cannot be excluded a priori) would deserve close attention and thorough examination, in the first place, in terms of nuclear stability against fission.

Now we turn to the problem of extracting information about the fission probabilities and barriers from experimental data on βDF branching ratios. So far, this problem has received much less attention than it deserves. The βDF branching ratio is determined by the evident relation

$$P_{\beta-df} = \frac{\sigma_{\beta-df}}{\sigma_\beta} = \frac{\int_0^{Q_\beta} F(Q_\beta - E) S_\beta(E) G_f(E)\, dE}{\int_0^{Q_\beta} F(Q_\beta - E) S_\beta(E)\, dE}, \qquad (5.1)$$

in which the product of the (fairly well known) statistical Fermi function $F(Q_\beta - E)$ and the β-decay strength function $S_\beta(E)$ regulates the population probability of the levels of the daughter nucleus in the excitation energy range of $(E, E+dE)$, while the fission probability of the daughter nucleus, $G_f(E) = \Gamma_f/\Gamma_{tot}$ describes in the same energy range the competition between fission and other modes of decay of the daughter nucleus. All information about the fission barrier of the nucleus being investigated concentrates in the a priori unknown $G_f(E)$, whereas the product $K(Q_\beta, E) \equiv F(Q_\beta - E) \times S_\beta(E)$ has only a formal relation to fission, although it may very strongly influence the resulting $P_{\beta-df}$. Therefore, we shall first consider these two main functions separately and then discuss their joint effect on the result.

From (5.1) it immediately follows that βDF is, first of all, a method for determining the probability of fission $G_f(E)$ – a physical quantity which itself is of great interest – and, in the second place, a tool for determining the parameters of the fission barrier, in particular, its amplitude B_f^{max}. This fact has not so far been noted in the literature clearly. In fact, if to illustrate this we assume that $S_\beta(E)$ is the Dirac delta function, then as $F(Q_\beta - E)$ and $G_f(E)$ are rather smooth functions, from (5.1) it immediately follows that

$$P_{\beta-df} = G_f(E_0), \qquad (5.2)$$

at

$$S_\beta(E) = \delta(E - E_0).$$

Despite this, no attempts have so far been made to extract $G_f(E)$ from the experimental $P_{\beta-df}$ values, although a number of papers [13-17] contain the results of different solutions of the more complicated problem - the straightforward determination of B_f^{max} by using numerous, including rather rough, assumptions, the validity of each of which is not always justified and, in any case, needs a special investigation.

It is also clear that unless $S_\beta(E)$ is the delta function and unless the experiment makes use of the (β-f) or any other coincidence technique, and (5.1) corresponds to this particular folded case, then the problem of extracting information about the unknown $G_f(E)$ on the basis of the experimental $P_{\beta-df}$ values and its errors $\Delta P_{\beta-df}$ gives a typical example of the inverse problem of mathematical physics. Specifically, it reduces to the solution of a homogeneous Volterra (or Fredholm) integral equation of the first kind *

$$\lambda \int_C^{Q_\beta} K(Q_\beta, E)\, G_f(E)\, dE = P_{\beta-df}(Q_\beta) \qquad C \le E \le Q_\beta \le b \qquad (5.3)$$

with the kernel $K(Q_\beta, E) = F(Q_\beta - E) \times S_\beta(E)$, where $\lambda = 0.693/T_{1/2\beta}$ is the decay constant, $C > 0$ is a constant, and $b > 0$ is a positive number. We note that at $E > Q_\beta$ the kernel of (3) vanishes. Even if the function $K(Q_\beta, E)$, i.e., the strength function $S_\beta(E)$, is well known, problem (5.3) is rather complicated and one should be careful in solving it. This problem has only an approximate solution at least due to the fact that $P_{\beta-df}(Q_\beta)$ is determined experimentally and, consequently, is burdened with errors of measurements. In addition, the kernel of (5.3) is not known exactly either; essentially, it should also be determined from a separate experiment. Therefore, we would like to emphasize that the experimental determination of the β-decay strength function for nuclei undergoing βDF is of extreme importance. It goes without saying that neither the sense nor the concrete content of problem (5.1) depends on whether we, on the basis of (5.1) or (5.3), first extract information on the function $G_f(E)$ and then derive the fission barrier parameters, or extract the barrier parameters directly. In the latter case, however, the true character of the problem remains rather vague. At the same time, the classification of the problem is the first important step toward its solution. It is noteworthy that, in the general case, the integral equations of the first kind provide a typical example of "incorrect-

* For a definition, see, e.g., ref. [96].

ly posed problems" and the main method of investigating integral equations of the first kind is the regularization method (see, e.g., ref. [97]).

On the other hand, as a whole, the connection of the fission probability $G_f(E)$ with the fission barrier parameters in the region of $E \lesssim B_f^{max}$ has been elaborated rather well and even in detail by many authors (see, e.g., [1,54] and references therein). Therefore, bearing in mind the βDF problem only, we shall stress some of the most important aspects of the correlation between $G_f(E)$ and B_f^{max}, which have not yet been discussed before, or have not been formulated in a properly clear way.

(A) The potential energy surface associated with fission is multi-dimensional and has quite a complex structure perturbed by shell effects. Therefore, in extracting the barrier parameters, in particular, the amplitude B_f^{max}, on the basis of $P_{\beta-df}$ or $G_f(E)$, one should inevitably use a certain amount of concrete a priori information (we denote this amount by J) about the landscape of the energy surface in the vicinity of its stationary points. The problem of minimizing J is very important and, strictly speaking, unsolved, and the problem of the influence of the quantity J on the uncertainty of the sought result, in particular, of B_f^{max}, is still little investigated.

(B) In determining the fission barrier parameters on the basis of $P_{\beta-df}$ the approximation $G_f(E) \approx \Gamma_f/(\Gamma_f + \Gamma_\gamma)$ is commonly used, the validity of which is far from being positive and needs a serious additional investigation in every concrete case.

First of all, in the region of neutron-deficient nuclei it is necessary to consider thoroughly the decay probabilities for the states populated via EC (β^+)-transitions along competing channels such as proton and/or α-particle emission, i.e., to determine the partial widths Γ_p and, particularly, Γ_α.

In particular, in the considered region of nuclei with $Z \gtrsim 80$ and $N \lesssim 126$, where the Q_α values are rather large and where high-lying states with $E \lesssim Q_{EC}$ can be populated via EC (β^+) decay with high probabilities, it is very likely that $\Gamma_\alpha \sim \Gamma_\gamma$. We note that the β-delayed α-particle emission with different probabilities has been observed experimentally in various nuclear regions, e.g., ref. [98] The contribution of Γ_α and/or Γ_p to the total decay width Γ_{tot} can decrease noticeably the value of $G_f(E)$ and change the character of its energy dependence and, consequently, reduce $P_{\beta-df}$ as well, especially if in the region of $Z \gtrsim 85$ and $N \lesssim 126$ a significant increase in the B_f^{max} values occurs. These factors should particularly be taken into account in the analysis of experimental data if, despite the high experimental sensitivity, the sought βDF effect proves to be very weak or even undetectable at all.

The direct experimental determination of $\Gamma_\alpha/\Gamma_{tot}$, which can be carried out with a sensitivity not lower than that of detecting βDF fragments, is of great interest. A measurement of the energy spectrum of β-delayed α-particles (and/or β-delayed protons) could provide valuable information on the structure of $S_\beta(E)$. As a whole, the emission of β-delayed α-particles by the nuclei undergoing βDF can give rise to a number of new, interesting and important questions in the problem under discussion. These questions will be considered elsewhere. As to the radiation width Γ_γ, the use for its estimation of the statistical expressions of the type /99/ containing such parameters as the temperature and neglecting the real structure of the states, between which the transition occurs, appears to be insufficiently justified at comparatively low energies $E \lesssim Q_{EC}$.

We now turn to consideration of the β-decay strength function $S_\beta(E)$. The analysis of β-delayed processes is usually made under the following main assumptions about $S_\beta(E)$.

(i) $S_\beta(E) = \mathrm{const}$ /100/;

(i) $S_\beta(E) \propto \rho(E)$, where $\rho(E)$ is the level density in the daughter nucleus (ref. /101/); and

(iii) $S_\beta(E)$ determined within the context of the gross-theory of β-decay /102/. So far, in determining, on the basis of (5.1), the fission barrier heights B_f^{max} from the experimental $P_{\beta-df}$ values one used /13-17/ only the assumption of $S_\beta(E) = \mathrm{const}$ over the entire range of accessible energies or above cut-off energy $E \geq C = 26^{-1/2}$ MeV /103/ (for even-even daughter nuclei) given by pairing effects. It is clear that the assumption of $S_\beta(E) = \mathrm{const}$ substantially simplifies (5.1): in this case $S_\beta(E)$ is left out of consideration at all. At the same time, the large number of experimental and theoretical studies shows /104/ that, contrary to assumptions (i)-(iii) about the "smooth" shape of $S_\beta(E)$, the Gamow-Teller β-decay strength function may have, especially in the case of spherical and almost spherical nuclei, a pronounced resonance structure due to collective states such as core-polarized and back spin-flip states. The detailed consideration of a large variety of aspects concerning the shape of the strength function $S_\beta(E)$ and its consequences for nuclear physics and astrophysics is contained in the collection of papers /104/ edited by Klapdor, to which we refer for further bibliography and details.

For the nuclei undergoing βDF, the microscopic calculations of the Gamow-Teller strength function $S_{EC}(E)$ were performed by

Klapdor et al.[105,106] and Naumov et al.[107]. In particular, for the neutron-deficient nuclei ^{232}Am, ^{240}Bk, $^{244,248}Es$, and ^{248}Md that undergo fission following EC, it was shown[106,107] that the strength function $S_{EC}(E)$ has a narrow main maximum at an excitation energy $E_0 \sim 2.5$ MeV (the Q_{EC} values lie in the range 4-5 MeV for all of these nuclei), the amplitude of other peaks in $S_{EC}(E)$ being smaller by several tens of times. The location of the main maximum of E_0, in general, depends sooner on the N rather than Z of the nucleus; in paricular, E_0 increases as one moves farther from the line of β-stability in the direction of neutron deficit but it remains by about 1.5-2.5 MeV below the value of Q_{EC} [107]. However, these calculations have been performed neglecting deformation. Of course, the above- mentioned nuclei are deformed in the ground state ($\beta_2 \approx 0.2$-0.3) and, consequently, it can be expected that taking into account the deformation effect will lead to a spreading of the peaks in $S_{EC}(E)$, so that the FWHM will be $\sim$1-2 MeV. Nevertheless, for deformed nuclei, the ratio of the "peak" to "background" areas in $S_{EC}(E)$ is also expected to be sufficiently large, of the order of 100 or more [106].

It now should be noted that macroscopic-microscopic calculations (see, e.g., [108]) predict a spherical ground state for nuclei with Z close to 82 and/or N close to 126. Therefore, in the nuclear region proposed for investigation, the main maximum of $S_{EC}(E)$ may be rather narrow. If this is really the case, the problem of extracting $G_f(E)$ and then B_f^{max} on the basis of (5.1) will again be simplified to some extent and the resulting value of $G_f(E)$ will be referred to a comparatively narrow excitation energy range around E_0.

In general, the shape of the real strength function may strongly differ from the assumption of $S_\beta(E) = \mathrm{const}$. For the time being, there are no results of the determination of fission barrier heights from experimental $P_{\beta-df}$ values taking into account the real structure of the strength function. Therefore, the problem of the concrete quantitative influence of different assumptions concerning the shape of $S_\beta(E)$ on the uncertainty of the sought result, strictly speaking, remains open. So far, only separate comments have been made on this problem and a qualitative conclusion [106,107] has been drawn that the fission barrier heights extracted from the present βDF measurements taking the resonance structure $S_\beta(E)$ into account do not contradict those calculated by the macroscopic-microscopic method.

In summary, our consideration shows that the experiments aimed at studying the βDF of nuclei with $Z > 80$ and $N < 126$ can be carried out with a very high sensitivity. However, in terms of data analysis, βDF as a method of the experimental determination of the fission barrier amplitudes, needs a considerable development in many respects, as has been emphasized in this section above. Finally, the detection of the βDF of nuclei with $Z \geq 80$ and $N \leq 126$ with sufficient probability may offer a unique possibility for a wider investigation of the characteristics of low-energy fission for those nuclei that are rather exotic in their nucleon composition. These characteristics comprise the fission fragment kinetic energy distributions, the distributions of prompt fission neutron numbers, etc. In view of the considerable changes expected to occur in the fission barrier shape, especially valuable is determination of the mass distributions of fission fragments and their correlations with the shape of the fission barrier. However, a more detailed discussion of this interesting possibility seems to be somewhat premature.

6. GENERAL CONCLUSIONS

The two unique circumstances - the nearness to the closed proton or neutron shell and, at the same time, a substantial remoteness (by about 20 or more neutrons) from the line of β-stability - attract extremely great interest to the experimental and theoretical investigations of the fission probabilities and barriers of nuclei lying in the region of $Z > 80$ and $N \leq 126$. These investigations are very important to clarify the isospin dependence of the macroscopic parts of the saddle-point and ground-state nuclear masses, the influence of the closed nucleon shell on the fission barrier heights of cold nuclei, the fissility of heated nuclei, the cross sections of (HI, xn) reactions, etc. In particular, they allow us to expose the closed ($Z=82$) proton shell effects in very proton-rich nucleus (like ^{188}Pb) and the ($N=126$) neutron shell effects in a very neutron-deficient nucleus (like ^{216}Th). The whole complex of these unsolved problems is closely related to the synthesis and investigation of the properties of SHE's in the region of $Z > 110$ and $N=184$.

We have shown that the most valuable possibilities for obtaining information on the fission barriers of very neutron-deficient nuclei with $Z \geq 80$ and $N \leq 126$ are provided by the experimental βDF investigations. In the given region of nuclei, such experiments can be performed with a sensitivity characterized by minimum cross sections of the order of $10^{-36}\,cm^2$ in detecting βDF fragments, or by minimum βDF branching ratio values of the order of 10^{-9} in favourable cases.

The other approach based on the analysis of experimental data on the excitation functions of the (HI, xn) reactions yields, in general, much less direct information about the fission barrier heights. Moreover, if such an analysis is grounded only on the effective $<\Gamma_n/\Gamma_f>$ quantities averaged over the neutron cascade, as in ref. /58/, the conclusions concerning the fission barriers and shell effects on Γ_n/Γ_f and other quantities become ambiguous especially in the case where the number of cascade stages $x = 4$-5. We have pointed out positively that within the framework of such an approach, most informative is the analysis of the energy dependence of the Γ_n/Γ_f or $(\Gamma_n/\Gamma_f)_i$ values for each stage i of the neutron cascade, extracted from the experimental excitation functions of xn-reactions at minimum x values, $x \leq 2$.

In particular, to obtain information about the fission barrier heights in the region of transfermium elements this possibility so far seems to be the only one. For these elements, one should use experimental data on excitation functions of (HI, xn) reactions induced by ions with $A_I > 40$ /73-81/, such as Pb(^{40}Ar, 1-2n) Fm, Pb(^{48}Ca,1-2n)102 , Pb(^{50}Ti,1-2n)104 and others. Similar reactions leading to slightly excited nuclei can be materialized also in the region of the lighter nuclei including those with $Z \gtrsim 80$ and $N \lesssim 126$.

The βDF is a valuable tool for obtaining information about the fission probabilities $G_f(E) = \Gamma_f/\Gamma_{tot}$ at low excitation energies ($E \leq 10$ MeV), about the fission barrier amplitudes B_f^{max}, and, in principle, about the mass and energy distributions and other properties of fission from quite uncommon nuclei. The advantages of βDF can be used most fully in investigations of those nuclei which, in Z or N, are close to the magic ones and, at the same time, are considerably removed from the line of β-stability, and this just takes place in the region of $Z \gtrsim 80$ and $N \lesssim 126$ proposed by us for investigation. The methods of the analysis of experimental data on the βDF branching ratios, however, need a considerable development in many respects. In particular, it is necessary to determine the partial widths of the competing decay channels of the states populated via EC (β^+) transitions, such as the emission of β-delayed α-particles and/or protons. On the other hand, the measurement of the energy spectra of these particles could allow one to obtain information on the β-decay strength function direct for the nuclei undergoing βDF. The real structure of $S_\beta(E)$ should be taken into account in the extraction of $G_f(E)$ and B_f^{max} from the experimental βDF branching ratios, and therefore, further theoretical and, particularly, experimental investigations of the β-decay strength functions are of great interest. Then there will be a hope that by investigating βDF one can obtain information about fission barriers with an accuracy characteristic of experiments aimed at studying the fission probabilities in direct reactions.

We take pleasure in thanking Professor G.N.Flerov for his attention to the present work and for stimulating discussions, and Dr. Yu.V.Naumov for discussions of various problems concerning the β-decay strength function and its role in the analysis of β-delayed phenomena. We are also grateful to Dr. V.V.Pashkevich for numerous discussions of the theoretical aspects of the fission barrier problem. Thanks are also due to L.Pashkevich for her help in preparing an English version of this paper.

REFERENCES

1. Britt H.C. Physics and Chemistry of Fission. Proc. Symp. Jülich, 1979, vol.1, p.3, Vienna, IAEA, 1980; Nukleonika, 1980, 25, p.71.
2. Moretto L.G. et al. Phys.Lett., 1972, 38B, p.471.
3. Ignatyuk A.V. et al. Yad.Fiz., 1975, 21, p.1185; Yad.Fiz., 1977, 25, p.25.
4. Zhukova O.A. et al. Yad.Fiz., 1977, 26, p.473.
5. Istekov K.K. et al. Yad.Fiz., 1979, 29, p.1156.
6. Ignatyuk A.V. et al. Physics and Chemistry of Fission. Proc.Symp. Jülich, 1979, vol.1, p.421, Vienna, IAEA, 1980.
7. Türck D., Clerc H.-G., Träger H. Phys.Lett., 1976, 63B, p.283.
8. Drake T.E., Pai H.L., Nascimento I. Nucl.Phys., 1976, A259, p.317.
9. Kernohan A. et al. Phys.Rev., 1977, C16, p.239.
10. Specht H.J. Revs.Mod.Phys., 1974, 46, p.773; Phys.Scripta, 1974, 10A, p.21; Nukleonika, 1975, 20, p.717.
11. Kuznetsov V.I., Skobelev N.K., Flerov G.N. Yad.Fiz., 1966, 4, p.279; Yad.Fiz., 1967, 5, pp.271,1136.
12. Skobelev N.K. Yad.Fiz., 1972, 15, p.444.
13. Gangrsky Yu.P. et al. Yad.Fiz., 1978, 27, p.894.
14. Batist L.Kh. et al. Preprint of the B.P. Konstantinov Institute of Nuclear Physics in Leningrad, No.363, Leningrad, 1977.
15. Habs D. et al. Z.Phys., 1978, A285, p.53.
16. Kuznetsov V.I. Yad.Fiz., 1979, 30, p.321.
17. Gangrsky Yu.P. et al. Yad.Fiz., 1980, 31, p.306.
18. Strutinsky V.M. Yad.Fiz., 1966, 3, p.614.
19. Strutinsky V.M. Nucl.Phys., 1967, A95, p.420; Nucl.Phys., 1968, A122, p.1.

20. Myers W.D., Swiatecki W.J. Nucl.Phys., 1966, 81, p.1; Berkeley Report UCRL-11980, Berkeley, 1965.
21. Myers W.D., Swiatecki W.J. Ark.Fysik, 1967, 36, p.343.
22. Brack M. et al. Revs.Mod.Phys., 1972, 44, p.320.
23. Bohr N., Wheeler J.A. Phys.Rev., 1939, 56, p.426.
24. Frenkel Ya.I. Phys.Rev., 1939, 55, p.987; J.Phys. (USSR), 1939, 1, p.125.
25. Frankel S., Metropolis N. Phys.Rev., 1947, 72, p.914.
26. Swiatecki W.J. Phys.Rev., 1956, 101, p.651; Phys.Rev., 1956, 104, p.993.
27. Strutinsky V.M. ZhETF, 1962, 42, p.1571.
28. Strutinsky V.M., Lyaschenko N.Ya., Popov N.A. ZhETF, 1962, 43, p.584; Nucl.Phys., 1962, 46, p.639.
29. Cohen S., Swiatecki W.J. Ann.Phys.(N.Y.), 1962, 19, p.67; Ann.Phys.(N.Y.), 1963, 22, p.406.
30. Strutinsky V.M., Tyapin A.S. ZhETF, 1963, 45, p.960.
31. Strutinsky V.M. ZhETF, 1963, 45, p.1891.
32. Myers W.D., Swiatecki W.J. Ann.Phys. (N.Y.), 1969, 55,p.395; Ann.Phys.(N.Y.), 1974, 84, p.186.
33. Hasse R.W. Ann.Phys. (N.Y.), 1971, 68, p.377.
34. Myers W.D. Preprint LBL-3428, Berkeley, 1974.
35. Myers W.D. Droplet Model of Atomic Nuclei. New York, IFI/Plenum, 1977.
36. Krappe H.J., Nix J.R. Physics and Chemistry of Fission. Proc. Symp. Rochester, 1973,vol.1, p.159, Vienna, IAEA,1974.
37. Krappe H.J., Nix J.R., Sierk A.J. Phys.Rev., 1979, C20, p.992.
38. Nix J.R. Ann.Rev.Nucl.Sci., 1972, 22, p.65.
39. Möller P., Nix J.R. Physics and Chemistry of Fission. Proc. Symp. Rochester, 1973, vol.1, p.103, Vienna, IAEA, 1974.
40. Pauli H.C., Ledergerber T. Nucl.Phys., 1971, A175, p.545.
41. Schrøder B. Phys.Lett., 1980, 90B, p.339.
42. Adeyev G.D., Filipenko L.A., Cherdantsev P.A. Yad.Fiz., 1979, 30, p.933.
43. Möller P. Physics and Chemistry of Fission. Proc. Symp. Jülich, 1979, vol.1, p.283. Vienna, IAEA, 1980.
44. Howard W.M., Nix J.R. Physics and Chemistry of Fission. Proc. Symp. Rochester, 1973, vol.1, p.145. Vienna, IAEA, 1974.
45. Möller P. Invited Talk at the Int. Meeting on Reactions of Heavy Ions with Nuclei and on Synthesis of New Elements, Dubna, 13-16 December, 1977.
46. Muzychka Yu.A., Pashkevich V.V., Strutinsky V.M. Yad. Fiz., 1968, 8, p.716.

47. Nilsson S.G. et al. Nucl.Phys., 1969, A131, p.1.
48. Bolsterli M. et al. Phys.Rev.Lett., 1971, 27, p.681; Phys.Rev., 1972, C5, p.1050.
49. Pauli H.C. Phys.Rep., 1973, 7, p.35.
50. Fiset E.O., Nix J.R. Nucl.Phys., 1972, A193, p.647.
51. Randrup J. et al. Phys. Scripta, 1974, 10A, p.60.
52. Delagrange H. et al. Phys.Rev.Lett., 1977, 39, p.867.
53. Delagrange H., Fleury A., Alexander J.M. Phys.Rev., 1978, C17, p.1706.
54. Vandenbosch R., Huizenga J.R. Nuclear Fission. Academic Press, New York, 1973.
55. Plasil F., Blann M. Phys.Rev., 1975, C11, p.508.
56. Beckermann M., Blann M. Phys.Rev.Lett., 1977, 38, p.272; Phys.Rev., 1978, C17, p.1615.
57. Gilat J. et al. Phys.Rev., 1977, C16, p.694.
58. Schmidt K.-H. et al. Physics and Chemistry of Fission. Proc. Symp., Jülich, 1979, vol.1, p.409, Vienna, IAEA,1980.
59. Sikkeland T. Ark.Fys., 1967, 36, p.539.
60. Sikkeland T., Ghiorso A., Nurmia M.J. Phys.Rev., 1968, 172, p.1232.
61. Iljinov A.S., Cherepanov E.A. Preprint of the Institute of Nuclear Research of the USSR Academy of Sciences P-0090, Moscow, 1978.
62. Strutinsky V.M., Kolomiets V.M. Lectures "Nuclear Shell Structure and Fission", Proc. of the 8th Winter School of the Physics of the Nucleus and Elementary Particles. Part II, pp.483-594. Leningrad, the B.P.Konstantinov Institute of Nuclear Physics, 1973.
63. Strutinsky V.M. Yad.Fiz., 1974, 19, p.259.
64. Istekov K.K. et al. Pis'ma ZhETF, 1978, 27, p.135.
65. Hasse R.W., Stocker W. Phys.Lett., 1973, 44B, p.26.
66. Sauer G., Chandra H., Mosel U. Nucl.Phys., 1976, A264, p.221.
67. Iljinov A.S., Cherepanov E.A., Chigrinov S.E. Z.Phys., 1978, A287, p.37.
68. Pik-Pichak G.A. ZhETF, 1958, 34, p.341; ZhETF, 1962, 42, p.1294; ZhETF, 1962, 43, p.1701.
69. Cohen S., Plasil F., Swiatecki W.J. Ann.Phys. (N.Y.), 1974, 82, p.557.
70. Aberg S. et al. Physics and Chemistry of Fission. Proc. Symp. Jülich, 1979, vol.1, p.303, Vienna, IAEA, 1980.
71. Faber M.E., Faessler A., Ploszajczak M. Physics and Chemistry of Fission. Proc. Symp., Jülich, 1979, vol.1, p.361, Vienna, IAEA, 1980.
72. Bjørnholm S., Bohr A., Mottelson B.R. Physics and Chemistry of Fission. Proc. Symp.Rochester, 1973, vol.1, p.367, Vienna, IAEA, 1974.

73. Oganessian Yu.Ts. et al. Nucl.Phys., 1975, A239, p.353.
74. Oganessian Yu.Ts. et al. Nucl.Phys., 1975, A239, p.157.
75. Oganessian Yu.Ts. et al. Nucl.Phys., 1976, A273, p.505.
76. Flerov G.N. et al. Nucl.Phys., 1976, A267, p.359.
77. Oganessian Yu.Ts. et al. Pis'ma ZhETF, 1974, 20, p.580.
78. Gaeggeler H. et al. Z.Phys., 1979, A289, p.415.
79. Orlova O.A. et al. Yad.Fiz., 1979, 30, p.618.
80. Oganessian Yu.Ts. et al. Physics and Chemistry of Fission. Proc. Symp., Jülich, 1979, vol.1, p.129, Vienna, IAEA, 1980.
81. Münzenberg G. et al. Contribution to the Int. Symp. on the Synthesis and Properties of New Elements, Dubna, 23-27 September, 1980; D7-80-556, Dubna, JINR, 1980, p.27.
82. Barashenkov V.S. et al. Nucl.Phys., 1973, A206, p.131.
83. Goldstone P.D. et al. Phys.Rev.Lett., 1977, 38, p.1262.
84. Sun-Tsin-Yan G.Ya., Druin V.A., Trofimov A.S., Yad.Fiz., 1971, 14, p.1297.
85. Le Beyec Y., Lefort M., Sarda M. Nucl.Phys., 1972, A192, p.405.
86. Le Beyec Y. et al. Phys.Rev., 1976, C14, p.1038.
87. Iljinov A.S., Oganessian Yu.Ts., Cherepanov E.A. JINR, P4-80-356, Dubna, 1980.
88. Pai H.L., Andrews D.G. Can.J.Phys., 1976, 54, p.2056.
89. Epherre M. et al. Nucl.Phys., 1980, A340, p.1.
90. Viola V.E., Jr., Swant J.A., Graber J. Atomic Data and Nucl.Data Tables, 1974, 13, p.35.
91. Möller P. Nucl.Phys., 1975, A192, p.529.
92. Möller P., Nix J.R. Nucl.Phys., 1974, A229, p.269.
93. Valli K., Treytl W., Hyde E.K. Phys.Rev., 1968, 167, p.1094.
94. Cabot C. et al. Nucl.Phys., 1975, A241, p.341.
95. Le Beyec Y. Preprint IPNO-RC-76, Orsay, 1976.
96. Trikomi F.G. Integral Equations. New York, Interscience, 1957.
97. Tikhonov A.N., Arsenin V.Ya. Methods for Solution of Incorrectly Posed Problems (in Russian), 1st ed. "Nauka", Moscow, 1974.
98. Jonson B. et al. Proc. 3rd Int. Conf. on Nucl. Far from Stability, Cargese, Corsica, 1976, pp.277-298. CERN Report 76-13, Geneva, 1976.
99. Di Toro M. Lett.Nuovo Cim., 1978, 22, p.608.
100. Duke C.L. et al. Nucl.Phys., 1970, A151, p.609.
101. Shalev S., Rudstam G. Nucl.Phys., 1974, A230, p.153; Nucl.Phys., 1974, A235, p.397; Nucl.Phys., 1977, A275, p.76.

102. Takahashi K. Progr.Theor.Phys., 1972, 47, p.1500.
103. Kratz K.-L., Herrmann G. Z.Phys., 1973, 263, p.435.
104. The Shape of the Beta-Decay Strength Function and Consequences for Nuclear Physics and Astrophysics (Collection of Papers), Klapdor H.V., ed. Report MPI-H-V13, Max-Planck-Institut für Kernphysik, Heidelberg, 1979.
105. Klapdor H.V. et al. Phys.Lett., 1978, 78B, p.20.
106. Klapdor H.V. et al. Z.Phys., 1979, A292, p.249.
107. Naumov Yu.V. et al. JINR, D7-80-556, Dubna, 1980, p.55.
108. Möller P., Nilsson S.G., Nix J.R. Nucl.Phys., 1974, A229, p.292.
109. Möller P., Nix J.R. Preprint LA-UR-80-1996, Los Alamos, 1980 (to be published in Nucl.Phys.A).

Received by Publishing Department
on August 26, 1980.